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SIZE EFFECTS ON STIRLING CYCLE MICRO ENGINE

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Abstract: Prior to a micro Stirling engine design, size effects on global performances are investigated by examining each individual loss associated with the heat transfers, regenerator, pressure drop and combining their outcomes. The ideal performances of a Stirling engine taking into account the dead volume consequences are governed by the classical Schmidt analysis. The generic Newton's law governs the thermal transfers through the heat exchangers and thermal losses. Hence, a global analytical model is obtained. From the GPU-3 engine features, the results from scaling process put emphasis notably on the growing impact of conduction loss as the size decreases.

Key words: Stirling engine, size effect, scaling process

1. INTRODUCTION

The theoretical interest of the Stirling cycle for micro engines is based firstly on its thermal efficiency. It appears to be suitable for miniaturization with the addition of the profitable use of thermal regeneration and its simple technological achievement. However, one must study further the well known theoretical results to aim at more realistic values consistent with practical machines.

In the many parameters to be taken into account, dead volumes as well as non-ideal regeneration have the highest influence on the performances. Stirling machine with dead volume can be studied using the Schmidt analysis approach as far as isothermal evolutions and ideal regeneration are assumed [1]. Following a second order approach according to Martini [2] classification, the Schmidt results can be completed by an energetic balance including a non ideal regeneration this time. In addition to the previous limitations two types of effects in relation with thermal transfers are taken into account. It is shown that they induce great thermal losses eventually. The first one is related to heat transfers to the working fluid at the hot and cold ends of the machine. Consequently, a thermal gradient appears in the engine chambers and in its internal components such as the regenerator or the displacer as well. Thus the second effect is a direct thermal energy flow which exists between the hot and cold ends without any conversion in usable mechanical work.

Mechanical losses related to pressure drops, joints and seals are finally assessed trough the Senft's central theorem on the efficiency of reciprocating heat engines [3]. Figure 1 shows a generic diagram of the Stirling engine model which underlines thermal and mechanical effects described previously.

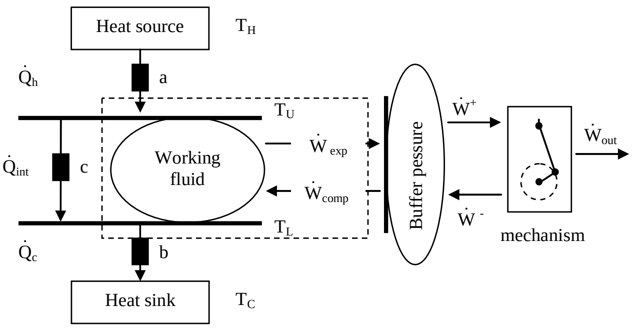


Fig. 1: Generic Stirling machine scheme

In this study, an analytical model has been developed. It mixes the losses and handles them simultaneously. The model developed has been tested using the data from the General Motor GPU-3 widely studied in the literature. Geometrical as well as operating parameters are summarized in Table 1.

Table 1: GPU-3 main parameters

Source temperature T_H	700°C	Sink temperature T_C	15 °C
Piston-Displacer phase angle α	120°	Piston-Displacer swept vol. ratio κ	1.01
Displacer swept volume V_d	120.88 cm^3	Working fluid	Helium
Pressure p	69 bars	Operating frequency f	50 Hz
Heat transfer ratio ρ = c/a	1	Heat transfer ratio δ = b/a	1
mechanical efficiency η_{mec}	0.9	Swept volume V_{sw}	

Firstly, analytical Schmidt results are recalled.

With the addition of the thermal flaws, energetic characteristics of a Stirling engine are set out. Then, the heat exchangers and the regenerator efficiencies dependant of fluid flow rate are added in order to find optimal operating point.

On the assumption that the swept volume V_{sw} is representative of the Stirling engine global size, a scale analysis is performed. The scaling procedure is the one described by Organ in [4]. However, the technological break between power MEMS and more conventional power machine can not be assessed by a linear scaling of components. Consequently, the effect of the miniaturization on the engine behavior is finally underlined studying the consequences of heat transfer coefficient especially.

The model can be used to design micro Stirling engines including their dead volumes, imperfect regenerator, thermal interfaces such as exchangers and internal thermal losses. It provides the theoretical size limits of Stirling engines.

