NONLINEAR EFFECTS IN PIEZOELECTRIC VIBRATION HARVESTERS WITH HIGH COUPLING FACTORS

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Abstract: The nonlinear effects of piezoelectric coupling on piezoelectric harvesters are experimentally examined and compared to the approximate solutions of a simple nonlinear model. In this paper we apply a nonlinear version of the piezoelectric equations proposed in [1] to model the nonlinear behavior of a piezoelectric harvester. It turns out that the general nonlinear effects, i.e. the dependence of the resonance frequency on the amplitude, as well as the non-constant voltage-to-excitation ratio can be explained by this model, but the overall behavior of the harvester suggests that the model must be modified to accurately describe the experiment.

Keywords: piezoelectric energy harvester, nonlinear effects

INTRODUCTION

The main challenge in designing piezoelectric energy harvesters is to gain the maximum output power from a given vibration source. Most groups are specialized on vibration sources with one dominating frequency, e.g. electro motors, to exploit the high output power of piezoelectric vibration harvesters in resonance.

These harvesters are typically modeled as linear devices [2]. This linear modeling leads to the following result: The output power in resonance is optimized for low parasitic damping, high effective mass and high device coupling factors k_{dev} [3]. We focus on the nonlinear effect of high electro-mechanical coupling. Though the piezoelectric effect is typically described as a linear effect it is inherently nonlinear. This nonlinearity has an increasing impact on the harvester characteristics with increasing coupling factor k_{dev} and excitation amplitude a_{ex}.

NONLINEAR EFFECTS

PZT is typically used as the piezoelectric material in piezoelectric harvesters due to its high material coupling factor. As a high device coupling factor is generally desirable, most of the vibrating structure should be fabricated from the piezoelectric material. Therefore the behavior of the harvester will be dominated by the electro-elastic behavior of the piezoelectric material.

It is well known that PZT like any other piezoelectric material shows a significantly nonlinear behavior at high electric fields, like e.g. the butterfly curve.

As harvesters typically only produce small electric fields these nonlinearities should not be significant in these devices. But as demonstrated in this paper piezoelectric harvesters can show strong nonlinearities even under short circuit conditions, e.g. the resonance frequency and the ratio of vibration and excitation amplitude depend on the excitation amplitude. This effect can be interpreted as a consequence of a displacement-dependent Young`s modulus and coupling constant of the piezoelectric material. The structure of the harvester investigated in this study is shown in figure 1.

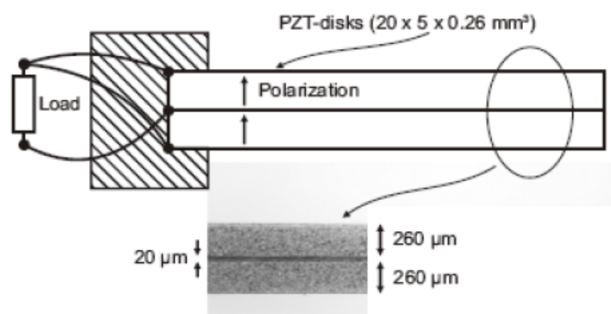

Fig. 1: Sketch and micrograph of our high coupling harvester.

The whole bending element is made of the piezoelectric material, only a small glue layer is situated in the center. Hence we expect that the electro-mechanical behavior is solely described by the piezoelectric material equations.

MODELLING

Our modeling is based on the material equations derived in [1].

$$
\begin{aligned}
Y &= Y_0 + Y_1 \cdot S + Y_2 \cdot S^2 + \ldots \\
d &= d_0 + Y_1 \cdot S + d_2 \cdot S^2 + \ldots
\end{aligned}
\qquad (1)
$$

Here Y denotes the Young´s modulus and d is the piezoelectric coupling constant. Both depend on the strain S. We have omitted the indices indicating the direction for clarity. Our beam-like harvester is subjected to bending stresses in 1-direction and an electric field in 3-direction. Therefore Y_{11} and d_{31} are the relevant parameters in the common notation.

