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Experimental friction factor study for microchannel flows

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Abstract

Friction factor calculations are presented for three different microchannels machined inside a silicon substrate and compared with previous work. Experimental results are obtained for the pressure drops across micro channels with uniform rectangular cross section. The silicon channels dimensions are 3000 x 360, 1500 x 180, 750 x 90 with depth 360, all units in micrometer. Deviations of the experimental values are seen for Reynolds numbers in the range of 100 – 1000. Rarefaction of the air in the micro channel flow is verified.

Key words: Friction factor, Rarefaction effect, Wave Rotors, Micro scale effects

1 Introduction

Over the past two decades, micro fluidic systems have been studied and developed using the available micromachining technologies. Experimental and theoretical studies have been made to establish the flow behavior and fluid characteristics in microchannels. Micro fabrication techniques have been used to design and manufacture more complex structures like gas turbines and other turbo components [1, 2] for generating power at the micro level, Power MEMS. Also studies in wave rotor have shown considerable increased interest over the past few years. A wave rotor is an unsteady flow device that utilizes shock waves occurring across the flow to obtain a pressure rise [3, 4]. Using the same principle, and independent, torque generating wave engine devices have been conceptualized [5].

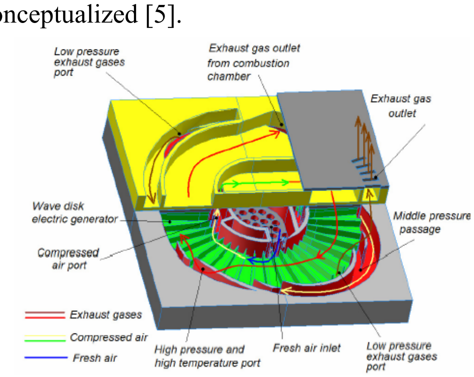

Fig1 Details of the one step compression wave engine construction. [5]

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Friction factor studies are made to simulate and verify the flow behavior occurring in silicon microchannels as they occur in such MEMS devices. Some of the first studies on friction factor were performed by Wu and Little [6] where Darcy friction factors were calculated for microchannels etched in silicon. The observed friction factors were larger than the predicted by the classical macro scale theory. The findings here verify these results as the experimental values are greater by around 22% than the theoretical values for hydraulic diameters 114µm, 240µm and 360µm. Figure 2 shows the microchannels etched inside the Si substrate and topped with glass.

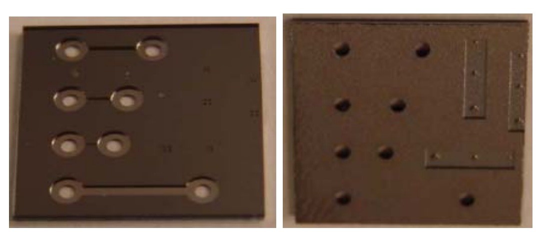

Fig2 Silicon microchannels: Left shows single die – front view, Right shows single die – back view, [7]

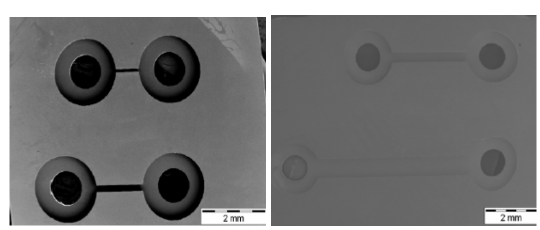

Fig3 Channels visualized with SEM: a) 90µm and 180 µm wide channels; b) 360 µm and 720 µm wide channels,[7]

Standard properties of air were taken at one end of the channel. The temperature is assumed to be constant throughout the channel (isothermal flow). This assumption has been made considering the high coefficient of thermal conductivity of silicon, 148W/mK. Although, a recent study shows no significant effect of heat transfer on the compression efficiency in silicon microchannels [7].

2. Theory

The two non dimensional flow parameters are often used to characterize the fluid flow are the Reynolds number *Re* and the Darcy friction factor *f*. The viscous effects of the flow are characterized by *Re*.

$$\mathrm{Re} = \frac{\rho V_{avg} D_h}{\mu} \qquad (1)$$

where V_{avg} is the average velocity of the fluid, D_h is the hydraulic diameter of the channel, ρ is the density of fluid and μ is the dynamic fluid viscosity that was assumed to be constant throughout at 25ºC.

