

OPTIMIZATION OF MAGNETIC VIBRATIONAL ENERGY HARVESTING SYSTEMS USING EQUIVALENT CIRCUIT REPRESENTATIONS

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Abstract: This paper investigates an enhanced equivalent electrical circuit model for magnetic vibrational energy harvesting systems. The model incorporates nonlinear relationships to more accurately model the dynamic behavior of the system, namely a nonlinear spring and position-dependent electromagnetic coupling factor. Implementation of the model and optimization of several parameters is discussed and implemented using PSPICE simulation. These results demonstrate the flexibility and adaptability of equivalent circuit modeling methods for design and optimization of vibrational energy harvesting systems, even in the presence of nonlinear mechanical and electromechanical elements.

Key Words: vibrational energy harvesting, equivalent circuit modeling, electromagnetic transduction, design optimization

1. INTRODUCTION

Energy scavenging from the environment has advanced rapidly during past few years [1-3], and vibrational energy has been found to be a promising energy source that is widely available. Magnetically-based vibrational energy harvesting systems, with their potential for superior energy density, have been widely studied.

Equivalent circuit modeling offers an effective tool for interpretation of the underlying physics of a multi-energy domain energy harvesting system. Moreover, once an equivalent circuit model has been developed, there are a host of software simulation tools (e.g. PSPICE) that can be utilized to study the system dynamics in conjunction with any associated power electronic circuits. A linear equivalent circuit model approach was presented in [4]. This original model assumes the resistance, capacitance, or inductance of each of the mechanical circuit elements is constant. However, the physical system may exhibit substantial nonlinearities, e.g. a nonlinear spring behavior or position-dependent electromagnetic coupling.

Consequently, in this paper, an enhanced equivalent circuit modeling method is introduced. This method models the nonlinearity of the spring and position-dependent electromagnetic coupling using lookup tables, which are feasible in PSPICE using the Analog Behavior Model (ABM).

Furthermore, optimization procedures based on this model are implemented in PSPICE. Parametric sweeps are used to study the system performance and overcome the difficulty in analytical optimization of system with nonlinear components.

2. ENHANCED EQUIVALENT CIRCUIT MODEL

2.1 Linear Equivalent Circuit Model

A schematic of a typical magnetic vibrational energy harvester is shown in Fig.1. A proof mass of mass m is connected to a frame via a spring of stiffness k (assuming linear spring in this section). A dashpot of damping factor b is used to represent of the mechanical damping. The moving magnet (mass) and the coil attached to the frame form the electromagnetic coupling. Here the coupling is considered constant with a factor of K .

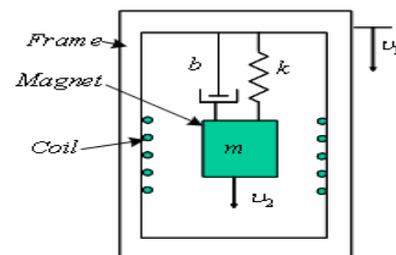


Fig. 1: Schematic of a typical magnetic vibrational energy harvester.

The linear equivalent circuit model from [4] is shown in Fig. 2. Lumped element methods are used to determine each equivalent circuit parameter. For example, an inductor, a capacitor and a resistor are used to represent the mass, the spring and the dashpot, respectively, of the mechanical system. The electromagnetic coupling is modeled using an electrical gyrator.

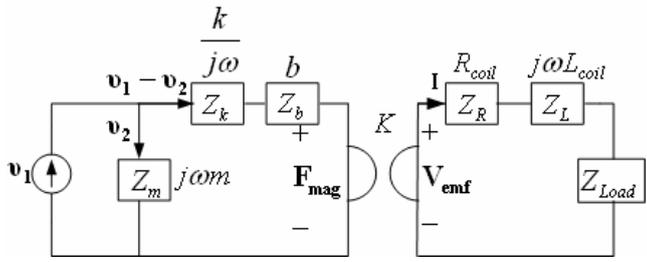


Fig. 2: Linear equivalent circuit model of the magnetic vibrational energy harvester [4].

2.2 Enhanced Nonlinear Equivalent Circuit Model

Without losing the advantages such as good physical insight and ease of simulation, the enhanced circuit model enables the electrical representations of nonlinear spring and position-dependent electromagnetic coupling.

The relationship between deformation, X , and force, F , of a linear spring is:

$$X = \frac{1}{k} F \quad (1)$$

where k is the spring constant. Thus,

$$v = \frac{1}{k} \frac{dF}{dt} \quad (2)$$

where v is the time rate of change in deformation.