As discussed before the device will behave equivalent to the material. Therefore two equations equivalent to the formulation in eq. (1) can be established.

$$\begin{aligned} F &= K_0 \cdot Z - \Theta_0 \cdot U + K_2 \cdot Z^3 - \Theta_2 \cdot Z^2 \cdot U \\ Q &= \Theta_0 \cdot Z + C^T \cdot \left(1 - k^2\right) \cdot U + \frac{1}{3} \cdot \Theta_2 \cdot Z^3 \end{aligned} \qquad (2)$$

Here F denotes the force acting on the harvester and Z symbolizes the displacement, while U and Q are the voltage across and the charge on the electrodes of the piezoelectric material. C^T stands for the free capacitance of the harvester while k stands for the device´s coupling factor.

$$k^2 = \frac{\Theta_0^2}{K_0 \cdot C^T} \qquad (3)$$

K_0 and Θ_0 are the linear spring constant and coupling constant of the device, while K_2 and Θ_2 are the nonlinear corrections of smallest order. K_1 and Θ_1 vanish due to symmetry of the device. Throughout this paper any capital letter stands for a dimensional variable or constant, while small letters are used for dimensionless quantities.

In our simple setup the force consists of an inertial part due to the displacement Z and the sinusoidal excitation Z_{ex} acting on an effective Mass M_{eff} and a damping force proportional to the velocity. The electrical circuit consists of a single resistive load R.

$$\begin{aligned} F &= -M_{eff} \cdot \left(\ddot{Z} + \ddot{Z}_{ex}\right) - \Gamma \cdot \dot{Z}; \\ Z_{ex} &= Z_0 \cdot \sin(\Omega \cdot T + \varphi); \\ U &= -R \cdot \dot{Q} \end{aligned} \qquad (4)$$

For further analysis the nonlinear system of eq. (2) to (4) is transformed into the dimensionless version eq. 5, with the dimensionless quantities defined in eq. 6.

$$\begin{aligned} &\ddot{z} + \delta\dot{z} + z + \alpha z^3 + \frac{k^2}{\omega_{el}}\left(\dot{q} + \chi z^2 \dot{q}\right) \\ &= \omega^2 \sin(\omega t + \varphi) \\ &q + \frac{1-k^2}{\omega_{el}}\dot{q} = z + \frac{1}{3}\chi z^3 \end{aligned} \qquad (5)$$

$$\begin{aligned} &t = \sqrt{\frac{M_{eff}}{K_0}} \cdot T; \quad \omega = \sqrt{\frac{K_0}{M_{eff}}} \cdot \Omega; \\ &z = \frac{Z}{Z_0}; \quad q = \frac{Q}{Z_0 \cdot \Theta_0}; \\ &\omega_{el} = \frac{1}{R \cdot C^T \cdot \sqrt{\frac{M_{eff}}{K_0}}}; \quad \delta = \frac{1}{\sqrt{K_0 \cdot M_{eff}}} \cdot \Gamma; \\ &\alpha = \frac{K_2 \cdot Z_0^2}{K_0}; \quad \chi = \frac{\Theta_2 \cdot Z_0^2}{\Theta_0} \end{aligned} \qquad (6)$$

The nonlinear system eq. (5) can approximately be solved for small nonlinearities α and χ. The solution will be virtually sinusoidal. Therefore the phase angle φ can be chosen such that the displacement z is a pure sine-function. The charge q is then also sinusoidal function with a sine and a cosine part.

$$\begin{aligned} z &\approx z_0 \cdot \sin(\omega \cdot t); \\ q &\approx q_S \cdot \sin(\omega \cdot t) + q_C \cdot \cos(\omega \cdot t) \end{aligned} \qquad (7)$$

The nonlinear terms are linearized by developing into a sine respective cosine part and neglecting the high frequency parts.

$$\begin{aligned} &\sin^3(\omega \cdot t) = \\ &\frac{3}{4}\sin(\omega \cdot t) - \frac{1}{4}\sin(3\omega \cdot t) \approx \frac{3}{4}\sin(\omega \cdot t); \\ &\sin^2(\omega \cdot t) \cdot \cos(\omega \cdot t) \\ &= \frac{1}{4}\cos(\omega \cdot t) - \frac{1}{4}\cos(3\omega \cdot t) \approx \frac{1}{4}\cos(\omega \cdot t) \end{aligned} \qquad (8)$$

