# POWER PROCESSING CIRCUITS FOR VIBRATION-BASED ENERGY HARVESTERS: AN INTEGRATED APPROACH

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**Abstract:** The analysis given in this paper shows that an integrated approach is needed for the design of power conditioning circuits. The efficiency of the conditioning circuit as well as the effect of the circuit on the output power flow of the harvester has to be taken into account if optimal transformation of vibration input energy to useful output energy is aimed at. Two power conditioning circuits are considered, a standard buck converter and a converter using 'switched charge extraction'. Their influence on the harvester output power is determined, and an analytic loss model is set up of both circuits.

**Key words:** vibration-based energy harvesting, power conditioning, AC-DC conversion

## 1. INTRODUCTION

Motion energy or vibrations are an attractive source for powering miniature energy harvesting generators. A block diagram of an autonomous system with a vibration-based energy harvester as power source is given in Fig. 1. The electronic load is decoupled from the source by means of a buffer, e.g. a (super)capacitor or a battery, as the energy demand of the load is generally not constant over time. A power conditioning circuit is not only needed to perform an AC to DC voltage conversion, but has to maximize the energy extraction from the harvester as well. In order to obtain an efficient transformation of input vibration energy to useful output power, both the efficiency as well as the influence on the harvester output power flow of the conditioning circuit has to be taken into consideration while designing the circuit.

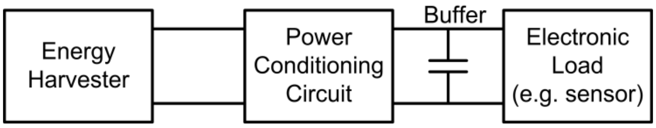


*Fig. 1: Block Diagram of a vibration powered harvester system.*

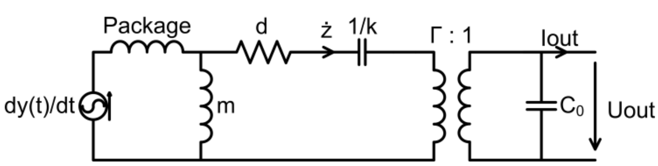


*Fig. 2 : Electrical equivalent model of energy harvester*

The model, shown in Fig. 2, of the energy harvester is used to calculate the influence on the harvester behavior of different power conditioning circuits. The inertial harvester is modeled as a mass-spring-damper system with input vibration y(t), the displacement of the mass is represented by z(t). The generator transforming the mechanical movement into electrical energy is modeled as a transformer, with an output capacitor $C_0$ [1].

## 2. POWER CONDITIONING CIRCUITS

Two different power conditioning circuits are considered in this paper. The first circuit is a standard buck converter, see Fig. 3. In [2] is shown that an optimal DC load for the harvester can be calculated for every operating frequency. The input voltage of the buck converter, controlled through the duty cycle of the switching element, is set to be the optimal DC load voltage. A fixed output voltage $U_{out}$ of the converter is assumed.

