# A NOVEL ENERGY HARVESTING DEVICE EMBEDDED IN A ROTATING WHEEL

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**Abstract:** This paper proposes a novel device of energy harvesting from a rotating wheel. The device is composed of a proof mass made of permanent magnets, two springs, a coil and an energy storage circuit. The rotating wheel produces a centrifugal force while the proof mass is subjected to a pull force by one spring and a push force by another. The proof mass vibrates along the transverse direction due to the variations of the gravity. For a specific ratio of spring constant of the two springs, the natural frequency of the spring-mass system can be adjusted automatically by the centrifugal force of the rotating wheel. This allows the proof mass vibrates with large velocity and displacement. A numerical study reveals that the amplitude of the displacement is more than 1 *mm* and the converted electrical power is more than 100 $\mu W$.

**Key words:** Energy harvesting, TPMS, vibration

## 1. INTRODUCTION

Increasing demands for embedding a tire pressure measurement system (TPMS) in all wheels as a car safety device have drawn attentions from academia and industries. However, the power supply of TPMS still relies on batteries, which has many drawbacks such as low durability, large weight, and, most importantly, inferior sustainability to environmental impact. An alternative approach for the replacement of the battery in TPMS is to harvest vibration energy from the environment [1-4].

In this paper, a concept of novel energy harvesting device embedded in a rotating wheel is proposed. The device is composed of a proof mass made of permanent magnets, two springs, a coil and an energy storage circuit. The spring-mass system can vibrate due to the gravity variation while the wheel rotates. The magnet, coil and energy storage circuit form a magnetic power generator. For a specific ratio of spring constant of the two springs, the natural frequency of the spring-mass system can be adjusted automatically by the centrifugal force of the rotating wheel. This allows the proof mass vibrates with large velocity and displacement. This paper is focused on the analysis of the vibration system. Both linear and non-linear vibration equations are established. The effect of the coil can be modeled as an electrical damping and the converted energy can be estimated.

## 2. THE SPRING-MASS CONFIGURATION AND ITS DYNAMIC EQUATION

Fig. 1(a) shows an energy harvesting device embedded in a rotating wheel. The device is mounted on the rim. The detail configuration of the device is shown in Fig. 1(b). It is composed of a proof mass made of permanent magnet, two springs, a coil and an energy storage circuit. The device is orientated such that the centrifugal force is along the longitudinal direction, i.e., the $y$ axis. The angle $\phi$ is changed as the wheel rotates, i.e., $\phi=\omega t$, where $\omega$ is the angular velocity of the wheel and $t$ is the time. This leads the transverse component of the gravity changes when the wheel rotates. Due to the variation of the gravity, the proof mass vibrates along the transverse direction.

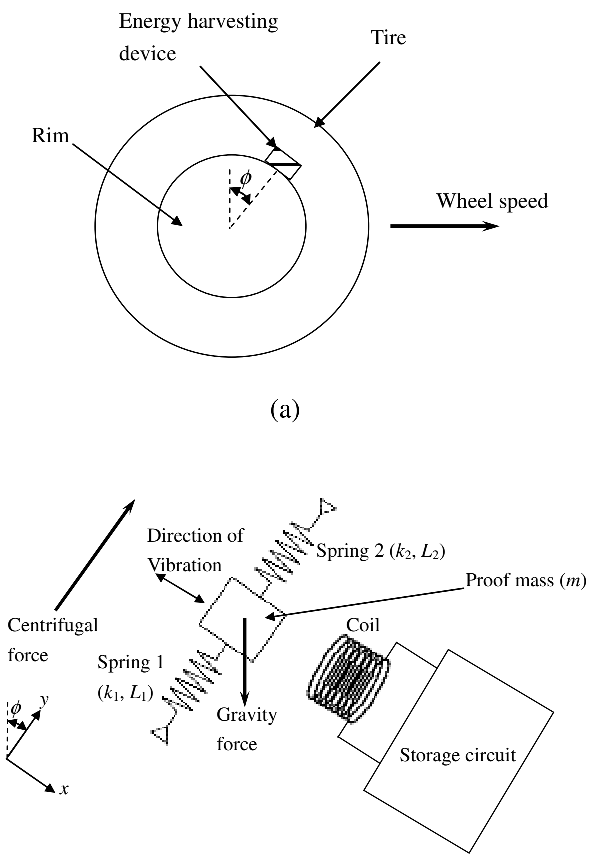
The spring is free of stretch when the wheel is at rest. When the wheel rotates at an angular velocity $\omega$, the centrifugal acceleration is $a=\omega^2 r$, where $r$ is the distance between the wheel center and the proof mass. The centrifugal acceleration is along the longitudinal direction of the spring-mass system. Assume that the gravity is much less than the centrifugal acceleration, so that the gravity has no contribution to the longitudinal displacement $\delta$. The free body diagram of the spring mass system is shown in Fig. 2(a). The longitudinal displacement $\delta$ can be rewritten as