With these approximations eq. (5) can be converted into the familiar form of a harmonic oscillator.

$$\ddot{z} + \delta_{eff}\dot{z} + \omega_0^2 z = \omega^2 \sin(\omega t + \varphi) \qquad (9)$$

The resonance frequency ω_0 and the effective damping ratio δ_{eff} are defined in eq. (10)

$$\delta_{eff} = \delta + \frac{k^2 \cdot \omega_{el}}{\omega_{el}^2 + \omega^2 \cdot (1-k^2)^2} \cdot \left(1 + \frac{1}{4} \cdot \chi \cdot z_0^2\right)^2$$
$$\omega_0 = \sqrt{1 + \frac{3}{4}\alpha z_0^2 + \frac{k^2\omega^2(1-k^2)}{\omega_{el}^2 + \omega^2(1-k^2)^2}\left(1+\frac{1}{4}\chi z_0^2\right)\left(1+\frac{3}{4}\chi\omega z_0^2\right)} \quad (10)$$

The so-called Lorentz approximation to eq. 9 yields a good approximation for the frequency response.

$$z_0(\omega) = \frac{\omega^3 \delta_{eff}}{\left(\omega^2 - \omega_0^2\right)^2 + \omega^2 \delta_{eff}^2} \quad (11)$$

As eq. (11) is an implicit equation, i.e. ω_0 and δ_{eff} still contain the amplitude z_0 it must be solved numerically to get the frequency response curve. Once the solution is found the generated charge can be calculated from eq. (12).

$$q_0(\omega) = z_0(\omega) \frac{\left(1 + \frac{1}{4}\chi \cdot z_0^2(\omega)\right)}{\sqrt{1 + \left(\frac{\omega \cdot (1-k^2)}{\omega_{el}}\right)^2}} \quad (12)$$

The proposed model is of quite approximate character but should lead to appropriate results at least for small nonlinearities. The first results can directly be drawn from eq. (10). The damping is only influenced by the nonlinearity of the coupling constant. A positive ratio χ, i.e. an increased coupling with increased stress leads to higher damping with higher excitation amplitude. The resonance frequency ω_0 depends on both nonlinear parameters α and χ. The dependence on χ is load-dependent, while the contribution of α is constant. The nonlinear correction to the resonance frequency is of the order z_0^2 for both effects.

EXPERIMENTS

The harvester introduced in figure 1 was subjected to a sinusoidal excitation with a constant excitation amplitude on a shaker (Tira TV 51110). The frequency response of the generated voltage was recorded with a data acquisition card (Meilhaus Electronic ME AB-D78M) for eleven equally spaced acceleration levels from 2.2 m/s² to 24.2 m/s². This experiment was conducted for a load of 10 MΩ ("open loop") and 1 kΩ. The measured voltage amplitudes are shown in figures 2 and 3.

