# PARAMETRIC STUDY OF A CANTILEVERED PIEZOELECTRIC ENERGY HARVESTER

**Rupesh Patel[1*], Atanas A. Popov[1] and Stewart McWilliam[1]**
[1]Materials, Mechanics and Structures Research Division, Faculty of Engineering,
University of Nottingham, Nottingham, UK

**Abstract:** This paper presents the theoretical findings from a parameter study on a conventional, unimorph, piezoelectric energy harvester. An analytical model has been developed which provides accurate representation of various geometrical changes, allowing confident predications in the power generated over a resistive load. One analysis has been carried out in this paper whereby the position, length and thickness of the piezoelectric layer are varied to maximise the power output of an energy harvester. The dimensions of the substrate layer are fixed at $50\times5\times0.7$ mm. Through a parameter study, for an acceleration of 9.81 ms$^{-2}$, the power across a 100 kΩ resistor was increased by 67.6 % - from 0.734 mW to 1.23 mW.

**Keywords:** Energy harvesting, Energy scavenging, Piezoelectric, Parameter study, Finite element method

## INTRODUCTION

The last decade has experienced a rapid growth in research within the field of vibrational energy harvesting. Devices which convert vibrational energy into electrical energy are highly appealing as battery replacements for numerous applications. The three electromechanical transducers which accomplish this are piezoelectric [1], electromagnetic [2], and electrostatic [3]. The majority of publications on energy harvesting are related to piezoelectric devices, the conventional type being a unimorph. There is a large amount of literature and ongoing research on improving the energy harvester design [4, 5], however, a lack of work on a fundamental cantilever parameter study. A basic parameter study could help improve the efficiency of devices and increase the generated power.

Parameter studies have been carried out in the past but the majority of models used for the energy harvester ignore coupling between the electrical and mechanical aspects of the system. Eggborn [6] has performed a comparable parameter study to the one presented here but the model created was limited to a very thin layer of piezoelectric material and ignored electromechanical coupling. Recent efforts on modelling a non-uniform beam for energy harvesting were undertaken by Liao and Sodano [7]. The developed model has similar capabilities to the one proposed, however, the authors did not use their model to carry out a dimensional parameter study.

## THEORY

Two variables which are vital in obtaining reliable predictions for the generated power are the natural frequencies and mode shapes of the structure. While changing certain structure geometries, for example the position of the piezoelectric layer, the beam becomes irregular and therefore modelling it as a uniform beam is inaccurate. In the presented analytical model, the finite element (FE) method is used to obtain the natural frequencies and mode shapes. The beam is modelled as three large sections, each comprising of smaller elements, (Fig. 1).

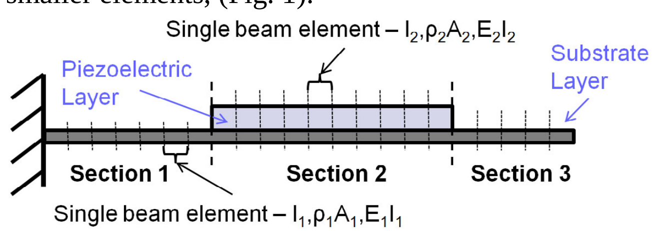


*Fig. 1: Non-uniform beam created by moving the piezoelectric layer.*

Each element is defined using the following mass and stiffness matrices:

$$\left[m_i\right]=\frac{\rho_i A_i l_i}{420}\begin{bmatrix}156 & 22l_i & 54 & -13l_i\\ 22l_i & 4l_i^2 & 13l_i & -3l_i^2\\ 54 & 13l_i & 156 & -22l_i\\ -13l_i & -3l_i^2 & -22l_i & 4l_i^2\end{bmatrix} \tag{1}$$

$$\left[k_i\right]=\frac{E_i I_i}{l_i^3}\begin{bmatrix}12 & 6l_i & -12 & 6l_i\\ 6l_i & 4l_i^2 & -6l_i & 2l_i^2\\ -12 & -6l_i & 12 & -6l_i\\ 6l_i & 2l_i^2 & -6l_i & 4l_i^2\end{bmatrix} \tag{2}$$

The matrices for each element are then assembled to produce the global mass and stiffness matrix for the entire structure. The model has been validated using Abaqus CAE and the percentage difference in the natural frequency, for various beam configurations, was found to be below 1.5 %.