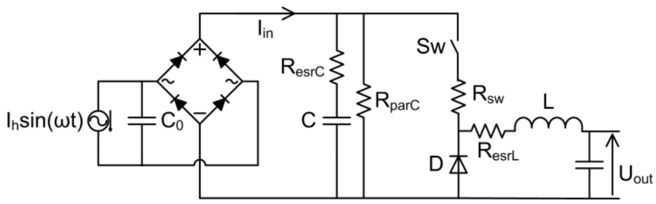


*Fig. 3: Buck converter.*

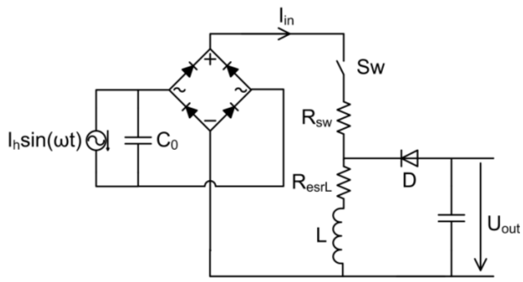


*Fig. 4: Synchronous Charge Extraction Circuit.*

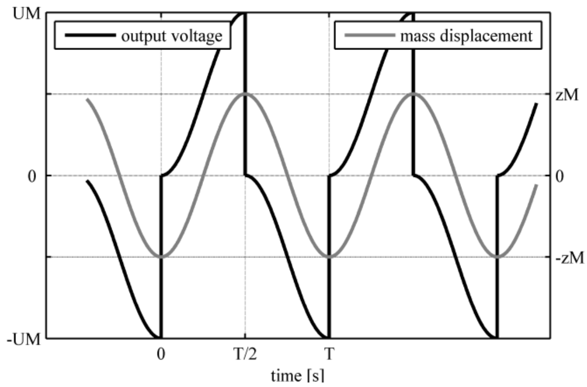


*Fig. 5: Output voltage and mass displacement of harvester with SCHE.*

The second circuit, shown in Fig. 4, operates according to the principle of synchronous charge extraction (SCHE) [3][4]: the switch closes when the output voltage on the output capacitor $C_0$ of the harvester reaches a maximum, the energy on $C_0$ is then transferred to the inductor L, the switch re-opens when the charge on $C_0$ is completely removed, the energy stored in the inductor is then transferred to the output capacitor. A fixed output voltage of the converter is assumed. In fig. 5 the output voltage of the harvester while using SCHE is shown. As the harvester operates in open-circuit condition most of the time, the harvester behavior is expressed by:

$$m\ddot{y} = m\ddot{z} + d\dot{z} + kz + \frac{\Gamma^2 z}{C_0} \qquad (1)$$

Working out (1) gives:

$$-ma\cos\varphi = -m\omega^2 z_M + kz_M + \frac{\Gamma^2 z_M}{C_0} \qquad (2)$$

with *a* the input acceleration, $z_M$ the displacement amplitude of the mass and $\varphi$ the phase difference between the input vibration y(t) and the displacement of the mass z(t).

The energy balance during half a vibration period is:

$$\int_0^{T/2} m\ddot{y}\dot{z} = \frac{C_0 U_M^2}{2} + \int_0^{T/2} \dot{z}(m\ddot{z} + kz + d\dot{z}) \qquad (3)$$

With $U_M$ the harvester output voltage at the switching instant. Working out (3) gives:

$$maz_M \frac{\pi}{2}\sin\varphi = dz_M^2\omega\frac{\pi}{2} + \frac{2\Gamma^2 z_M^2}{C_0} \qquad (4)$$

From (2) and (4), $z_M$ can be calculated. The power output of the harvester connected to a SCHE converter is then calculated as follows:

$$P_{SCHE} = \frac{2\omega\Gamma^2 z_M^2}{\pi C_0} \qquad (5)$$

The optimal output power flow of the harvester for both load cases depends on the coupling coefficient $\kappa = \Gamma / \sqrt{\Gamma^2 + kC_0}$ of the harvesting device and is shown in Fig. 6. In can be seen that for larger coupling coefficients, the output power of the harvester connected to the buck converter tends to the maximal reachable power output $P_{lim} = a^2m^2/8d$, determined by the acceleration, damping and the mass of the harvester [5]. When SCHE is used, the power output limit $P_{lim}$ is reached for smaller coupling coefficients, and the output power decreases with larger coupling coefficients.

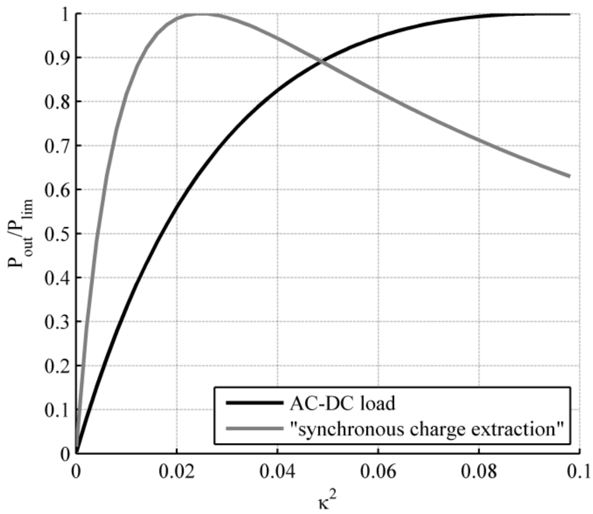


*Fig. 6: Optimal harvester output power versus squared coupling coefficient for both load cases.*

