AN ANALYTIC TREATMENT OF THE TRANSDUCTION IN A SHUTTLE TYPE VIBRATION ENERGY HARVESTER

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Abstract: In this paper we present an idealized model of a shuttle type energy harvester. If we assume that the shuttle moves at a constant velocity, the system is described by a simple, linear differential equation. The solution gives us an analytic expression of the generated output waveform as a function of time. We use this waveform to calculate the output power and the mechanic force on the shuttle. This force varies in value and direction over one period, but the fluctuation is significantly reduced in a device with two outputs of opposite phase.

Key Words: Energy harvester, analytic model, fluctuating electric reaction force.

1. INTRODUCTION

Shuttle type energy harvesters have attracted much attention the past. Different designs and measurement results from fabricated devices have been presented. [1]-[4]. We use a simple idealized model and are able to find an analytic expression for the time-function of the generated voltage and electric force on the shuttle. To our knowledge this has not been done before.

2. MODEL

The present model is not a complete model of a shuttle harvester. It does not account for the mechanical dynamics in the seismic mass – suspension system and it assumes a constant shuttle velocity. But the simple, analytic solutions it gives us, makes it possible to study in detail relations in the electrostatic transduction. These properties will also be relevant and important for the design of a complete energy harvester system.

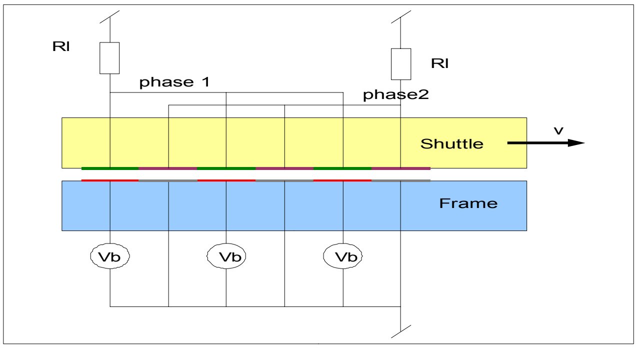

Fig. 1: Electrode structure.

Figure 1 shows the two-phase electrode arrangement on the frame and shuttle surfaces. The frame has two sets of fingers at V_b and ref potential respectively. The shuttle has also two sets of n fingers, phase 1 and phase 2. Each phase is connected to ref through a load resistor, R_l. During operation fingers belonging to one phase have its capacitance to one set of frame electrodes linearly changed from 0 to C_{max} and to the other set changed from C_{max} to 0. At the same time fingers belonging to the other phase have their capacitance change in the inverse manner. As the shuttle moves at a velocity v across the frame, a periodic voltage, $V_o(t)$, is generated across the load resistors. In the model v is a parameter. The assumption of a constant shuttle velocity may be justified if the shuttle traverses several finger electrodes during each stroke. In the frequency domain, we can say that the typical excitation frequency must be much smaller than the frequency of the output waveform.

The capacitance between the fingers is based on a plate capacitor model with variable overlap. Fringing fields and parasitic capacitance between the working electrodes, phase 1 and phase 2, as well as between the working electrodes and ref, is not included. Clever design can minimize these effects, but they will always be present and degrade the transduction efficiency in a real harvester. The justification for not including them in our model is simply that this would remove the linearity of the electric model and thereby the simplicity of the solution for the time function of the generated waveform. Our results will not be quantitatively accurate, but we believe that they will exhibit a qualitatively correct dependence on the design parameters included in the model. The strength of our approach is that we can analyse a wide range of situations simply by varying the value of the parameters. In the present analysis we have used the parameter values listed in table 1.

shuttle length	shuttle width	finger pitch	gap	bias	shuttle vel.	load res
6mm	6mm	30μm	5μm	100V	0-1m/s	0-1MΩ

Table 1: Parameter values used in our analysis.

An electric equivalent circuit for each phase of the idealized structure is shown in figure 2.