$$\delta = \frac{ma}{k_1 + k_2} \qquad (1)$$

where $k_1$ and $k_2$ are the spring constants of the two springs. If the mass is subjected to a transverse excitation $mg\sin\omega t$, the mass would have a transverse displacement $x$, as shown in Fig. 2(b). The restoring force due to two springs is $k_1\delta_1 \sin\theta_1 - k_2\delta_2 \sin\theta_2$. After some elementary trigonometric geometry, the dynamic equation of the 1-d.o.f system can be written as

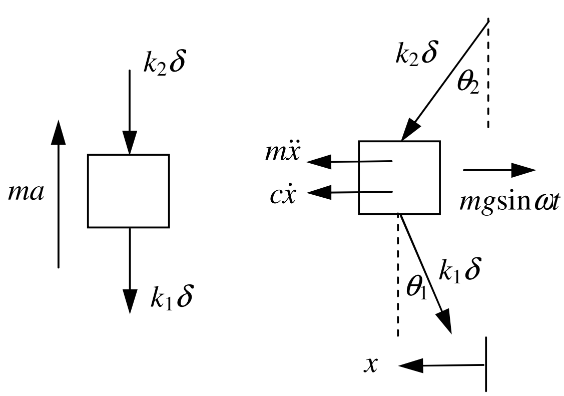
$$m\ddot{x} + c\dot{x} + f(x) = mg \sin \omega t \tag{2}$$

where $c$ is the electrical damping due to the relative velocity between coil and proof mass, and




(b)

Fig. 1 An energy harvesting device embedded in a rotating wheel.




(a) (b)

Fig. 2 The free body diagram for (a) longitudinal direction and (b) transverse direction.

$$f(x) = x\left[k_1\left(1 - \frac{L}{\sqrt{x^2 + (L+\delta)^2}}\right) + k_2\left(1 - \frac{L}{\sqrt{x^2 + (L-\delta)^2}}\right)\right] \tag{3}$$

with $L$ being the length of the springs. Eq. (2) is a non-linear differential equation and can be solved by numerical method. If the transverse displacement $x$ is small compared with $L$, Eq. (3) can be reduced to

$$f(x) = \left[k_1\left(1 - \frac{L}{L+\delta}\right) + k_2\left(1 - \frac{L}{L-\delta}\right)\right]x \tag{4}$$

Substitution of Eq. (4) into (2) gives a linear vibration equation:

$$m\ddot{x} + c\dot{x} + kx = mg \sin \omega t \tag{5}$$

where $k$ is the effective spring constant and can be written as

$$k = k_1\left(1 - \frac{L}{L+\delta}\right) + k_2\left(1 - \frac{L}{L-\delta}\right) \tag{6}$$

In the following, both linear and non-linear differential equation will be discussed.

## 3. LINEAR MODEL

The general solution of Eq. (5) is

$$x(t) = C_1 \cos \omega_d t + C_2 \sin \omega_d t + X \sin(\omega t - \varphi) \tag{7}$$

where $\omega_d$ is the damped natural frequency, $X$ is the amplitude of the steady-state vibration and $\varphi$ is the phase angle due to damping. The expressions of the parameters mentioned above can be found in elementary vibration textbook. For small damping ratio, $\omega_d$ is closed to natural frequency $\omega_n$, which is

$$\omega_n = \sqrt{\frac{k}{m}} \tag{8}$$

Substitution of Eq. (6) into (8) gives

$$\omega_n = \omega\sqrt{\frac{r}{k_1 + k_2}\left(\frac{k_1}{L+\delta} - \frac{k_2}{L-\delta}\right)} \tag{9}$$

Usually we choose the spring constants and the proof mass such that $\delta$ is much less than the spring length $L$, that is, Eq. (9) can be approximated as

$$\omega_n \cong \omega\sqrt{\frac{r(k_1 - k_2)}{L(k_1 + k_2)}} \tag{10}$$

For the purpose that $\omega_n \cong \omega$, we have

$$\sqrt{\frac{r(k_1 - k_2)}{L(k_1 + k_2)}} \cong 1 \tag{11}$$

For the design parameters listed in Table 1, the left hand side of Eq. (11) is 1.27, which is close to unity. If we use the more accurate expression, Eq. (9), the results is plot in Fig. 3. It is seen that the natural frequency of the spring-mass system can match the rotating frequency of the wheel. This allows the proof mass vibrates with large velocity and displacement.