2. THEORY

2.1 Schmidt analysis

Performances of a Stirling machine can be analytically established by the classical Schmidt analysis [1]. It applies to the conventional schematic representation of a thermodynamic machine (dashed line frame in figure 1) and relies on the following assumptions:

- Working fluid within the engine chambers stays at constant temperatures.
- The temperature of the working fluid within heat exchangers volumes V_h and V_k is T_U and T_L respectively.
- Temperature within the regenerator can be described by a linear evolution between T_U and T_L.
- The regenerator behavior is symmetrical.
- Ideal gas law is adopted.
- Pressure is the same throughout the machine for each considered position of both the piston and the displacer.

Consequently, analytic expressions of the performances are:

- Heat added for one cycle

$$Qe = p_{mean} \frac{\pi V_d}{\beta} \left(1 - \frac{1}{\sqrt{1-\beta^2}}\right) \sin\theta \qquad (1)$$

- Heat rejected for one cycle

$$Qc = - p_{mean} \frac{\kappa V_d}{\beta} \left(1 - \frac{1}{\sqrt{1-\beta^2}}\right) \sin(\alpha-\theta) \qquad (2)$$

- Efficiency

$$\eta_i = 1 - \frac{T_L}{T_U} \qquad (3)$$

- Indicated power

$$P_{thermo} = \frac{f\, p_{mean} V_d \pi \kappa (\tau-1) \sin\alpha}{\beta \sqrt{\kappa^2+\tau^2+2\kappa\tau\cos\alpha}} \left(1-\sqrt{1-\beta^2}\right) \qquad (4)$$

With:

$$p_{mean} = \frac{M r}{s} \frac{1}{\sqrt{1-\beta^2}} \qquad (5)$$

$$\beta = \frac{\sqrt{2\tau\kappa\cos\alpha + \tau^2 + \kappa^2}}{2\nu + \kappa + \tau} \qquad (6)$$

$$\tan\theta = \frac{\kappa \sin\alpha}{\kappa\cos\alpha + \tau} \qquad (7)$$

$$\tau = T_L / T_U \qquad (8)$$

$$V_{sw} = V_d \sqrt{2\kappa\cos\alpha + 1 + \kappa^2} \qquad (9)$$

Although the predictions of the Schmidt calculations are optimistic (practical engines show no more than 30 to 60 % of the predicted power and efficiency), this approach is an efficient tool at a first stage design.

2.2 Thermal approach

Cycle average power can be given in an alternative way. According to a power balance of the machine described in figure 1, the cycle average power is $P_i = a\,(T_H-T_U) – b\,(T_L-T_C)$, which can be put in the form:

$$P_i = a\, T_H (1 + \delta\Gamma - \xi – \xi\delta\tau) \qquad (10)$$

Where $\xi = \frac{T_U}{T_H}$ is the temperature ratio between the heat source and the highest temperature of the engine.

The thermal efficiency of the engine reveals the ratio of the available power by the added power taking into account the thermal losses:

$$\eta_{th} = \frac{P_i}{\dot{Q}_h + \dot{Q}_T + \dot{Q}_R} \qquad (11)$$

$\dot{Q}_h$ is the heat power delivered by the source, $\dot{Q}_T$ is the internal thermal loss through the regenerator and the solid parts of the machine whereas $\dot{Q}_R$ stands for the reheat loss due to regenerator inefficiency. Hence, it is easy to express the thermal efficiency as a function of the adimensionnal variables:

$$\eta_{th} = \frac{1+\delta\gamma - \xi - \xi\tau\delta}{1 - \xi - (\rho + (1-e)\rho_R)\,\xi\,(1-\tau)} \qquad (12)$$

Where $\rho_R = \frac{\dot{m}_R C_v}{a}$

2.3 Heat transfer coefficient calculation

Thermal performances of the exchangers partly rely on fluid flow rate through them and are eventually dependant on the machine parameters (operating frequency and piston-displacer phase). a and b heat exchange coefficients are deduced from the simple convection model relations:

$$a = h_h A_{wh} C_p \quad (13)$$
$$b = h_k A_{wk} C_p \quad (14)$$

Where h_h and h_k relate to the convective heat transfer, A_{wh} and A_{wk} are the wetted areas and C_p is the constant pressure specific heat capacity of the working fluid.