## 3. CONVERTER EFFICIENCIES

To assess the effectiveness of the system consisting of a harvester and power processing circuit, not only the output power flow of the harvester needs to be looked at, but the efficiency of the power circuit itself needs to be considered as well. Therefore, an efficiency analysis is made for both power processing circuits in which the energy losses in the switch, the diode and the parasitic losses in the inductor due to its equivalent series resistance $R_{esrL}$ are taken into account. The conduction losses of the switch are modeled through resistor $R_{sw}$. The input filter losses of the buck converter are modeled as the losses in the equivalent series resistance $R_{esrC}$ and parallel resistance $R_{parC}$.

### Buck converter

The input voltage of the converter is controlled through the duty-cycle. Because of the very low power processed by the circuit, the converter is most likely to operate in discontinuous conduction mode. The harvester is modeled as a current source to simplify the determination of the voltages and currents in the converter, see Fig.3. The input current of the converter $I_{in}$ is defined by the optimal harvester output power $P_{UOpt}$ and the optimal fixed output voltage $U_{ccopt}$ :

$$I_{in} = \frac{P_{UOpt}}{U_{ccOpt} + 2U_D} \tag{6}$$

with $U_D$ the forward voltage drop of the rectifier diodes.

When the switch is closed, the inductor current $i_L$ is given by:

$$\frac{-L}{R_{parC}}\left(\frac{R_{esrC}}{R_{parC}}+1\right)\frac{d^2 i_L}{dt^2} + \left(-\left(\frac{R_{esrC}}{R_{parC}}+1\right)\left(\frac{R_{sw}+R_{esrL}}{R_{parC}}+1\right)-\frac{L}{CR^2_{parC}}+1\right)\frac{di_L}{dt} - \left(\frac{R_{sw}+R_{esrL}}{R_{parC}}+1\right)\frac{L}{CR_{parC}} i_L = \frac{-I_{in}+U_{out}}{CR_{parC}} \tag{7}$$

The current through the inductor with the switch open is:

$$i_L(t) = -\frac{U_D + U_{out}}{R_{esrL}} + \left(i_L\left(\frac{\delta}{f_s}\right) + \frac{U_D + U_{out}}{R_{esrL}}\right)e^{\frac{-R_{esrL}}{L}t} \tag{8}$$

Using the expressions for the inductor current, the input voltage of the converter can be calculated. The duty cycle $\delta$ needed to obtain the optimal $U_{ccopt}$ is then found through iteration.

The losses in this circuit are listed:

*1) Mosfet Loss:* The Mosfet loss $P_{mos}$, is the sum of the conduction loss $P_{on}$ , the switching loss $P_{sw}$ and the gate-drive loss $P_{drv}$. The switch conduction loss $P_{on}$ can be obtained from the RMS current through the switch and the switch on-resistance $R_{sw}$. The switching loss is:

$$P_{sw} = \frac{1}{2} U_{in} i_L\left(\frac{\delta}{f_s}\right) t_{off} f_s + U_{in}^2 C_{ds} f_s \tag{9}$$

As only discontinuous conduction is considered, no switching loss occurs during on-switching of the switch. The second term in (9) refers to the drain-source capacitance loss. The gate-drive loss is given by:

$$P_{drv} = U_{gs} Q_g f_s \tag{10}$$

$U_{gs}$ and $Q_g$ are the gate-source voltage at the on-state and the gate charge.

*2) Diode Loss:* The diode loss can be obtained using (11) with $i_{avg\_out}$ the average output current of the converter.

$$P_{diode} = U_D i_{avg\_out} \tag{11}$$

*3) Inductor Loss:* The loss in the inductor due to its series resistance is calculated using the RMS inductor current.

