

# Hydroinertia Gas Bearings for Micro Spinners

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## Abstract

Externally pressurized gas bearings with large bearing clearance are successfully used in ultra high speed micro spinners. For example, a micro spinner 4mm in diameter is stably operated more than 20krps and its whirl ratio exceeds 20. In such bearings, inertia effect of the gas flow in bearing clearance becomes predominant and its Mach number exceeds 1. As the results, gas pressure in the bearing clearance becomes negative. These bearings are called as hydroinertia gas bearings contrasted with the conventional hydrostatic gas bearings. Static characteristics of hydroinertia gas bearings are analyzed by considering the viscous effect of gas as wall friction, and the optimum design of hydroinertia gas bearings is showed. Experimental results on micro spinners, trial design of hydroinertia gas bearings for micro gas turbines and its experimental results are also discussed.

*Keywords: Gas bearing, Inertia effect, High-speed operation, Negative pressure, Micro spinner*

## 1 INTRODUCTION

Conventional externally pressurized hydrostatic gas bearings have so small clearance that the viscous effect of gas flow in a bearing clearance is predominant to inertia effect. But in case of large bearing clearance, inertia effect of the gas flow becomes predominant and its Mach number exceeds 1. As the results, gas pressure in the bearing clearance becomes negative<sup>1)</sup>. In this paper, these bearings are called as hydroinertia gas bearings contrasted with the conventional hydrostatic gas bearings.

Up to the present, the load capacity of the gas bearings is caused only by positive pressure, hydroinertia gas bearings have been thought to be useless. Recently, hydroinertia gas bearings are successfully used in ultra high-speed micro spinners. For example, a micro spinner 4mm in diameter supported by hydroinertia gas bearings is stably operated more than 20krps and its whirl ratio exceeds 20.

Static characteristics of hydroinertia gas bearings are analyzed by considering the viscous effect of gas as wall friction, and the optimum design of hydroinertia gas bearings is showed. Experimental results of hydroinertia gas bearings are well agreed with the calculated values. Dynamic characteristics of micro spinners and trial design of hydroinertia gas bearings for micro gas turbines are also discussed.

## 2 STATIC CHARACTERISTICS OF HYDROINERTIA GAS BEARINGS

### 2.1 Circular Thrust Bearings

Figure 1 shows a model of a hydroinertia circular thrust gas bearing. The bearing has a gas supply hole of inherent orifice

type at the center. Its bearing clearance  $h$  and the diameter  $d$  of the supply hole are relatively large compared with the bearing radius  $R_0$ . The inertia effect of gas flow in a bearing clearance is assumed to be predominant to viscous effect. The flow area of gas in the bearing clearance is the minimum at the cylindrical surface imagined at the exit of the supply hole and the flow is choked at this point. Gas supply pressure is  $p_s$ , ambient pressure is  $p_a$  and the coordinate  $x$  is taken along the bearing radius.

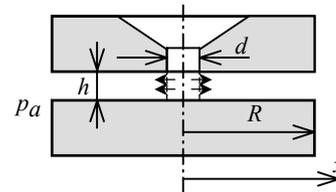


Figure 1. Hydroinertia circular thrust gas bearing

Supposing a compressible adiabatic flow with wall surface friction and applying its equations<sup>2)</sup> for the steady condition to the flow in the bearing clearance, distributions of Mach number and of pressure are given by following equations.

$$\frac{dM}{dx} = \frac{A}{1-M^2} \quad (1)$$

$$\frac{dp}{p} = \frac{\kappa M^2}{1-M^2} \frac{dA}{A} + \frac{\kappa M^2 [(\kappa-1)M^2 + 1]}{2(M^2-1)} \left( \frac{\lambda}{4m} \right) dx \quad (2)$$

where,  $A$  is

$$A \equiv M \left[ 1 + \frac{\kappa-1}{2} M^2 \right] \left[ \frac{\kappa M^2}{2} \left( \frac{\lambda}{4m} \right) - \frac{1}{A} \frac{dA}{dx} \right] \quad (3)$$

and  $\kappa$  is adiabatic index,  $A$  is flow area at Mach number  $M$ , and  $m$  is hydraulic mean depth and is given by  $m=h/2$ .