A simplified model relates for a given type and geometry of an exchanger, the thermal parameter h of heat exchangers to fluid flow characteristics through the Reynolds number Re and the Colburn J-factor J_h. Consequently, the thermal parameters can be calculated from the mass flow rates in the machine estimated by the Schmidt analysis.

Extra heat called reheat loss and denoted $\dot{Q}_R$ above occurs when a non-ideal regenerator can not provide an outlet fluid temperature equals to the adjacent chamber temperature. It follows that this heat must be provided by the source which decreases the efficiency of the engine. The regenerator effectiveness e is related to the general adimensional number NTU:

$$NTU = \frac{h A_w G}{C_p \dot{m}} = \frac{h A_w}{A_{ff} C_p} \quad (15)$$

Once, again, the thermal parameters can be calculated from the mass flow rates in the machine.

2.4 Scaling analysis

The scaling process is initiated by using dimensionless parameters for the geometrical factors. Swept volume is chosen as a representative or reference parameter for volumes. Hence, $A_{ref} = V_{sw}^{2/3}$ and $L_{ref} = V_{sw}^{1/3}$ are representative area and length respectively. A scale coefficient σ is introduced such as each geometrical parameter can be expressed as $L = \sigma L_{ini} / L_{ref}$. Consequently, the geometry of a Stirling engine is a function of σ.

The scaling process for fluid and thermal processes underlines the role of dimensionless parameters such as the Reynolds or the Mach number. Its application to Stirling engine by Organ [4] leads to the following numbers in particular:

Stirling number $\quad SG = p_{ref} / \omega \, \mu_{ref} \quad (16)$

Mach number $\quad Ma = \omega L_{ref} / \sqrt{R T_{ref}} \quad (17)$

The equality of these numbers between the initial and the derivative machine ensures the same performances.

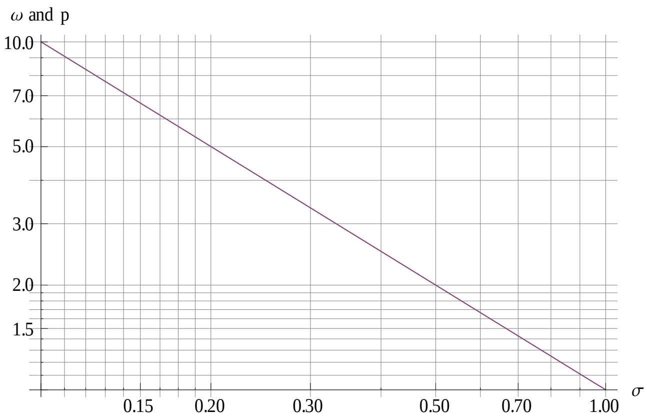


Fig. 2 Evolution of w and p with respect to the scaling factor with constant SG and Ma

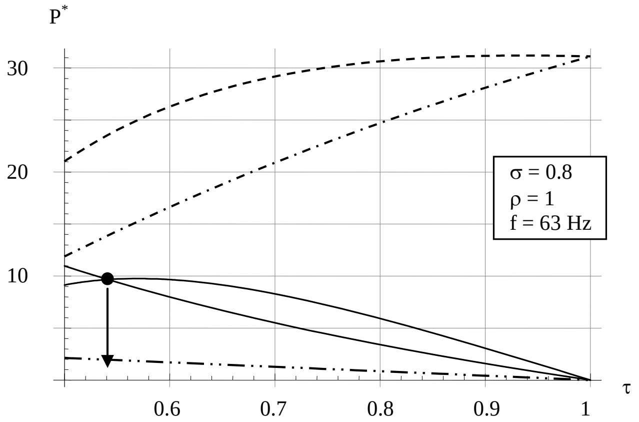
As the operating frequency and the pressure vary inversely as σ (Fig. 2), a scaling ratio of 1/5 leads to a virtual pressure twice as the initial pressure. From the GPU3, it leads to the huge value of 345 bars. Thus, strict similarity will not lead to a viable design.