*4) Bridge Rectifier Loss:* The loss in the bridge rectifier is:

$$P_{rect} = 2 i_{avg\_in} U_D \tag{12}$$

*5) Input filter loss:* The loss in the input filter capacitor $P_{filter}$ is calculated as the loss in the series and parallel parasitic resistors $R_{esrC}$ and $R_{parC}$.

The efficiency of the converter is then expressed as :

$$\eta = \frac{P_{UOpt}}{P_{UOpt} + P_{mos} + P_{rect} + P_{diode} + P_L + P_{filter}} \tag{13}$$

**SCHE converter**

As with the buck converter, the harvester is modeled as a current source with the harvester output capacitor $C_0$ connected in parallel, to simplify the determination of the voltages and currents in the converter, see Fig.4. The output voltage of the harvester while the switch is on is calculated using:

$$\frac{d^2 u_c}{dt^2} = -\frac{u_c}{LC_0} + \frac{2U_D}{LC_0} - \frac{R_{esrL} + R_{sw}}{L}\frac{du_c}{dt} \tag{14}$$

The current through the inductor is then:

$$i_L = -C\frac{du_c}{dt} \tag{15}$$

While the switch is off, the current through the inductor is determined by:

$$L\frac{di_L}{dt} = -U_D - R_{esrL} i_L - U_{out} \tag{16}$$

Using equations (13)-(15), the on-time of the switch is calculated. The losses in this circuit are listed:

*1) Mosfet Loss:* The switch conduction loss $P_{on}$ is obtained from the RMS current through the switch and the switch on-resistance $R_{sw}$. The switching loss is:

$$P_{sw} = U_D + U_{out} \; I_{L\max} t_{off} \frac{f}{2} + U^2_{C_0 \max} C_{ds} 2f \tag{17}$$

The vibration frequency f of the harvester determines the switching frequency $f_s$ = 2f of the SCHE converter. The gate-drive loss is given by (10).

*2) Diode Loss:* as with the buck converter, the diode loss is obtained using (11).

*3) Inductor Loss:* The loss in the inductor is calculated using the RMS inductor current.

*4) Bridge Rectifier Loss:* The loss in the bridge rectifier is:

$$P_{rect} = 2 i_{avg\_in} U_D \tag{18}$$

## 4. EXAMPLE

To illustrate previous analysis, a piezo bimorph element is taken as energy harvesting device. The measured model parameters of the device are given in Table 1, as well as the theoretical optimal output power of the harvester with buck converter load $P_{UOpt}$ and with synchronous charge extraction $P_{sche}$. Note that the output $P_{sche}$ is much larger than $P_{UOpt}$, because of the rather low coupling coefficient of the device, see Fig. 6. An output voltage of 3 V is chosen for both power processing circuits. The efficiency of both converters is calculated for varying inductance, using the optimal load conditions of the bimorph harvesting device as input (see Table 1). Figure 7 shows a plot of the calculated efficiencies versus inductance of both circuits. The efficiency of the buck converter is given for varying switching frequency. The highest efficiency of the buck converter, 67.4%, is reached using a switching frequency of 0.5 kHz and an inductance value of 68 µH. The duty cycle in the optimal point is 0.056%. The highest efficiency, 61.5%, of the SCHE-converter is reached using an inductance value of 33 µH. The calculated on-time of the switch at this point is 248 ns.

Although the efficiency of the buck converter is higher, the overall effectiveness $\eta_{eff}$, defined as $\eta_{eff} = \eta_{circuit} * P_{harvester} / P_{lim}$ , of the harvesting system, will be higher using SCHE, see Table 1.

This example shows that it is important to take the efficiency of the power processing circuit, as well as the effect of the processing circuit on the power flow of the harvester into consideration while designing a power conditioning circuit.