$\lambda$  is the coefficient of wall surface friction and is given by following equations depending on Mach number<sup>2)</sup>. For  $M < 1$ ,

the gas flow in the bearing clearance is thought as laminar,  

$$\lambda = 96\mu / 2h\rho u = 96 / Re \quad (4)$$

for  $M \geq 1$ , the flow is thought as turbulent,

$$Re = \sqrt{8/\lambda} e^{0.41(\sqrt{8/\lambda} - 2.4)} \quad (5)$$

where  $Re$  is Reynolds number defined by following equation using  $2h$  as representative length,  $u$  as flow speed and  $\nu$  as dynamic viscosity.

$$Re = 2hu/\nu \quad (6)$$

Mach numbers  $M_1$ ,  $M_2$  and pressures  $p_1$ ,  $p_2$  ahead of and behind the shock are given by following equations.

$$M_2^2 = \frac{(\kappa - 1)M_1^2 + 2}{2\kappa M_1^2 - (\kappa - 1)} \quad (7)$$

$$\frac{p_2}{p_1} = \frac{2\kappa M_1^2 - (\kappa - 1)}{\kappa + 1} \quad (8)$$

As the flow is choked at the cylindrical surface imagined at the exist of the supply hole, flow rate  $\dot{m}$  of the bearing gas is given by

$$\dot{m} = \frac{p_s A_c}{\sqrt{RT_s}} M \sqrt{\kappa} \left( 1 + \frac{\kappa - 1}{2} M^2 \right)^{\frac{\kappa + 1}{2(\kappa - 1)}} \quad (9)$$

where,  $p_s$  is the supply pressure,  $T_s$  is the gas temperature,  $c$  is the flow coefficient of inherent orifice and  $R$  is the gas constant.

Mach number at the exit of the bearing is given by

$$(\kappa - 1)M_e^4 + 2M_e^2 - \frac{2\dot{m}^2 RT_s}{\kappa A_e^2 p_a^2} = 0 \quad (10)$$

where,  $A_e$  is the flow area at the exit of the bearing and  $p_a$  is the ambient pressure.

Distribution of Mach number is obtained by solving equation (1), and then with equation (2) pressure distribution is obtained. Load capacity  $W$  is obtained by integrating the gauge pressure over the bearing surface. Force generated by the impact of gas flow from the supply hole against the bearing wall should be added to the load capacity. This force  $F$  is given by following equation<sup>3)</sup>.

$$F = \dot{m} M (\kappa RT)^{1/2} \quad (11)$$

The procedure of the calculation is as follows. As equation (1) diverges when  $M=1$ , then, at first, the Mach number at the bearing exit is calculated. With this Mach number, Mach number distribution is numerically calculated using equation (1). Supposing the breaking position of shock wave, backward Mach number  $M_2$  is calculated by equation (7). Then calculation of equation (1) is continued. This calculation is repeated until to find the breaking position of shock wave at which the Mach number at the exit of supply hole converges to 1. Once the distribution of Mach number is obtained, the pressure distribution in the bearing clearance is calculated by equation (2).

Figure 2 shows an example of pressure distribution of a hydroinertia circular thrust gas bearing. The outer radius of

the bearing  $R_o=5\text{mm}$ , diameter of the inherent orifice  $d=1\text{mm}$ , bearing clearance  $h=200\mu\text{m}$  and supply pressure  $p_s=400\text{kPa}$ . The pressure in the bearing clearance becomes partially negative.

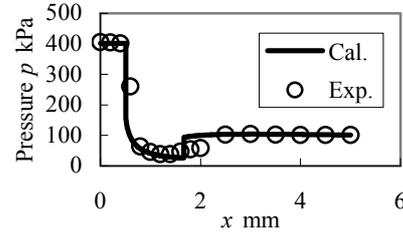


Figure 2. Pressure distribution of a hydroinertia circular thrust gas bearing

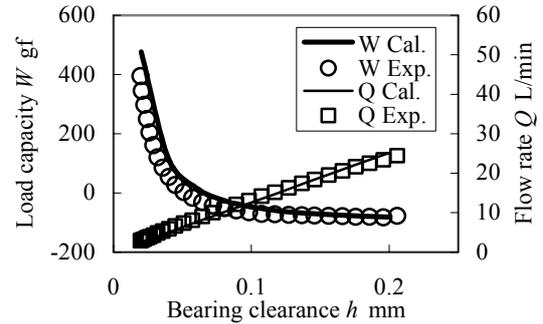


Figure 3. Load capacity and flow rate of a hydroinertia circular thrust gas bearing

Figure 3 shows the load capacity  $W$  and volumetric flow rate  $Q$  of the bearing. The load capacity decreases with increase in bearing clearance and it becomes negative, that means load capacity is generated by suction force, when bearing clearance exceeds about  $70\mu\text{m}$ . Over  $100\mu\text{m}$  of bearing clearance, load capacity becomes almost constant and bearing stiffness  $k$  becomes almost 0.