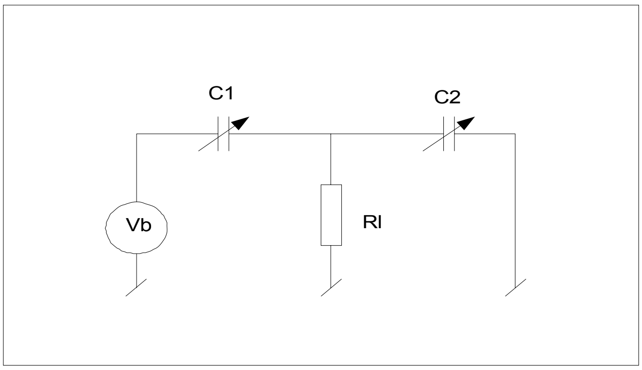

Fig. 2: Electric equivalent circuit.

The diagram includes two variable capacitors. Normally this leads to a nonlinear differential equation describing the circuit. In our case the symmetry of both the circuit and the variation of the capacitors has the effect of cancelling out the nonlinear terms, and we end up with the following differential equation for the generated waveform for the rising half period:

$$\tfrac{d}{dt}V_o + \frac{V_o}{\tau} = \frac{V_b}{T_e} \qquad (1)$$

V_o is the output waveform, V_b is the bias voltage, $\tau = C_{max} \cdot n \cdot R_l$, the electric time constant while $T_e = p/v$, p being the pitch of the fingers, is half the period of the output waveform. A similar equation describes the falling half period. This simple equation can be solved by a number of methods. The generated voltage is given by:

$$V_o(t) = V_{ini} e^{-\frac{t}{\tau}} - V_b \tfrac{t}{\tau}(1 - e^{-\frac{t}{\tau}}) \qquad (2)$$

The initial value, $V_{ini,}$ is found if we require that the output has no DC component. This is obvious, since R_l is fed through capacitors.

The first-order response, $V_o(t)$, calculated at the listed set of parameters, is shown in figure 3. The two curves are the rising and falling half-period of the waveform.

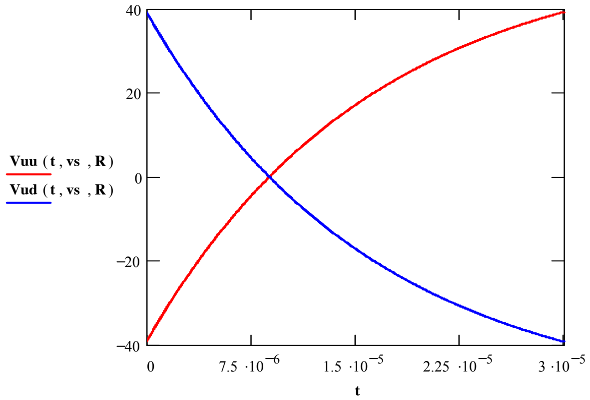

Fig.3: Output waveform of the two half periods at the listed set of parameters, v=1m/s and R_l=0.5MΩ.

From the analytic expression we can easily calculate the RMS power, P, delivered by the harvester into the Ohmic load, R_l, connected to each phase. Figure 4 shows a plot of P vs R_l and shuttle velocity, v.

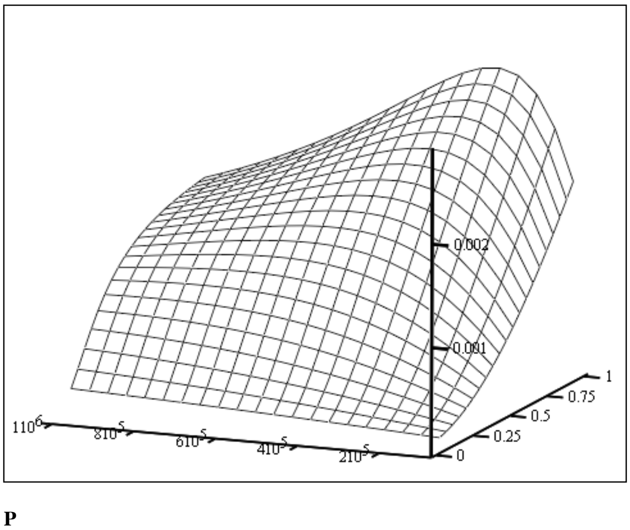

Fig 4: Power vs load resistance, 0-10^6Ω, and shuttle velocity, 0-1m/s.

