LINEAR AND NONLINEAR POWER PROCESSING OF VIBRATION BASED ENERGY SCAVENGERS

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Abstract: In this contribution, the power output of a vibration-driven energy scavenger is optimized when connected to a resistive, linear load and a nonlinear load, an AC-DC converter. The optimization of the power output is constrained by the limited generator size. The optimal loading is calculated for both load cases in various operating conditions, and the optimal generated power is compared. It is shown that the difference in power output for both load cases is very small. This conclusion is important for the design of the power management circuit, required to transform the scavenger output voltages to a suitable form.

1. INTRODUCTION

The current advances in performance and functionality of micro- and nano-systems have stimulated the development of intelligent networks of autonomous systems. The demand for a small, mobile and reliable energy supply for each autonomous network node has led to the development of a new type of generators, as the use of conventional electrochemical batteries is not always an option because of their limited lifespan, the need for replacement, and the volume-dependency of the amount of stored energy. Motion energy or vibrations are an attractive source for powering miniature energy harvesting generators [1].

In this paper, the optimal power output of a vibration-based energy scavenger is calculated for a resistive, linear load, and for a nonlinear load, an AC-DC converter. The vibration-driven generator is considered as being of the inertial type: a proof mass m is suspended within a frame, and energy is extracted by a transducer that damps the motion of the mass. A schematic overview of the generator is shown in Figure 1: the mass m is modeled as being suspended by a spring with spring constant k, while its motion is damped by a parasitic damping d due to friction and air. The mass is also damped by the generator, exerting a reaction force F_g. The displacement of the mass $z(t)$ is restricted to z_{MAX}, the size of the device. The amplitude of the motion of the package $y(t)$ is Y_0. An electric circuit equivalent of the same generator is shown in Figure 2, this circuit equivalent is valid for

electrostatic, electromagnetic and piezo-electric devices [2][3]. In this electric circuit, the voltages represent forces, and charges represent displacements. The electrical model is further used to analyze the behavior of the scavenger with different loads.
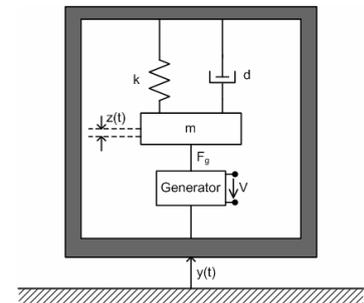

Fig. 1: General model of an inertial vibration-driven energy scavenger.
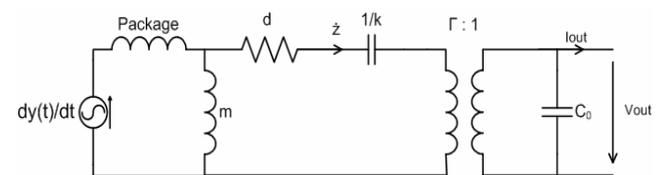

Fig. 2: Electrical circuit equivalent of an inertial vibration-driven energy scavenger.

2. LINEAR LOAD

In literature, a lot of work has been done to study the behavior of a scavenger with a linear (resistive) load (see a.o.[2][4]). The output power of the scavenger with resistive load can be calculated analytically using the electrical model of Fig. 2. It can be shown that an optimal resistive

load R_{opt} exists for every operating frequency [2][3], given by eq. (1). For simplicity reasons the parasitic damping d is neglected. In this equation, $\omega_1 = \sqrt{k/m}$ refers to the mechanical resonance frequency. Due to the coupling of the mass with the generator, a second resonance frequency is introduced: $\omega_{1*} = \omega_1 \sqrt{\Gamma^2 + C_0 k}/\sqrt{C_0 k}$. The coupling between the electrical and the mechanical part of the system is represented by $\alpha = (C_0 k)/(C_0 k + \Gamma^2)$.

$$R_{opt} = \frac{1}{\omega C_0} \frac{\alpha - (\frac{\omega}{\omega_{1*}})^2}{1 - (\frac{\omega}{\omega_{1*}})^2} \qquad (1)$$

However, using the optimal resistive load value can violate constraints imposed by the system, since the displacement of the mass should be limited to the dimensions of the package. If using the optimal load means that the displacement limit z_{MAX} is exceeded, a larger resistance should be chosen so that the motion amplitude is reduced just below the imposed limit. This resistive load R_{const} needed to fulfill the displacement constraint is calculated as follows. The laplace-form of the differential equation describing the scavenger behavior with a resistive load is given by:

$$ms^2(\mathcal{Y}(s) - \mathcal{Z}(s)) = k\mathcal{Z}(s) + \frac{\Gamma^2 R_{load} s \mathcal{Z}(s)}{1 + s C_0 R_{load}} \qquad (2)$$

