PARAMETRIC STUDY OF ZIGZAG MICRO-STRUCTURES FOR VIBRATIONAL ENERGY HARVESTING

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Abstract: The natural frequency drop achieved using the zigzag structure is characterized using a dimensionless vibration analysis. The zigzag structure was proposed to reduce the natural frequencies of the MEMS vibrational energy harvesters to the typical ambient vibrations frequency range. A dimensional analysis is performed first to identify that six dimensionless parameters fully describe the problem. The governing equations and the boundary conditions are nondimensionalized and the 6 dimensionless parameters are identified in the process. The dimensionless vibration problem is numerously solved to result in the dimensionless curves describing the variation of the natural frequency due to the changes in each of the remaining five parameters. These curves can be a great design tool for future researchers since they can just read the values of natural frequencies for their specific design, without the need to perform the vibrational analysis. The bending energy ratio parameter is defined next to characterize if the vibrations are dominantly torsional or bending. Since we are mostly interested in bending vibrations for energy harvesting, the last analysis is a key factor in designing zigzag MEMS energy harvesters.

Keywords: Energy harvesting, MEMS, Piezoelectric, low frequency

INTRODUCTION

The MEMS vibrational energy harvesters can be a great step towards realization of mobile autonomous sensor nodes. The advances in microelectronics and the MEMS technology have resulted in small low power sensors. It remains to develop a small size energy harvester that can lift the necessity to change the batteries of these sensor nodes such that the sensors can be placed at far to reach places with minimal maintenance requirement. If we want to harvest energy from the ambient vibrations, we should make sure that the resonant frequency of the harvesters is close to the frequency of the ambient excitations. Many of the reported MEMS energy harvesters have extremely high natural frequencies, which are mostly in the kHz range [1-7]. The typical ambient vibrations frequency range is 0.1 to 100 Hz. The zigzag geometry was developed by the authors [8] to help reduce the fundamental natural frequency. The analytical study of the vibrations was conducted and the geometry was proven to be effective in reducing the frequencies. In this study the problem is put in non-dimensional form to facilitate the designing of harvesters by eliminating the need to perform the cumbersome calculations to derive the frequencies. Finally it has been identified how much bending is contributing in the deflections of the structure. This helps identify if there are enough normal stress in the structure to generate power, or more complicated electrodes should be implemented.

THE ZIGZAG STRUCTURE

The main part of proposed piezoelectric energy harvesting device is a flat zigzag spring illustrated in Fig. 1. The thin spring is fixed at one end and forms a cantilever structure. The plane that the zigzag structure lies in is called the main plane of the zigzag structure. The structure can deflect out of the main plane. The structure can be modeled as a few straight lateral beams, with rectangular cross sections, placed next to each other on the main plane. Each beam is connected to its neighbor beams at its ends. Each of the beams can bend out of the main plane and can twist. The portions of the structure which connects the elements are very small and can be modeled as rigid massless links. The torsion of each of the beams causes the next beam to move out of the main plane. The amount of relative motion is the torsion angle times the rigid arm length.

Each of the lateral beams is a uniform composite beam composed of a piezoelectric layer bonded to the substructure layer (this forms a Unimorph). The substructure can be made of Silicon Oxinytride which results is a residual stress free microstructure.

When the beams are deflected some strain is generated in the piezoelectric layer which generates electrical energy.

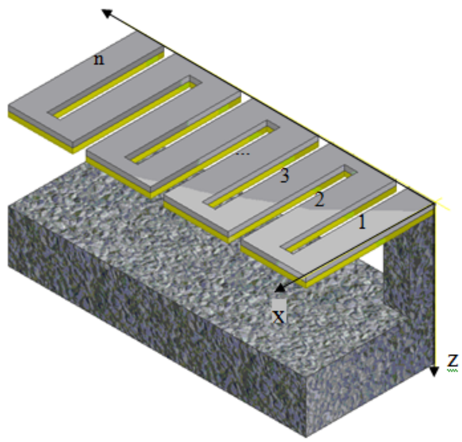

Fig. 1: the zigzag structure

DIMENSIONAL ANALYSIS

The structure goes though coupled bending-torsional vibration. The governing differential equations for each member of zigzag structure is [8]:

$$c^2 \frac{\partial^4 w_i}{\partial x^4} + \frac{\partial^2 w_i}{\partial t^2} = 0 \,, c = \sqrt{\frac{YI}{\rho A}} \qquad (1)$$

$$g^2 \frac{\partial^2 \beta_i}{\partial x^2} = \frac{\partial^2 \beta_i}{\partial t^2}, g = \sqrt{\frac{GJ}{I_p}} \qquad (2)$$

where w is out of plane deflection, β is the twist angle, YI is the bending stiffness, GJ is the torsional stiffness, ρA is mass per unit length and I_p is the mass axial moment of inertia per unit length. The clamped condition at the base and the free condition at the tip result six essential and natural boundary conditions:

$$w_1(0,t) = 0, \frac{\partial w_1(0,t)}{\partial x} = 0, \beta_1(0,t) = 0 \qquad (3)$$

$$\frac{\partial^2 w_n(x_{end},t)}{\partial x^2} = 0, \frac{\partial^3 w_n(x_{end},t)}{\partial x^3} = \mp \frac{m_{tip}}{EI} \omega^2 w_n(x_{end},t), \frac{\partial \beta_n(x_{end},t)}{\partial x} = 0 \qquad (4)$$

In the above equations x_{end} is the x-coordinate of the free end of the structure. The continuity and the equilibrium conditions written for all member-joints give the rest of the equations needed to solve the problem:

$$w_i(x^*,t) = d \times \beta_{i-1}(x^*,t) + w_{i-1}(x^*,t)\,, \quad \frac{\partial w_i(x^*,t)}{\partial x} = \partial w_{i-1}(x^*,t)/\partial x, \beta_{i-1}(x^*) = \beta_i(x^*) \qquad (5)$$

$$\frac{\partial^2 w_i(x^*,t)}{\partial x^2} = \frac{-\partial^2 w_{i-1}(x^*,t)}{\partial x^2}, \frac{\partial^3 w_{i-1}(x^*,t)}{\partial x^3} = -\frac{\partial^3 w_i(x^*,t)}{\partial x^3},$$

$$k \frac{\partial \beta_{i-1}(x^*,t)}{\partial x} = -k \frac{\partial \beta_i(x^*,t)}{\partial x} - \frac{\partial^3 w_i(x^*,t)}{\partial x^3} \times d$$

where the dimensionless parameter k is defined as $k = GJ/YI$.

It can be seen from the above equation that the natural frequencies of the zigzag structure are functions of the following parameters:

$$\omega_n = \omega_n(c, g, L, \frac{m_{tip}}{EI}, d, k, n) \qquad (6)$$

The dimensions of the parameters are L^2T^{-1}, LT^{-1}, L, LT^2, L,1 and 1 accordingly and the dimension of the natural frequencies is T^{-1}. Since there are 8 variables described by 2 dimensions a dimensional analysis [9] suggests restatement of the physics as a relation between $8 - 2 = 6$ dimensionless variables.

NONDIMENSIONALIZATION

Using separation of variables to convert Eqs. (1) and (2) to ODE's results

$$W_i^{(4)} - \frac{\omega_n^2}{c^2} W_i = 0 \,, B_i'' + \left(\frac{\omega_n}{g}\right)^2 B_i = 0 \qquad (7)$$

To put the above equations into the dimensionless form the following dimensionless parameters are introduced:

$$y = x/L \,, v_i = W_i(x)/L \qquad (8)$$

Substituting for W from Eq. (8) in Eq. (7) and applying the chain rule results

$$\frac{d^4 v_i(y)}{dy^4} - \lambda v_i(y) = 0 \qquad (9)$$

$$\frac{d^2 B_i(y)}{dy^2} + \lambda\, \gamma^2\, B_i(y) = 0 \qquad (10)$$

In the above governing equations for bending and torsion of each member, the dimensionless parameters λ and γ are defined as:

$$\lambda = \frac{\omega_n^2 L^4}{c^2}, \gamma = \frac{c}{gL} = \frac{\sqrt{\frac{YI}{\rho A}}}{L\sqrt{\frac{GJ}{I_p}}} \qquad (11)$$

So far we have derived two of the 6 dimensionless variables; λ is the dimensionless variable related to the natural frequency and γ is the torsional dimensionless variable.