As we would expect the highest power is harvested in a limited region in the v-R_l plane. At a given v there exists a range of R_l that gives the best match. This region becomes narrower as the shuttle velocity increases. With our parameter set and v=0.3m/s, the maximum power is 1mW.

From the power expression we can also find the average electrostatic force on the shuttle:

$$F_{el} = \frac{P}{v} \tag{3}$$

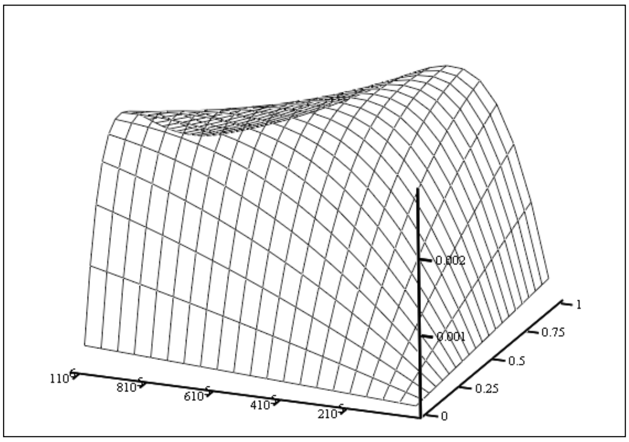

Fig5: Average electrostatic force vs load resistance and shuttle velocity.

Figure 5 shows a plot of F_{el} vs R_l and shuttle velocity, v. We observe that at a given R_l the average F_{el} is roughly proportional to v at low v-values. This average force is not so important, since we must expect the time function to fluctuate strongly as the fingers move. One of the benefits of our model is its ability to give us an analytic expression for $F_{el}(t)$.

At any time the power supplied from the external force and the bias supply must be equal to the sum of generated power and the rate of increase of the energy stored in the two capacitors, C1 and C2, in figure 2. For each finger belonging to a phase with increasing output voltage we have:

$$v \cdot F_{el1}(t) + V_b \tfrac{d}{dt} Q_1 = \frac{V_o^2}{n \cdot R_l} + \tfrac{d}{dt}\left[\frac{C1(t)(V_b - V_o)^2 + C2(t)V_o^2}{2}\right] \tag{4}$$

We express Q_1 by $C1\ (V_b$-$V_u)$ and rearrange:

$$v \cdot F_{el1}(t) = \frac{V_o^2}{n \cdot R_l} + \tfrac{d}{dt}\left[\frac{C1(t)(V_b - V_o)^2 + C2(t)V_o^2 - 2C1(t)(V_b - V_o)}{2}\right] \tag{5}$$

For all fingers in one phase this simplifies to:

$$F_{el}(t) = \frac{n}{v} \cdot \left[\frac{V_o^2}{n \cdot R_l} + \tfrac{d}{dt}\left[\frac{C_{max}V_o^2 - C1(t)V_b^2}{2}\right]\right] \tag{6}$$

Since C_{max} and V_b are constant, we get:

$$F_{el}(t) = \frac{n \cdot C_{max}}{v} \cdot \left[\frac{V_o^2}{\tau} + V_o \tfrac{d}{dt} V_o - \frac{V_b^2}{2T_e}\right] \tag{7}$$

When we analyze fingers belonging to a phase with decreasing output voltage, the result is identical to (7), except the last term has positive sign. The sum of both phases is:

$$F_{el}(t) = \frac{2n \cdot C_{max}}{v} \cdot \left[\frac{V_o^2}{\tau} + V_o \tfrac{d}{dt} V_o\right] \tag{8}$$

If we substitute the derivative term from the differential equation, (1), we get the end result:

$$F_{el,tot}(t) = \frac{2nC_{max}V_b}{vT_e} V_o(t) = \frac{\varepsilon_o nwV_b}{gap} V_o(t) \tag{9}$$

Figure 6 shows plots of $F_{el,tot}(t)$ at tree values of v, 0.1, 0.5 and 1m/s. The time range is 0-T_e in all tree plots. The labeling is only correct for v=1m/s.