The non dimensional hydraulic diameter is four times the ratio between the cross sectional area, *A* of the channel and the wetted parameter, *P*.

$$D_h = \frac{4 \times A}{P} \qquad (2)$$

The Darcy friction factor relates roughness effects to pressure drops in ducts and channels. Its defined as

$$f = \frac{8\tau_w}{\rho V_{avg}^{\ 2}} \qquad (3)$$

where τ_w is the shear stress at the walls. Using this relation and combining it with the energy and the momentum equation for the fully developed flow, the Darcy Weisbach equation is obtained, which holds for both laminar and turbulent flow.

$$h_f = f \frac{L V_{avg}^{\ 2}}{D_h 2g} \qquad (4)$$

The head loss (pressure loss) is denoted by h_f, with *L* being the channel length and *g*, acceleration due to gravity. Solving the continuity and momentum equation, the solution for the laminar flow in the Hagen-Poisseuille flow problem is found and is commonly known as the Darcy friction factor [8] as shown in Equation 5.

$$f = \frac{2 D_h \Delta p}{\rho L V_{avg}^{\ 2}} \qquad (5)$$

with Δp is the pressure difference within the channel. Analytically the laminar friction constants can be found out using the aspect ratio, α, which is the smaller of the ratios width/height and height/width so that $0 \leq \alpha \leq 1$.

The friction coefficient *C* can be represented as a function of α by the polynomial correlation

$$C = 96(1 - 1.35\alpha + 1.95\alpha^2 - 1.70\alpha^3 + 0.96\alpha^4 - 0.25\alpha^5)$$

where

$$C = f \times \mathrm{Re} \qquad (6)$$

The classical theory of laminar flow relates the friction coefficient to be proportional to the Reynolds number (Eq. 1). This relation is

$$f = \frac{64}{\mathrm{Re}} \qquad (7)$$

The experimental results show the variation from this constant value

3. Experimental Results

Ambient conditions were used for the channel entry. Pressure drop across the channel was determined using the experimental set up shown in Fig. 4. The flow exit velocity was computed by measuring the volume flow rate of the water.

Estimates of pressure drops can be made using the traditional theory and correlations. However, accurate prediction of pressure drops in microchannels for non-slip flows is not yet possible [9].

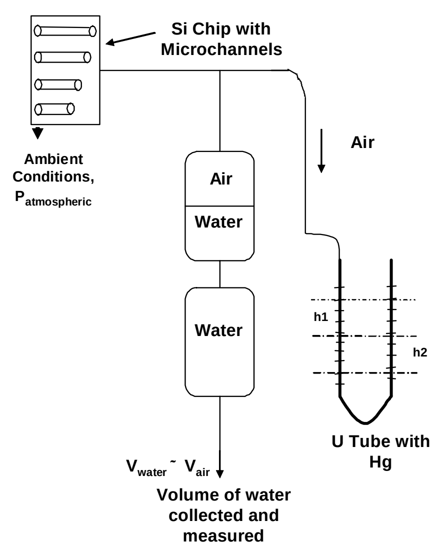

Fig 4 Schematic of the experimental setup

The results for Friction coefficients C are more conveniently shown using the normalized friction coefficient

$$C^* = \frac{f\,\mathrm{Re}_{experimental}}{f\,\mathrm{Re}_{theorectical}} \qquad (8)$$

Entry and exit losses were calculated based on the standard expression for minor losses in flow. Since the port will have a dimension larger than that of the channel, the entry losses are treated as losses due to variation of diameter in a pipe. The pressure drop is

$$\Delta p = k\rho \frac{V^2}{2} \qquad (9)$$

Where the entry loss coefficient, k is considered to be 0.5 for a contraction with sharp 90º corners [11]. The losses were computed as head losses at entry and exit and the sharp bends using the standard minor flow losses relationships [12] as shown by the equation below.

$$\Delta h_{tot} = h_f + \Sigma h_m = \frac{V^2}{2g}\left(\frac{fL}{d} + \Sigma K\right) \qquad (10)$$

The following table gives the experimental results for the pressure drops, friction factors, normalized friction coefficients and Reynolds number for all three channel sizes.

m(kg/s) E-6	ΔP(Pa)	f	C*	Re
1.147	3466	0.309	1.326	274
2.057	6933	0.179	1.375	491
2.403	8532	0.155	1.399	574
2.403	8532	0.155	1.399	574
2.859	12932	0.151	1.618	683
3.010	14665	0.149	1.674	719
3.307	17732	0.138	1.714	790
3.640	21065	0.125	1.703	871

Table 1: Experimental data for the smallest microchannel (750µm x 90µm x 360µm)

m (kg/s) E -6	ΔP(Pa)	f	C*	Re
4.753	5466	0.090	1.344	946
6.369	10398	0.086	1.716	1268
6.409	8532	0.073	1.458	1276
6.608	14132	0.100	2.067	1316
7.519	14532	0.079	1.851	1497

Table 2: Experimental data for (1500µm x 180µm x 360µm) microchannel

m (kg/s) E-6	ΔP(Pa)	f	C*	Re
9.636	7199	0.064	1.445	1439
8.299	5599	0.091	1.777	1239
6.994	4266	0.136	2.21	1037
5.863	3066	0.208	2.85	876

Table 3: Experimental data for (3000µm x 360µm x 360µm) microchannel

The plot in Fig. 5 shows the variation of the normalized friction factor, with different Reynolds Numbers. It is always greater than 1.