## 2.2 Hydroinertia Radial Bearings

A single admission radial bearing with inherent orifice shown in Figure 4 is chosen as an analytical model of hydroinertia radial bearings. Its radial clearance, the diameter of the supply holes and the number are relatively large compared with the bearing dimension.

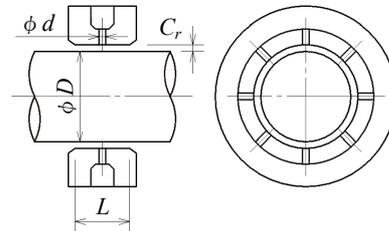


Figure 4. Single admission hydroinertia radial gas bearing

In case of radial bearings, it is difficult to determine the breaking position of shock wave to the flow from each supply hole by simple calculations. Then, radial bearings are replaced approximately with appropriate hypothetical circular thrust bearings of the same number as the supply holes shown in Figure 5.

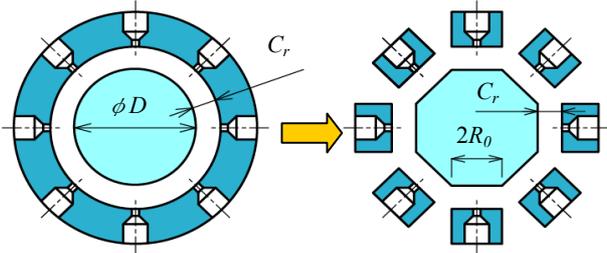


Figure 5. Approximation model for the radial bearing

Figure 6 shows the relation between a developed surface of the radial bearing covered by one supply hole and a hypothetical circular thrust bearing. In the radial bearing, gas from a supply hole flows radially to the radius  $x_i$  of which circumference equals to the outlet width of the radial bearing and then flows uniformly in constant cross sectional area. The radius  $R_0$  of the hypothetical circular thrust bearing is taken as the mean value of streamline length  $l_i$  supposed in the radial bearing and is calculated by the following equations.

$$R_0 = \frac{1}{n_k + 1} \sum_{i=1}^{n_k} l_i \quad (12)$$

$$l_i = \frac{L_b - \Delta L_a \cdot i}{\cos(\Delta\theta \cdot i)} \{1 - \sin(\Delta\theta \cdot i)\} + L_b \quad (13)$$

where,  $L_a = \pi D/2n$ ,  $L_b = L/2$ ,  $D$  is the bearing diameter,  $L$  is the bearing length,  $n$  is the number of the supply holes,  $n_k$  is the number of the supposed streamlines and  $\Delta\theta = \pi/n$ .

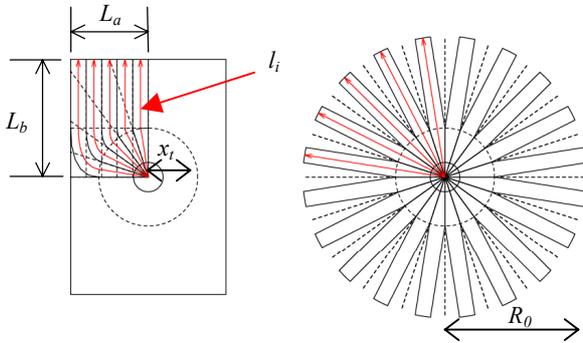


Figure 6. Developed surface of the radial bearing and a hypothetical circular thrust bearing

Radius  $x_i$  is corrected by expansion factor  $c_b$  because of the effect of flow inflection and is calculated by  $x_i = c_b \cdot D/n$ . The expansion factor  $c_b$  is taken as  $c_b = 0.8^4$ .

In eccentric conditions, the clearance  $h$  of each hypothetical circular thrust bearing is given by following equation.

$$h = C_r (1 - \varepsilon \cos \theta) \quad (14)$$

where,  $C_r$  is the radial clearance,  $\varepsilon$  is the eccentricity ratio and  $\theta$  is the angle between the normal to the surface of the hypothetical circular thrust bearings and the direction of eccentricity.