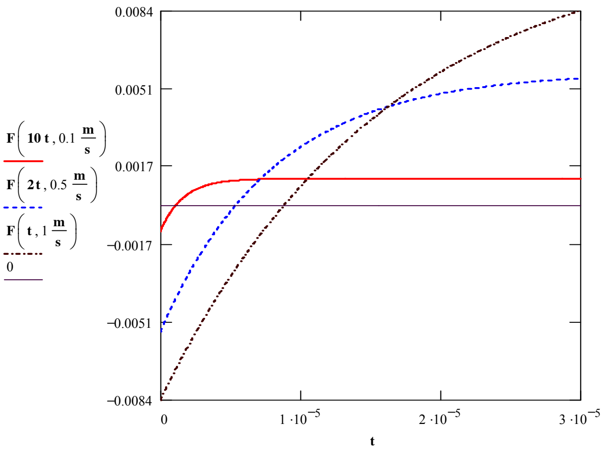

Fig.6: Plots of electrostatic force on shuttle at tree shuttle speed values.

From (9) we see that the electrostatic transduction force is proportional to the generated voltage. This simple relation was not expected, but we can verify it by calculating the average force from (9) and compare the result with the average force based on power, as shown in figure 5.

At v=0.1m/s and R_l=$6\cdot10^5\Omega$ we get P=120.6μW and $F_{el,av}$=1.206mN. Averaging (9) gives the exact same result:

$$F_{el,av} = \frac{\varepsilon_o nwV_b}{gap}\frac{1}{T_e}\int_0^{T_e} V_o(t)dt = 1.206mN \qquad (10)$$

3. DISCUSSION

Analysis of a shuttle harvester by the use of a realistic, numeric model is a demanding task. First the electrostatic picture has to be numerically solved for a fine mesh of relative positions between shuttle and frame for all geometries to be analyzed. Second the generated voltage and power has to be calculated by a dynamic simulation model at a range of excitation and load conditions. This may be quite time consuming and it need not give an immediate understanding of the effect of parameter variations. It will, on the other hand, give quantitatively correct results, provided that the model and the numeric approach are correct.

The present model is the opposite of this. Results for any new set of parameters are instantly available, but since we have used simplifications and only modeled one part of the harvester, the results will only be qualitative. Since the model consists of expressions for voltage, power and force, it gives a good understanding of the way parameter changes will affect the end results.

In the figures and examples we have used parameter values, listed in table 1, that are typical to known designs. We also need a range of v that will represent a given mass spring system and excitation situation, A shuttle with a vibration amplitude of 0.5mm at 200Hz has a RMS velocity around 0.45m/s. The frequency of the generated signal would be 30 times that of the vibration. This example seems very suitable for our model.

In addition to generated voltage and power the model also determines the time function of the electrostatic force on the shuttle. This gives us a good picture of the difference between a two phase and a single phase system. Equation (7) gives the force on the fingers in one phase. While the two first terms are the same in both half periods, increasing and decreasing output voltage, the third term changes sign. With two opposite phases this term will cancel out. In a single phase system, it will give rise to a square ripple waveform superimposed on the waveforms in figure 6. With a v=0.1 m/s, this ripple wave, which has zero average, has an amplitude that is 4.5 times the average, $F_{el,av}$.

4. CONCLUSIONS

We have developed a simple model of the electrostatic transduction in a shuttle type energy harvester. The model assumes a constant shuttle velocity. It also neglects parasitic effects that must be present in a real harvester. The model gives simple analytic solutions and may be used for rough assessment of power from a given design. It may also be used to study in some detail how parameter variations affect the transduction efficiency of a shuttle harvester. The model can be used to find an analytic expression for the time function of the electrostatic force on the shuttle. It turns out that shuttles with a single phase electrode system will have a square wave ripple force with zero average added. The magnitude of the ripple may be several times the average force. This ripple is cancelled out in two phase electrode arrangements.

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