The magnitude of the displacement of the mass, relative to the magnitude of the displacement of the package is then:

$$\frac{Z_0}{Y_0} = \frac{-\omega^2 \sqrt{1 + \omega^2 C_0^2 R_{load}^2}}{\sqrt{(\omega_1^2 - \omega^2)^2 + \omega^2 C_0^2 R_{load}^2 (\omega_{1*}^2 - \omega^2)^2}} \qquad (3)$$

R_{const} can be calculated from the rearrangement of eq. (3):

$$R_{const} = \sqrt{\frac{(\frac{z_{max}}{Y_0})^2 (\omega_1^2 - \omega^2)^2 - \omega^4}{-(\frac{z_{max}}{Y_0})^2 \omega^2 C_0^2 (\omega_{1*}^2 - \omega^2)^2 + \omega^6 C_0^2}} \qquad (4)$$

Γ	4.14e-3 N/V
C_0	170 nF
k	7.11e3 Nm^{-1}
m	57.1 g

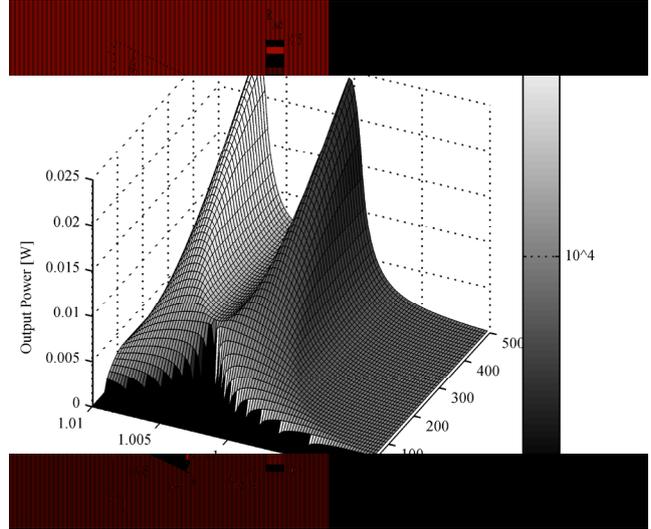*Table 1 : Model Parameters*

Fig. 3: Power output versus operating frequency and relative displacement limit with resistive load.
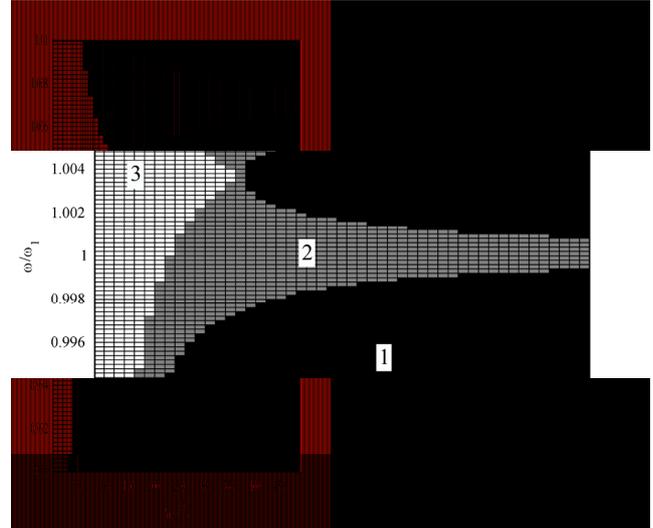

Fig. 4: Operating Region of Energy Harvester versus operating frequency and relative displacement limit with resistive load.

Figure 3 shows a surface plot of the optimal power output versus the operating frequency and the relative displacement limit. The generator parameters are taken from [5], and are given in Table 1. The height of the plot indicates the power consumed by the load resistor, the shading shows the optimal load resistance used in every point. If the necessary damping cannot be achieved by

using higher resistive values, because of the inherent system characteristics, the power output is set to be zero. Both the resonant frequencies ω and ω_{1*} are clearly visible. Three operating regions can be distinguished, shown in Fig. 4:

1) The system works optimally, the load resistor is the optimal one, given by eq.(1).
2) The device works in the 'constrained'-condition. The system would generate more power if the displacement amplitude could be increased beyond z_{MAX}. Equation (4) is applied.
3) It is not possible to damp the mass with an adequate load resistance, in order to prevent collisions of the mass against the package. The power output in this condition is set to zero.

3. NONLINEAR LOAD

Practical applications of vibration-based energy scavengers seldom show resistive behavior, a power processing circuit is therefore needed to convert the AC output voltage of the scavenger into a DC voltage of an appropriate level to supply the load. The behavior of the scavenger with a basic AC-DC converter, a diode rectifier with a fixed voltage at the output, as load is analyzed, using the scavenger model of Fig. 2, see Fig. 5.
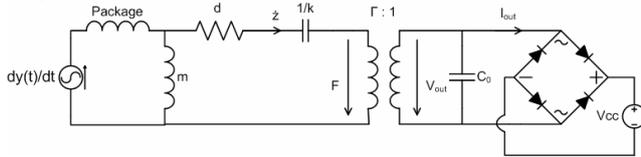

Figure 5: Electrical model of a general energy scavenger with non-linear load.