Table 1 Design parameters

| *L* (*m*) | $k_1$ (*N*/*m*) | $k_2$ (*N*/*m*) | *m* (*kg*) | *r* (*m*) | *R* (*m*) |
|---|---|---|---|---|---|
| 0.01 | 2000 | 1700 | 0.001 | 0.2 | 0.3 |

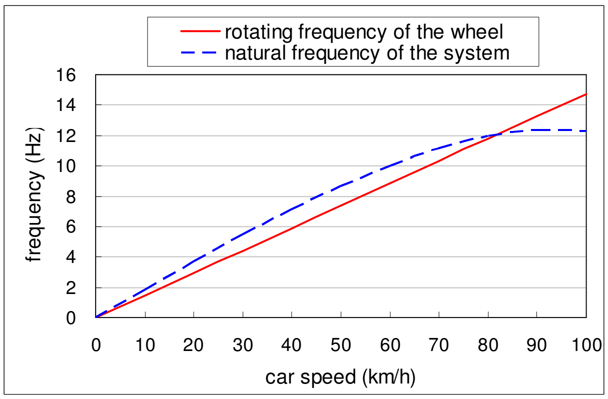


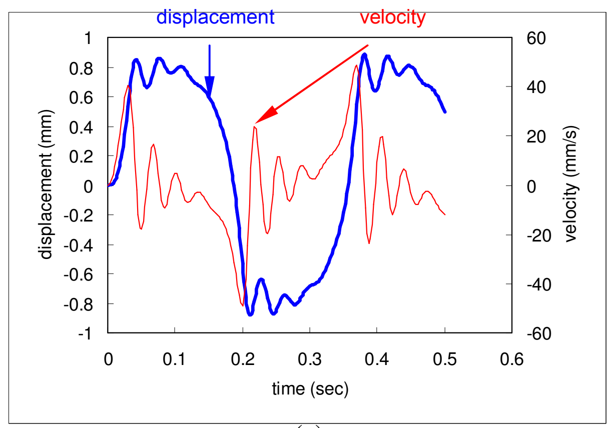

Fig. 3 The comparison of the rotating frequency of wheel and natural frequency of the system at various wheel speeds.

## 4. SOLUTIONS OF THE NON-LINEAR MODEL

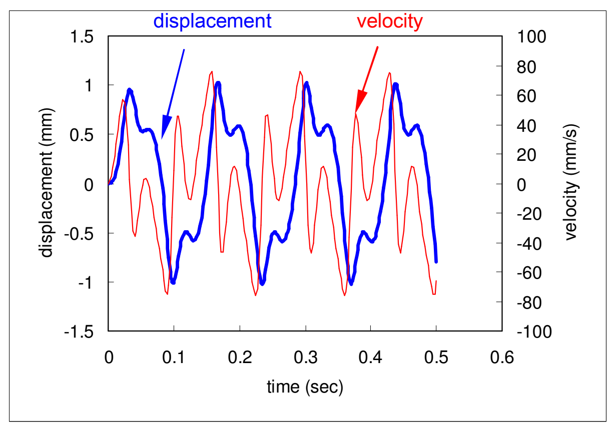
The solutions of the Eq. (2) can be computed by the standard procedure of Range-Kutta method. The damping coefficient is assumed as 0.05 *N-s*/*m*. The initial conditions are $x(0)=0$ and $\dot{x}(0)=0$ . The numerical results, including displacement, velocity of the mass and the power dissipated are shown in Figs. 4 and 5. The results indicate that the amplitude of the displacement is about 1 *mm*. For steady state condition, the maximum instantaneous power dissipated by the damper, which is converted to electrical power, is about 120 $\mu W$ for car speed 20 *km*/*h* and 300 $\mu W$ for car speeds 50 and 100 *km*/*h*.