Figure 7 shows an example of load capacity and flow rate of a hydroinertia radial gas bearing. The bearing diameter

$D=4\text{mm}$ , length  $L=2.4\text{mm}$ , diameter  $d$  of the supply holes  $d=0.3\text{mm}$  and number  $n$  of the supply holes  $n=0.8$  and supply pressure  $p_s=150\text{kPa(G)}$ . Load capacity becomes maximum  $C_r=28\mu\text{m}$  for  $\varepsilon=0.3$  and  $C_r=35\mu\text{m}$  for  $\varepsilon=0.5$ .

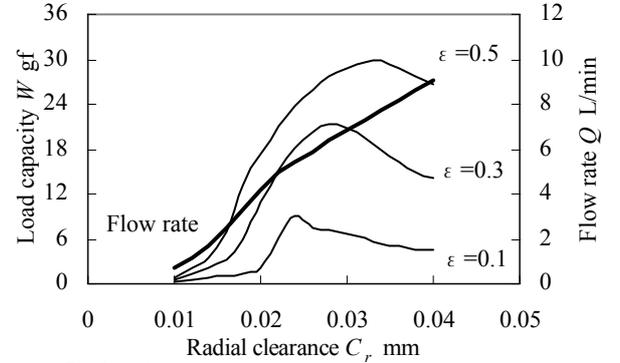


Figure 7. Load capacity and flow rate of a hydroinertia radial gas bearing

### 3 ROTATIONAL CHARACTERISTICS OF HYDROINERTIA GAS BEARINGS

Hydroinertia gas bearings are applied to a micro spinner. Figure 8 shows the schematic configuration of the trial micro spinner test rig. The diameter of the spinner (① of Figure 8) is 4mm and is driven by an impulse turbine (②) of the same diameter fitted at the end of the spinner. Two radial bearings (③) and a thrust bearing (④) set at the end surface of the spinner opposite to the turbine support the spinner. Rotational speed is measured by an optical sensor (⑤) and vibration by an eddy current type displacement meter (⑥). The radial bearings are the same type as shown in Figure 4 and their dimensions are almost the same as shown in Figure 7. The spinner is made of stainless steel (Japanese standard SUS420) and the radial bearings are made of ceramics (zirconium).

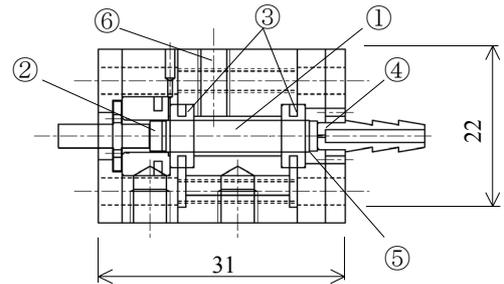


Figure 8. Test rig for the micro spinner

Figure 9 shows the 3-D display of the shaft vibration spectrum. The horizontal axis shows the frequency from 0 to 50kHz and the diagonal axis shows rotational speed from 10krps to 20krps. Two vibrations are observed, one is synchronous to the rotational speed (shown by (A) in the Figure 9) and the other is at the low frequency of about 1kHz (shown by (B)). The latter is the vibration by whirl motion and its frequency corresponds to the first resonance speed of this bearing-spinner system.

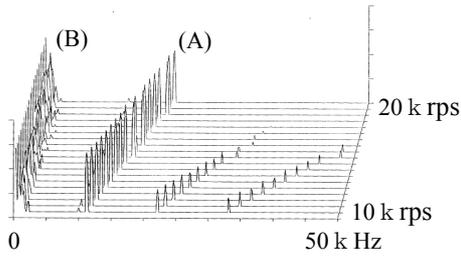


Figure 9. Vibration spectrum of the spinner

Figure 10 shows the waveforms of spinner at the rotational speed of 10krps and 20krps. In addition to the synchronous vibration, whirl vibrations can be apparently seen. At the rotational speed of 20krps, amplitude of the vibration fluctuates slightly but it does not diverge. The sensibility of the displacement meter is  $5.18\text{mV}/\mu\text{m}$ , the maximum total amplitude of the vibration corresponds  $2.94\mu\text{m}$ . As the radial clearance of the bearing is  $31\mu\text{m}$  (mean of the measured values), there are enough margins to the crash.