The expression for generated power is obtained by making modifications to Erturk and Inman's distributed parameter electromechanical model for cantilevered piezoelectric energy harvesters [8]. The final equation is shown below:

$$Power = \frac{\left( \dfrac{\sum_{r=1}^{\infty} \dfrac{i\omega N_r \varphi_r}{\left(\omega_r{}^2 - \omega^2\right) + \left(2\gamma_r \omega_r \omega\right) i}}{\left(i\omega + \tau_{ci}\right) + \sum_{r=1}^{\infty} \dfrac{i\omega R_r \varphi_r}{\left(\omega_r{}^2 - \omega^2\right) + \left(2\gamma_r \omega_r \omega\right) i}} \right)^2}{R_{load}} \tag{3}$$

Where $\gamma_r$ is the modal damping ratio, $\omega_r$ is the r<sup>th</sup> natural frequency and $\omega$ is the excitation frequency. $N_r$ is the modal forcing term given by:

$$Y\omega_r{}^2 \left( \int_0^{L_b} m(x).\phi_r(x) \right), \tag{4}$$

where $Y$ is the input amplitude, $L_b$ is the length of the beam, $m$ is the beam's mass per unit length and $\phi_r(x)$ is the r$^{th}$ eigenfunction. $\varphi_r$ is the forward modal coupling term given by:

$$\frac{-d_{31} E_p t_{pc} t_p}{\varepsilon_{33}^S L_p} \left[ \frac{d\phi}{dx} \right]_{x_1}^{x_2}, \tag{5}$$

where $E_p$, $L_p$ *and* $t_p$ are the piezoelectric Young's modulus, length and thickness respectively, $d_{31}$ is the piezoelectric constant, $\varepsilon^S_{33}$ is the permittivity, $t_{pc}$ is the distance between the centre of the piezoelectric layer and the neutral axis, and $\left[d\phi/dx\right]_{x_1}^{x_2}$ is the difference in the slope of the eigenfunction evaluated at the ends of the piezoelectric layer. $R_r$ is the backward modal coupling term given by:

$$\left( \frac{E_p b_p d_{31}}{2t_p} \left( h_c{}^2 - h_b{}^2 \right) \right) \left[ \frac{d\phi}{dx} \right]_{x_1}^{x_2}, \tag{6}$$

where $b_p$ is the piezoelectric layer width, $h_c$ is the distance between the neutral axis and the top of the piezoelectric layer and $h_b$ is the distance between the neutral axis and the bottom of the piezoelectric layer. $\tau_{ci}$ is the inverse time constant given by:

$$\frac{1}{R_{load} C} = \frac{t_p}{R_{load} \varepsilon_{33}^S b L_p}, \tag{7}$$

where $R_{load}$ is the resistive load and C is the piezoelectric capacitance.

## RESULTS AND DISCUSSION

The developed analytical model was used to determine the effects of various parameters on the generated power. In the following analysis steel is used for the substrate layer and PZT for the piezoelectric layer; the material properties and structure dimensions are provided in Table 1. All the results presented in paper are calculated at the first natural frequency of the structure and only the first vibrational mode is used in Eq. 3. This is a valid modelling step as, at the fundamental frequency, the contribution of other modes on the displacement is negligible. The input acceleration is kept constant at 1 g and the damping ratio is assumed to be 0.01.

*Table 1: Material properties, structural dimensions and electromechanical parameters used for the analysis.*

| | |
|---|---|
| *Substrate length (mm)* | 50 |
| *Substrate width (mm)* | 5 |
| *Substrate thickness (mm)* | 0.7 |
| *Young's modulus of the substrate (GPa)* | 210 |
| *Density of the substrate (kg/m$^3$)* | 7850 |
| *Young's modulus of the PZT (GPa)* | 66 |
| *Density of the PZT (kg/m$^3$)* | 7800 |
| *Piezoelectric constant (m/V)* | $-190\text{x}10^{-12}$ |
| *Permittivity (F/m)* | $15.83\text{x}10^{-9}$ |

### Effect of Length

The first variable discussed is the length of the PZT layer (Fig. 2). The width and thickness of the PZT layer remain constant at 5 mm and 1.02 mm, respectively.

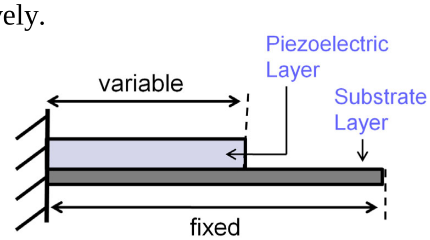


*Fig. 2: Adjusting the length of the piezoelectric layer.*

Fig. 3 shows that the trend between generated power and PZT length varies with the magnitude of the load resistance. For a load resistance of 41 kΩ the maximum power is 0.912 mW, which occurs when the PZT layer covers the entire beam length. If the load resistance is increased to 1 MΩ the maximum power is 0.260 mW which occurs for a much shorter, 8 mm, length of PZT.

The magnitude of the inverse time constant, $\tau_{ci}$, is mainly responsible for the varying trend. For a small resistance this term is dominant in the power expression (Eq. 3) and increasing the PZT length reduces the magnitude of $\tau_{ci}$ thereby benefiting the generated power. As the load resistance increases other terms have a greater significance and a different trend is observed.