## 5. CONCLUSION

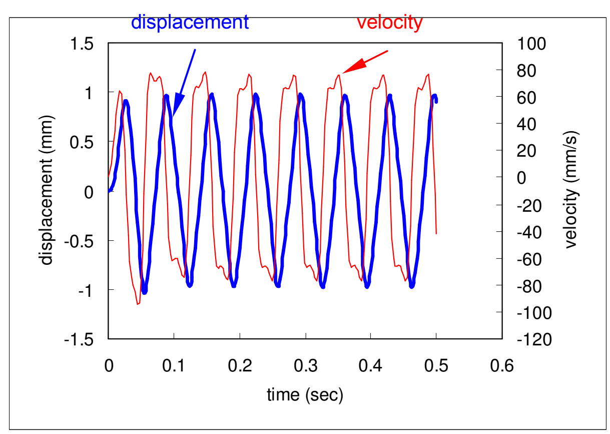
In this paper, a novel device of energy harvesting from a rotating wheel is proposed. The device is modeled as a spring-mass system composed of a proof mass, two springs, a coil and a storage circuit. The current study focuses on the dynamic behavior of the spring mass system. Because of the variation of gravity when the wheel rotates, the proof mass has a transverse vibration with a nature frequency close to rotating wheel frequency if the relative parameters are well designed. This allows the spring-mass system vibrates with resonant state in various wheel speeds. A further study of solving non-linear dynamic equation indicates that the converted power is more than 100 $\mu W$.




(a)




(b)




(c)

Fig. 4 The variations of displacement and velocity for various car speeds: (a)20 *km*/*h*; (b)50 *km*/*h*; (c)100 *km*/*h*.

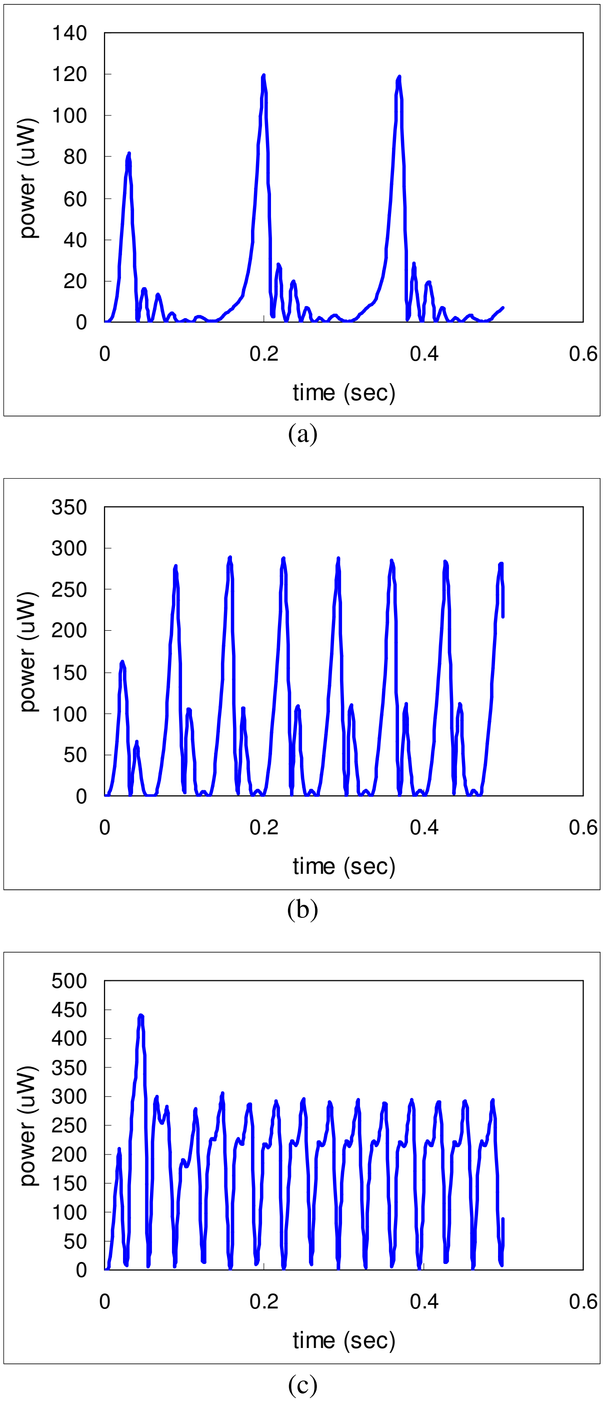



Fig. 5 The variations of converted power for various car speeds: (a)20 *km*/*h*; (b)50 *km*/*h*; (c)100 *km*/*h*.

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