Therefore, for nonlinear spring, whose spring constant is a function of deformation (or force, if $F(X)$ is a monotonic function),

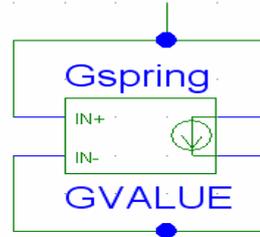
$$v = \frac{1}{k(F)} \frac{dF}{dt} \quad (3)$$

This relationship enables the implementation in PSPICE of the nonlinear spring, represented as a nonlinear capacitor. Recall that in the equivalent circuit representation, the dual of velocity is current, the dual of force is voltage, and the dual of $1/k$ is capacitance. As such, the capacitor is realized by a controlled current source G_{spring} , whose current is defined by

$$I = C(V) \frac{dV}{dt} \quad (4)$$

The time derivative $\frac{dV}{dt}$ is modeled by using

DDT() function in PSPICE, and the voltage dependent capacitance can be implemented by a lookup table [5]. An example of a nonlinear capacitor (representing the spring) is implemented in PSPICE as shown in Fig. 3.



TABLE(V(%IN+, %IN-), 0v, 1e-6, 20v, 1.24e-6) * DDT(V(%IN+, %IN-))
Fig. 3: Example of nonlinear capacitor implemented in PSPICE.

In a similar way, a position-dependent electromagnetic coupling can be modeled by using two current-controlled voltage sources, whose coupling factors are defined by a lookup table. In the lookup table, the coupling factor is a function of the displacement of the mass. In the model, this displacement is determined by the charge on the capacitor representing the spring, in this case the nonlinear capacitor G_{spring} .

The final circuit model of the system is shown in Fig. 4. In the circuit, G_{spring} is the nonlinear spring, E_{couple1} and E_{couple2} are the nonlinear electromagnetic couplings, and R_{s1} and R_{s2} are sampling resistors used to detect the current, they are small enough to avoid influence on the overall circuit.

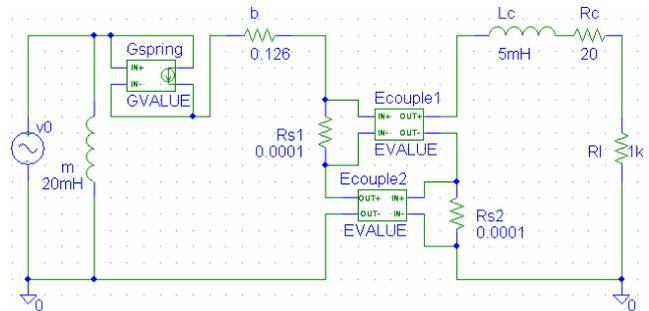


Fig.4: The circuit representation of the nonlinear system model with a 1 kΩ resistive load.

3. OPTIMIZATION

In this section, an optimization procedure for a prototype nonlinear energy harvester is investigated. An example nonlinear spring with spring constant vs. deformation characteristic exhibiting a spring stiffening behavior is shown in Fig. 5. The spring constant is based on an equation of a typical Duffing spring,

$$F = aX + bX^3 \quad (6)$$

where the values of a and b are 2,500 N/m and 5,000,000 N/m³, respectively. Additionally, a position-dependent electromagnetic coupling is assumed, with the coupling factor vs. displacement shown in Fig. 6. The coupling factor is based on actual measurements of the force between a magnet and current-carrying coil, where the relative position is changed for each measurement.

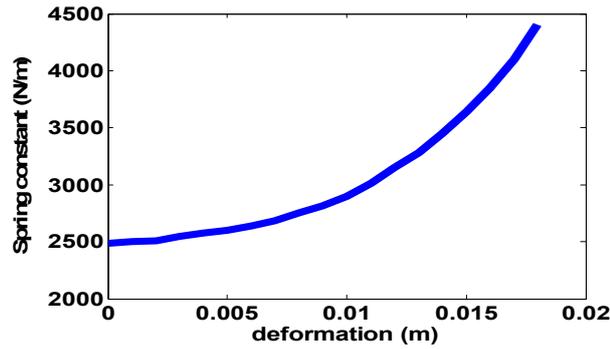


Fig.5: The characteristic of the nonlinear spring.

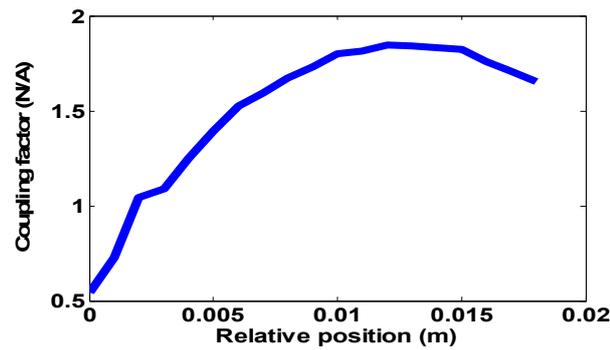


Fig.6: The characteristic of the coupling factor.

3.1 Resistive Load Matching

It has been shown that the optimal simple resistive load, R_{load_opt} , for maximum power is [4]

$$R_{load_opt} = \sqrt{\left(\frac{K^2}{b} + R_{coil}\right)^2 + (L_{coil}\omega)^2}, \quad (5)$$

where K is the electromagnetic coupling factor, b is the damping factor, R_{coil} is the coil resistance, L_{coil} is the coil inductance, and ω is the vibration frequency. However, for the nonlinear model, where the electromagnetic coupling factor is not a constant, the optimum load resistance cannot be expressed explicitly.