It can be shown that an optimal output voltage exists for every operating frequency, in [3] is shown that the optimal output voltage V_{cc}, and the corresponding output power P_{out} can be calculated from equations (5),(6) and (7).

$$-mY_0\omega^2\cos(\varphi) = -mZ_0\omega^2 + kZ_0 + V_{cc}\Gamma \qquad (5)$$

$$mY_0\omega^2 Z_0\sin(\varphi)\frac{\pi}{2} = 2V_{cc}(\Gamma Z_0 - V_{cc}C_0) \qquad (6)$$

$$P_{out} = 2V_{cc}\frac{\omega}{\pi}(\Gamma Z_0 - V_{cc}C_0) \qquad (7)$$

For reasons of simplicity, the parasitic damping d is neglected in eq. (5)-(7), and φ represents the phase-difference between the displacement of the package and the displacement of the mass.

Analogous to the linear-load case: using the optimal output voltage can end in a displacement constraint violation. If the displacement amplitude is higher than the imposed z_{max}, an output voltage is numerically chosen, giving the needed damping and the highest output power. Figure 5 shows a surface plot of the optimal power output versus the operating frequency and the relative displacement limit. The height of the plot again indicates the power output, the shading shows the fixed output voltage used in every point. Again, both the resonance frequencies are clearly visible.
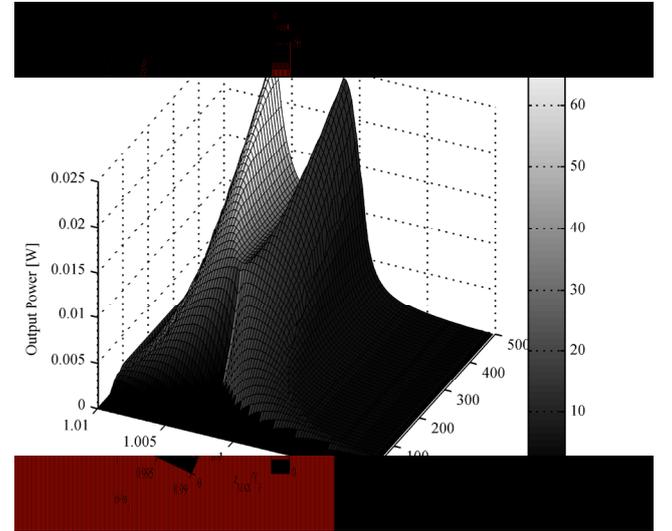

Fig. 6: Power output versus operating frequency and relative displacement limit with nonlinear load.

Three operating regions can be distinguished, shown in Fig. 6:

1) The system works optimally, the output voltage is the optimal one, calculated from eq. (5)-(7).
2) The device works in the 'constraint'-condition. The system would generate more power if the displacement amplitude could be increased beyond z_{max}.
3) It is not possible to damp the mass in these conditions, in order to prevent collisions of the mass against the package. The power output in this condition is set to zero.
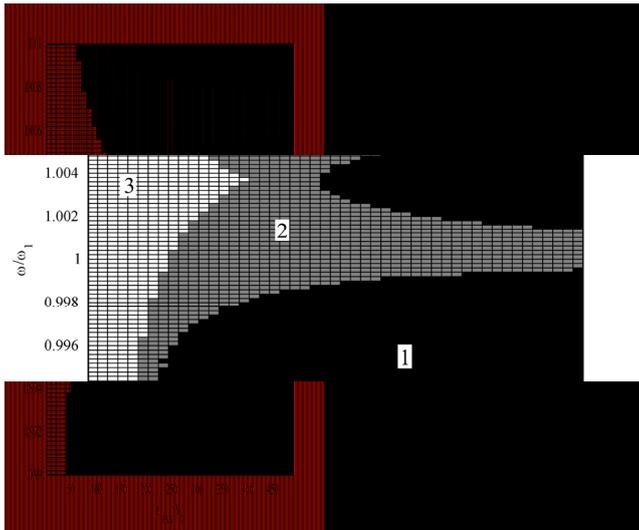
Figure 7 : Operating Region of Energy Harvester versus operating frequency and relative displacement limit with nonlinear load.

4. CONCLUSION

From Fig. 3 and Fig. 6 can be seen that the difference in output power at both resonance frequencies is rather small between both load cases, and it can be concluded that the load of a scavenger does not have to be resistive for the scavenger to behave optimal. This conclusion might have a big influence on the design of the power management circuit, required to transform the scavenger output voltages to an appropriate form for powering the electronic load. Power management circuits that have resistive input impedance are not straightforward to design for very low power ratings, and their efficiency is relatively low, especially at very high input vibration frequencies [6]. The fact that the input impedance of the power conversion circuit not necessarily needs to be resistive, but can be a non-linear impedance, for the scavenger to deliver relatively high power, might improve the overall effectiveness of the total system of scavenger and power management circuit.

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