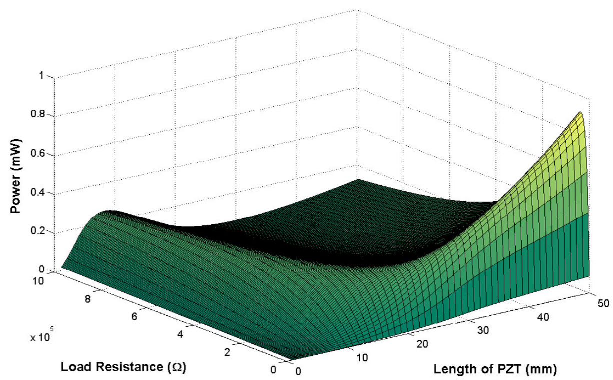


*Fig. 3: Trend in power output while modifying the piezoelectric length and the load resistance.*

**Effect of Position**

The position of the PZT layer is the next parameter to be examined. The width and thickness remain as before however the length is now fixed at 25 mm. The PZT layer is initially clamped along with the substrate layer and then moved to various locations on the beam.

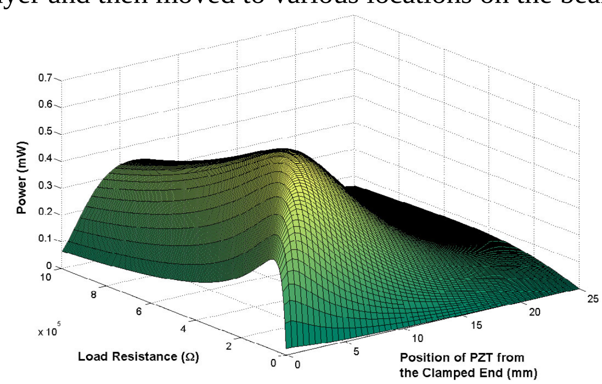


*Fig. 4: Trend in power output while modifying the piezoelectric position and the load resistance.*

Fig. 4 shows that, for a load resistance of 196 kΩ, the power can be increased 3 fold by attaching the PZT layer 4 mm from the clamped end rather than at the clamped end. Naturally, the strain experienced by the PZT layer reduces as it is moved towards the beam's free end. However, placing the layer only a small distance from the clamped end favours power generation since the stiffness of the structure is reduced allowing greater bending.

This can be expanded by examining the effect of increasing the length of the PZT layer from the free end. Fig. 5 shows the results obtained, and they indicate, for a load resistance of 121 kΩ, the power can be increased from 0.457 mW to 1.26 mW when the PZT layer is moved 2.5 mm from the clamped end.

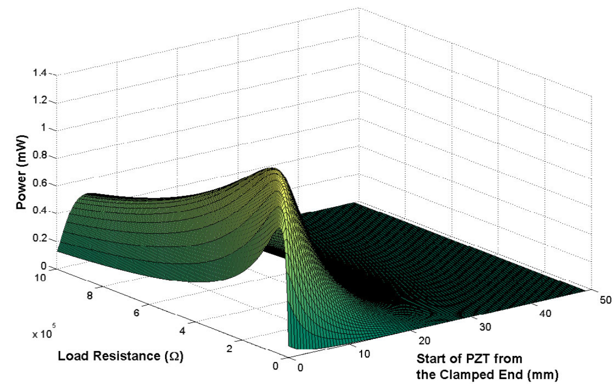


*Fig. 5: Trend in power output while altering the piezoelectric start position, thereby changing its length, and the load resistance.*

**Effect of Width**

Lastly, the effect of width is discussed. For this scenario, changes are made to both substrate width and PZT width. The results show over a small resistance it is highly advantageous to use a wider beam. For load resistance of 16 kΩ doubling the beam width from 2 mm to 4 mm increases the power from 0.172 mW to 0.536 mW. On the other hand, if the load resistance is large, for example 501 kΩ, the power only increases from 0.196 mW to 0.230 mW. In this case it is better to use multiple narrower beams rather than one wide beam.