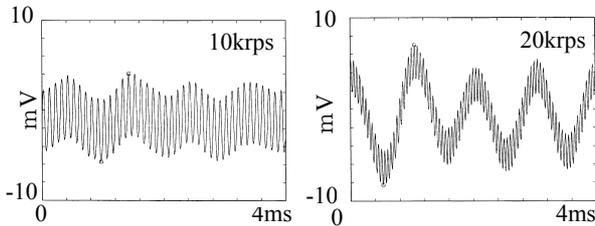


Figure 10. Wave forms of the spinner vibration

Figure 11 shows the vibration spectrum near the resonance frequency. Resonance frequency  $N_1$  for the cylindrical mode is given by

$$N_1 = (k/m_w)^{1/2} / 2\pi \quad (15)$$

where,  $k$  is the stiffness of the bearing and  $m_w$  is half of the spinner mass.

From the calculation results of load capacity, for  $\varepsilon < 0.1$ ,  $k$  is given as  $k=22.57\text{kN/m}$  at the supply pressure  $p_s=150\text{kPa(G)}$  and  $k=34.75\text{kN}/\mu\text{m}$  at  $p_s=300\text{kPa(G)}$ . Measured spinner mass is  $2m_w=1.071\text{g}$ , and calculated resonance frequency is  $N_1=1033\text{Hz}$  for  $p_s=150\text{kPa(G)}$  and  $N_1=1281\text{Hz}$  for  $p_s=300\text{kPa(G)}$ . These values well agree with the measured resonance frequency shown in Figure 11.

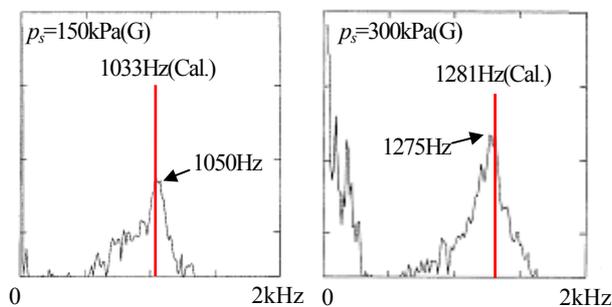


Figure 11. Vibration spectrum near the resonance frequency

Because of its high speed stability, hydroinertia gas bearings are applied to a micro gas turbine.

Figure 12 shows a bearing test rig for the micro gas turbine.

A compressor impeller is replaced by a 10mm diameter dummy wheel with equivalent moment. The diameter of the turbine is 10mm and the shaft diameter is 4mm. Two hydroinertia radial gas bearings support the shaft and a double action thrust ring hydroinertia gas bearing of 10mm diameter is set at the center of the shaft and control the axial position. For facility of assembly and to keep the dynamic balance of the shaft, the bearings are cut into two half cylindrical shapes.

Figure 13 shows the photo of the shaft and the bearings.

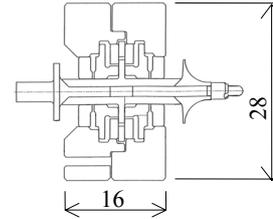


Figure 12. Bearing test rig for the micro gas turbine



Figure 13. Photo of the shaft and the bearings

#### 4 CONCLUSION

In this paper, externally pressurized gas bearings with relatively large bearing clearance and with big supply holes compared with conventional hydrostatic gas bearings are called as hydroinertia gas bearings.

Static characteristics of hydroinertia circular thrust gas bearings and radial gas bearings are analyzed. The optimum design of the gas bearings for a micro spinner is showed. The micro spinner 4mm in diameter supported by hydroinertia gas bearings are stably operated more than 20krps and its whirl ratio exceeded 20. Hydroinertia gas bearings are applied to a micro gas turbine and a bearing test rig for the micro gas turbine is made for trial.

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#### References

- [1] R. Comolet, Publications Scientifiques et Techniques du Ministère de L'air, No.334, 1957.
- [2] K. Matsuo, Compressible Fluid Dynamics, Rikogakusha, Co. Ltd., 1994 (in Japanese)
- [3] S. Kida and S. Yanagase, Dynamics of Turbulence, Asakura Shoten Co. Ltd., 1999 (in Japanese)
- [4] H. Mori, A. Mori and H. Doi, Trans JSME, 36, 283, 1970, 494 (in Japanese)