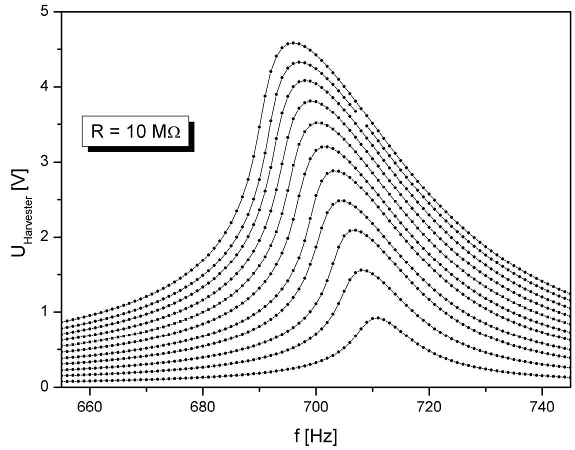

Fig. 2: Frequency response of the harvester with a resistive load of 10 MΩ

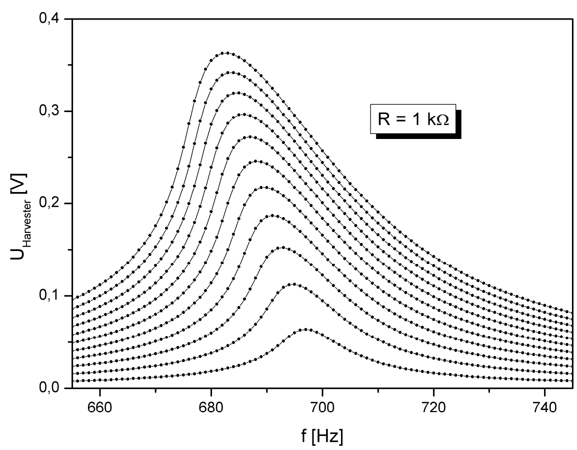

Fig. 3: Frequency response of the harvester with a resistive load of 1 kΩ

To analyze the curves in detail the resonance frequency versus excitation (fig. 4) and peak voltage versus excitation (fig. 5) curves are derived from the measurement. Figure 4 shows a virtually linear dependence of the resonance frequency on the excitation. The change in resonance frequency with excitation does not depend on the load condition. The offset between the low and high load measurements can be explained as a consequence of the high coupling factor, i.e. the open-loop and short-circuit stiffness differ significantly.

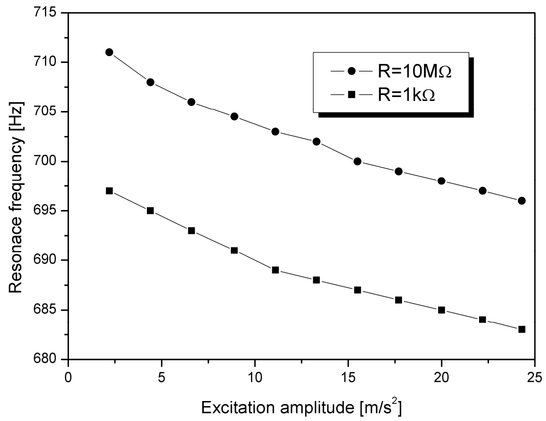

Fig. 4: Resonance frequency versus excitation for two load conditions

In figure 5 a clearly nonlinear relation of peak voltage to excitation can be observed. The voltage to excitation ratio is reduced by approximately 50% from low to high excitation amplitudes. Both curves show a similar trend on different scales, indicating a load-independent mechanism.

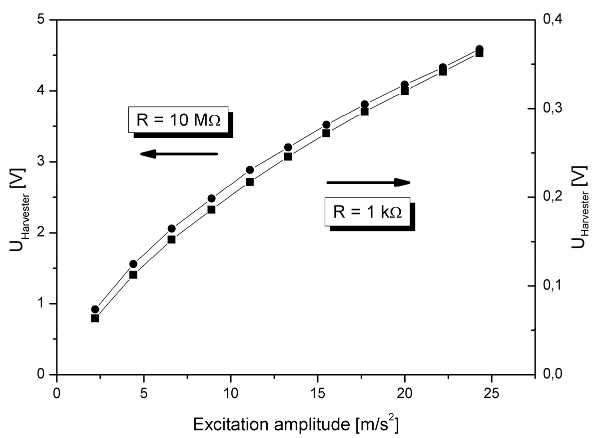

Fig. 5: Peak voltage versus excitation for two load conditions

DISCUSSION

The experiments indicate a purely mechanical reason for the nonlinearities of the harvester. The nonlinearities do not depend on the load, but this would be expected for a nonlinearity arising from a nonlinear coupling constant. In other words the nonlinear parameter χ can be neglected in our model.

The proposed mechanical nonlinearity parameterized by α does not explain our experiments accurately. Our model suggests a resonance frequency reduction proportional to the square of the excitation amplitude while the experiments show a linear dependence. This discrepancy cannot be overcome by any higher order modeling. Furthermore the nonlinearity in the peak-voltage to excitation ratio is much stronger than expected for reasonable values of α.

CONCLUSION

We conclude that a purely mechanic nonlinear mechanism that cannot be modeled by a simple dependence of the Young`s modulus on the strain must be found to get an accurate model for the nonlinear harvester. We propose the effect of stress strain hysteresis as the most promising candidate. This effect may also explain the nonlinear peak-voltage to excitation ratio, due to the energy dissipation in a hysteretic loop. Further research has to be conducted to gain a more detailed inside into the relevant non-linear mechanisms.

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