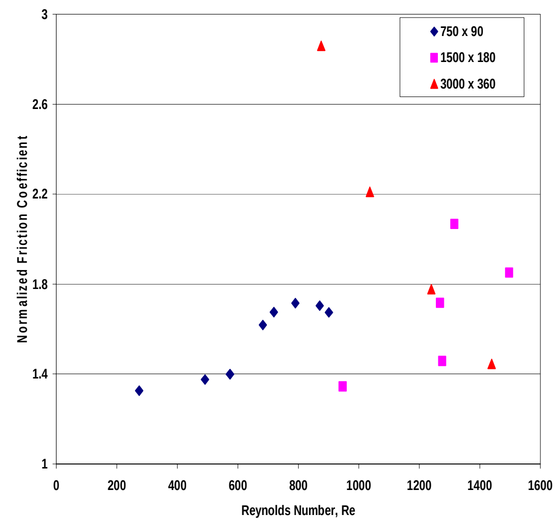

Fig5 Plot showing Reynolds number versus the Normalized friction coefficient for three different micro channel dimensions

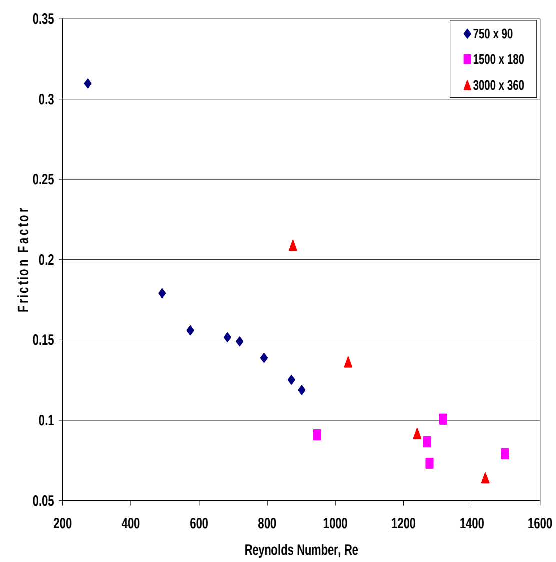

Fig6 Plot showing Reynolds number versus the experimentally calculated friction factor for three different micro channel dimensions

As shown in above figures, the normalized coefficients show a marked deviation form the laminar flow values. Another important factor verified with the experiment is the significant rarefaction effect occurring during flow over even short distance in a micro channel. A considerable change in density is observed due to the dominance of viscous boundary layers in the gaseous flow in micro channels. This reduction in density due to viscous dissipation is termed as the rarefaction effect. Figure 7 verifies the viscous dissipation and rarefaction studies of previous studies [13].

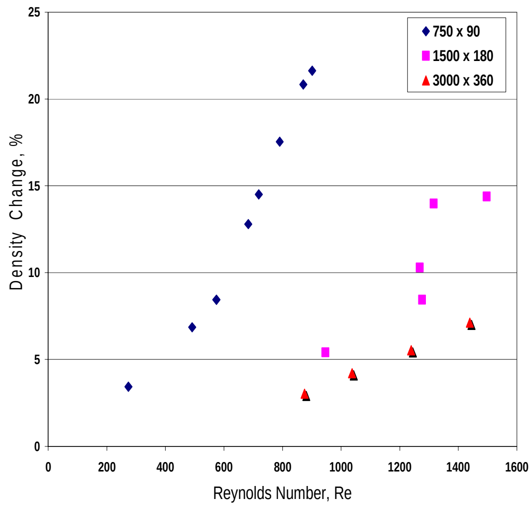

Fig7 Plot showing the percentage of density change for different channel lengths and for different Re

4. Conclusions

The study verifies experimental findings of previous investigations [8, 9]. A considerable variations of the laminar friction coefficients for a given pressure difference compared to the macro scale theoretical predictions is recorded. The trends do not show a uniform trend for the three investigated channel configurations. Effects of varying aspect ratio and developing laminar flow may be dominant and need further investigation to clearly identify and quantify these effects. The investigated flow regimes all fall into the developing laminar flow regimes with channel length of 10-20% of the entry length. However, this is typical for many microchannel investigations and applications. The higher sheer forces in the developing boundary layers cause greater pressure losses than in the case of fully developed laminar flow. This is seen as a major contribution to the normalized friction coefficient (C*>1).

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