3. RESULTS AND DISCUSSION

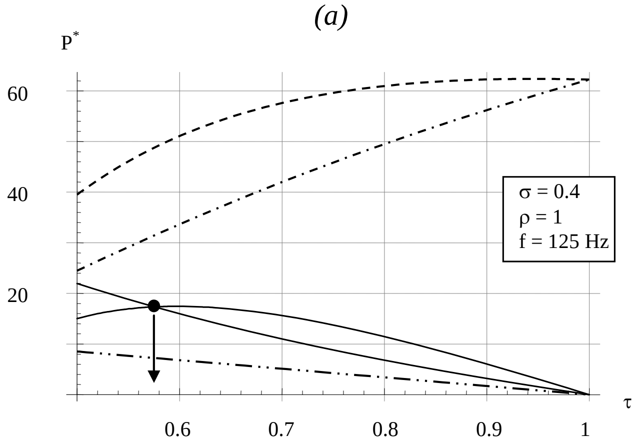
The following results aim at showing the effect of a scaling ratio of 1/10 on the engine performances. According to such a reduction a microengine with piston diameters of 20 mm and a representative stroke of 1 mm can be designed.

For technological purpose the pressure is limited to 69 bars whereas the operating frequency follows the similarity.

Fig. 3 (a) describes the evolution of the heat input (- - - -) heat rejected (- . - . -) and heat transfer by conduction (— . . —) with respect to the temperature ratio τ. The curves related to the indicated power from the Schmidt and the thermal analyses intersect at the operating point (—●—).



(a)

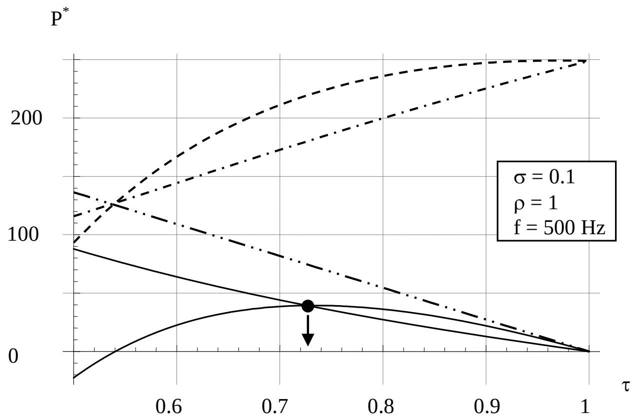


(b)

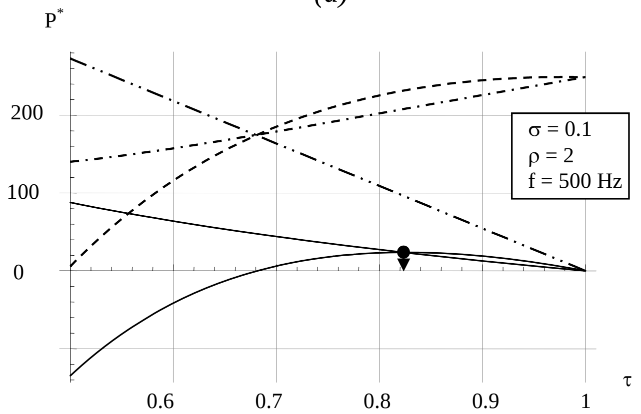
Fig.3 Power balance with respect to temperature ratio

As the size decreases specific power $P_i^* = P_i / (p_{mean} V_{ref})$ increases (Fig. 3 (a) and (b)).

However, usual technological solutions for macro size engines can not be applied to a micro engine. For example, the constraint of sealing and small friction at the piston and cylinder bore contact can not be easily handled. Consequently, membrane design often replaces the usual piston one. As a result, thickness which among others provides the thermal insulation goes to a fraction of its initial value. Thus, we choose to study the influence of the related value of the conduction coefficient ρ when it varies from 1 to 2 (Fig. 4 (a) and (b)).



(a)



(b)

Fig.4 Power balance with respect to temperature ratio

In the last case conduction loss rises to 48% of the input power, whereas the efficiency is 7.8% which represents about half the Carnot efficiency.

4. CONCLUSION

A Stirling engine model has been elaborated prior to design a micro Stirling engine. Unlike models generally used, thermal impedance related to heat exchangers which link heat source and sink to respectively the hot and cold parts of the engine are taken into account. Moreover, pressure drops and internal thermal loss through fixed and moving components are integrated in the model as many as internal defects of the machine. Thus, the influence of downsizing on the global engine performance can be assessed.

The major rule of conduction loss has been enhanced. Thus, new mechanical arrangements and material should be proposed and evaluated. Moreover, heat exchange at the cold end of a millimeter micro engine should have to drain about 250 Watts which consists in another design challenge.

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