*Table 1 : Measured model parameters of piezo bimorph harvesting device and used converter parameters.*

| | | | |
|---|---|---|---|
| m | $1.1e^{-3}$ kg | $C_{ds}$ | 5 pF |
| d | 0.067 Ns/m | $t_{off}$ | 5ns |
| $C_0$ | 750 pF | $U_D$ | 0.6 V |
| $\kappa^2$ | 0.03 | $R_{esrL}$ | 45 kΩ/H |
| $f_{resonance}$ | 300 Hz | $R_{parC}$ | 100 MΩ |
| a | $4 m/s^2$ | $R_{esrC}$ | 0.1 Ω |
| $P_{lim}$ | 37 µW | $Q_g$ | 0.3 nC |
| $P_{UOpt}$ | 26 µW | $U_{gs}$ | 5 V |
| $P_{sche}$ | 35 µW | $\eta_{buck}$ | 67.4% |
| $U_{ccopt}$ | 5.3 V | $\eta_{SCHE}$ | 61.5% |
| $U_{M\_SCHE}$ | 12.4 V | $\eta_{eff\_buck}$ | 47.4% |
| $R_{on}$ | 2 Ω | $\eta_{eff\_SCHE}$ | 58.2% |

## 5. CONCLUSION

The analysis given in this paper shows that an integrated approach is needed for the design of power conditioning circuits. The efficiency of the conditioning circuit as well as the effect of the conditioning circuit on the output power flow of the harvester has to be taken into account if optimal transformation of vibration input energy to useful output energy is aimed at.

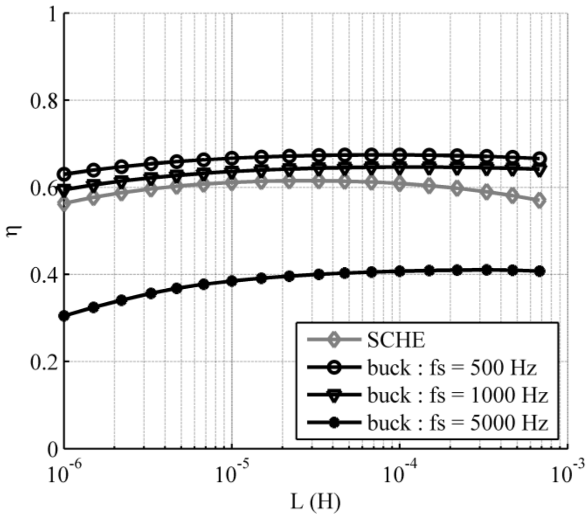


*Fig. 7 : Calculated efficiency of SCHE and buck converter for varying inductances and switching frequency.*

## REFERENCES

[1] Sterken T, Baert K, Van Hoof C, Puers R, Borghs G, and Fiorini P, 2007 Comparative modelling for vibration scavengers [MEMS energy scavengers] in *Sensors, 2004. Proceedings of IEEE* 1249– 1252.

[2] D'hulst R, Sterken T, Fiorini P, Puers R, Driesen J 2007 Energy Scavengers: Modeling and Behavior with Different Load Circuits *33rd Annual Conference of the IEEE Industrial Electronics Society IECON2007 (Taipei, Taiwan, November 5-8, 2007)* 2169-2174.

[3] Lefeuvre E, Badel A, Richard C, Petit L, Guyomar D 2006 A comparison between several vibration-powered piezoelectric generators for standalone systems *Sensors and Actuators A: Physical,* **Vol. 126, no. 2** 405-416.

[4] Xu S, Ngo K, Nishida T, Shung G, Sharma A 2007 Low Frequency Pulsed Resonant Converter for Energy Harvesting *IEEE Trans. on Power Electronics* **Vol 22, no. 1** 63-68.

[5] D'hulst R, Driesen J 2008 Power Processing circuits for Vibration Based Energy Harvesters *IEEE 39th Power Electronics Specialists Conference PESC08 (Rhodes, Greece, June 15-19).*

[6] Sodhi R, Brown S, Kinzer D 1999 Integrated design environment for DC/DC converter FET optimization *The 11th International Symposium on Power Semiconductor Devices and ICs, ISPSD '99.* 241–244.