**Changing Multiple Parameters**

One issue with altering geometrical parameters is that the natural frequency of the system also changes [9]. However, this problem can be overcome by altering multiple parameters simultaneously. When the piezoelectric layer is moved away from the clamped end, the stiffness of the structure reduces, causing a reduction in the natural frequency. This can be counterbalanced by increasing the thickness of the piezoelectric layer thus maintaining a constant natural frequency. The natural frequency of the system is 413.9 Hz when the piezoelectric layer covers the entire beam length. If the layer is moved 0.25 mm from the

*Table 2: Results from a parameter study in which the natural frequency of the system remains constant - 413.9 Hz*

| | **Power obtained when the PZT covers the entire substrate layer (mW)** | | | | | | |
| --- | --- | --- | --- | --- | --- | --- | --- |
| | *10kΩ* | *25kΩ* | *50kΩ* | *75kΩ* | *100kΩ* | *500kΩ* | *1MΩ* |
| | 0.582 | 0.866 | 0.899 | 0.819 | 0.734 | 0.239 | 0.128 |
| **Parameter change** | **Optimisation for maximum power over various resistance loads** | | | | | | |
| | *10kΩ* | *25kΩ* | *50kΩ* | *75kΩ* | *100kΩ* | *500kΩ* | *1MΩ* |
| *Start position of PZT layer from the clamped end (mm)* | 0 | 0.25 | 0.5 | 0.58 | 0.63 | 0.65 | 0.65 |
| *Length of PZT layer (mm)* | 50 | 49.75 | 49.5 | 49.42 | 49.37 | 49.35 | 49.35 |
| *Thickness of PZT layer (mm)* | 1.02 | 1.16 | 1.44 | 1.61 | 1.81 | 2.02 | 2.02 |
| **Power generated (mW)** | 0.582 | 0.886 | 1.08 | 1.17 | 1.23 | 0.882 | 0.538 |

clamped end, so its length is 49.75 mm, then the thickness needs increasing to 1.16 mm to maintain the natural frequency. Table 2 shows an optimisation of piezoelectric position, length and thickness while maintaining a constant natural frequency. For the majority of load resistances it is advantageous to slightly alter parameters in-order to maximise the power output. Taking a unimorph with a $50\times5\times0.7$ mm substrate layer covered by a $50\times5\times1.02$ mm piezoelectric layer the power generated over a 100 kΩ resistor, with an input acceleration of 1 g, is 0.734 mW. By attaching a $49.37\times5\times1.81$ mm piezoelectric layer 0.63 mm from the clamped end, 1.23 mW can be generated – equating to a 67.6 % increase.

## CONCLUSION AND FUTURE WORK

An analytical model with the capability to accurately predict the power generated, by a cantilevered piezoelectric energy harvester, while altering a number of geometrical parameters has been developed. One possible optimisation was performed and preliminary results show that an increase in power can be achieved, over 200 % in some cases, by solely making changes to the piezoelectric layer while maintaining the targeted natural frequency.

The model is currently being validated through experimental work and once this is complete the effects of further parameters will be investigated, including the consequences of adding a tip mass. The results presented have been obtained for the power generated over a load resistor. Since power levels obtained from energy harvesters are rather low and periodic, a storage capacitor is usually included in the circuitry. Future studies will be carried out on a more realistic storage circuit.

## REFERENCES


[1] Cook-Chennault K A, Thambi N, Sastry A M 2008 Powering MEMS portable devices - a review of non-regenerative and regenerative power supply systems with special emphasis on piezoelectric energy harvesting systems *Smart Mater. Struct.* **17** 043001 (33 pages)

[2] Glynne-Jones P, Tudor M J, Beeby S P, White N M 2004 An electromagnetic, vibration-powered generator for intelligent sensor systems *Sens. Actuators, A* **110** 344-349

[3] Chiu Y, Kuo C T, Chu Y S 2007 MEMS design and fabrication of an electrostatic vibration-to-electricity energy converter *Microsyst. Technol.* **13** 1663-1669

[4] Erturk A, Renno J M, Inman D J 2009 Modeling of piezoelectric energy harvesting from an L-shaped beam-mass structure with an application to UAVs *J. Intell. Mater. Syst. Struct.* **20** 529-544

[5] Soliman M S M, Abdel-Rahman E M, El-Saadany E F, Mansour R R 2008 A wideband vibration-based energy harvester *J. Micromech. Microeng.* **18** 115021 (11 pages)

[6] Eggborn T 2003 *Analytical Models to Predict Power Harvesting with Piezoelectric Materials* (Virginia Tech, U.S.A, Thesis - MEng)

[7] Liao Y, Sodano H A 2008 Model of a single mode energy harvester and properties for optimal power generation *Smart Mater. Struct.* **17** 065026 (14 pages)

[8] Erturk A, Inman D J 2008 A distribution parameter electromechanical model for cantilever piezoelectric energy harvesters *J. Vib. Acoust.* **130** 041002 (15 pages)

[9] Shen D, Ajitsaria J, Choe S Y, Kim D J 2005 The optimal design and analysis of piezoelectric cantilever beams for power generation devices *Materials and Devices for Smart Systems II 2005 (Mater. Res. Soc. Symp. Proc. 888, Warrendale, PA, 2006)* V03-04



*Contact author: Mr Rupesh Patel, Tel: +44 (0)115 8467683, Fax: +44 (0)115 9513800, E-mail: eaxrp@nottingham.ac.uk