Assuming an input acceleration of 1 g at 60 Hz, the optimum load resistance for the nonlinear model is determined using a parametric sweep in PSPICE. Fig. 7 shows the power transferred to the load for various load resistances. As can be seen, the model predicts an optimum resistive load at $\sim 22 \Omega$. In contrast, the linear model predicts 28 Ω , but if used with the nonlinear system, the power output would be 10% less than optimum.

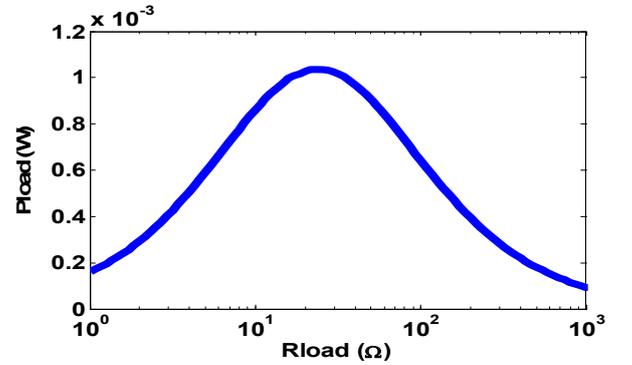


Fig.7: Resistive load matching.

3.2 Mass Optimization

With the conventional linear model, there is little room for mass optimization, since the ratio of the mass and spring constant are fixed by the desired natural frequency. However, if nonlinearity is considered in the model, parameter selection is not that straightforward, because the natural frequency of a nonlinear spring mass system is not well defined. Therefore, mass becomes a parameter that needs optimization.

Assuming a 1 g, 60 Hz excitation vibration, a parameter sweeping procedure was performed to find the optimum mass. For each mass, an optimum load resistance was selected, and corresponding average power was calculated. The mass vs. power curve is shown in Fig. 8. The optimum mass is predicted to be about 21 g. In contrast, if a linear spring is assumed, the optimum mass would be 17 g, in order to match the natural frequency. If the 17 g mass were used with the nonlinear system, the power output

would be 80% less that using the optimum mass. This illustrates significant differences in the design procedure when considering nonlinear mechanical and electromechanical elements.

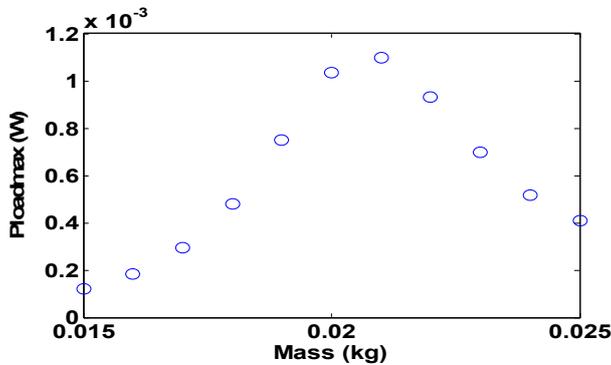


Fig. 8: Average power consumption on optimum resistive loads using different masses.

3.3 Coil Position Optimization

Another parameter that can be optimized is the initial relative position of the coil and the magnet, which determines the effective electromagnetic coupling factor. This can be done by simply adding an offset variable in the independent variable of the lookup table. By sweeping the offset variable, and finding the maximum average load power, an optimum initial coil position can be found, as shown in Fig. 9.

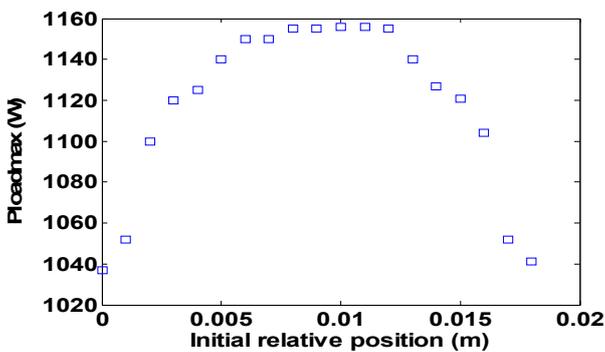


Fig. 9: Average power consumption on optimum resistive loads with different initial coil positions.

4. CONCLUSION

An enhanced equivalent circuit model is developed and implemented in PSPICE. In the enhanced model, nonlinearity in the spring and position-dependence in the electromagnetic coupling are appropriately modeled using nonlinear circuit elements. Based on the enhanced

model, optimization procedures for load resistance, mass, and coil position are explored. These results are specific to one particular device, but the methods described here can be generally applied to other similar devices. The ability to capture the nonlinearities in the model results in better optimization and design methods. Moreover, understanding the impact of nonlinearities can help to utilize them for improved system performance. For example, optimal spring nonlinearities can be found via simulation. Then, the desired nonlinear force vs. displacement characteristics may be implemented in the physical system, e.g. using interacting permanent magnets to add nonlinearity to a mechanical spring.

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