The next variable is derived by nondimensionalizing the tip mass natural boundary condition (Eq. (4-middle)):

$$\frac{d^3 v_n(y_{end})}{dy^3} = \mp \widehat{M} \lambda\, v_n(y_{end}) \qquad (12)$$

where the third dimensionless variable $\widehat{M}$ is defined as $\widehat{M} = m_{tip}/\rho AL$ and is the tip mass divided by the mass of a member.

The last new dimensionless variable is revealed when nondimensionalizing the continuity condition. The following equation relates the displacement of two consecutive members at their point of connection (Eq. (5)):

$$v_i(y^*) = D\, B_{i-1}(y^*) + v_{i-1}(y^*), D = \frac{d}{L} \qquad (13)$$

The four introduced dimensionless parameters plus the dimensionless variables k and n form the six dimensionless variables that fully describe the problem. The natural frequency can therefore be expressed as a function of the rest of the variables:

$$\lambda_n = \lambda_n(n, D, \widehat{M}, k, \gamma) \quad (14)$$

To find the natural frequencies we follow an approach similar to [8]. We consider the governing equations for the vibration of each of the members (Eqs.9 and 10) and write the general solution in exponential form:

$$v_i(y) = \sum_{j=1}^{4} A_{ij}\, e^{s_{ij}y}, s_{ij} = \pm\sqrt[4]{\lambda}, \pm i\sqrt[4]{\lambda}\,,\ j = 1,2,\dots,4 \quad (15)$$

$$B_i(y) = \sum_{j=5}^{6} A_{ij}\, e^{s_{ij}y}, s_{ij} = \pm i\gamma\sqrt{\lambda} \quad (16)$$

The unknown coefficients A_{ij} are calculated by observing the boundary conditions at the clamped and free ends and also by preserving the equilibrium and continuity conditions at the connection points of consecutive members. Two of these condition have already been discussed in Eqs. 12 and 13. The rest of these conditions are similar to those expressed in [8] and are not elaborated for brevity. Since all these $6 \times n$ conditions involve the coefficients A_{ij}, we can put all of them in a matrix relation:

$$[\boldsymbol{M}]_{6n\times 6n}[A_{11}, \dots, A_{16}, A_{21}, \dots, A_{26}, \dots, A_{n1}, A_{n6}]^T = \boldsymbol{0}_{6n\times 1} \quad (17)$$

The determinant of the matrix $[M]$ depends on the exponents S_{ij} which in turn are functions of λ. The natural frequency parameters λ_n are those specific values which make the *M*-matrix singular and therefore allow nontrivial responses.

RESULTS

The natural frequency dimensionless parameters are functions of five parameters: n, D, $\widehat{M}$, k and γ. The effects of individual parameters on the frequency are discussed in this section. The illustrations (Figs. 2-6) have been arranged to make the results easy to use.

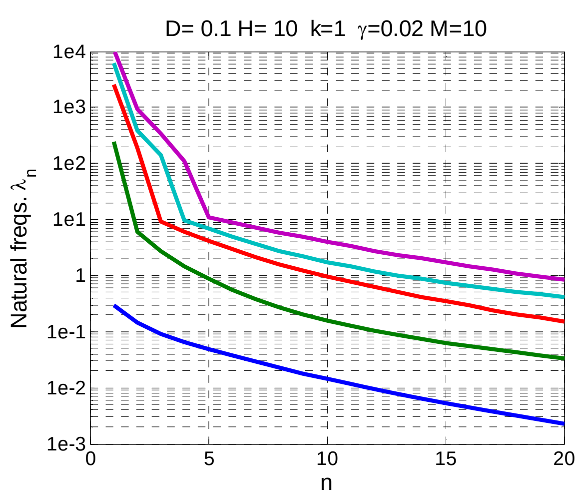

Fig. 2: effect of n on first five natural frequencies

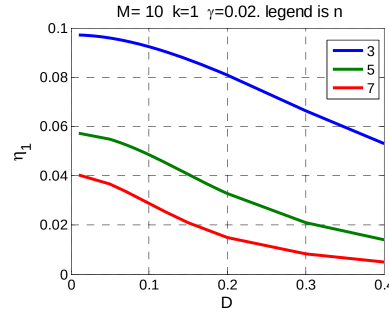

Fig. 3: The effect of D on the first natural frequency.

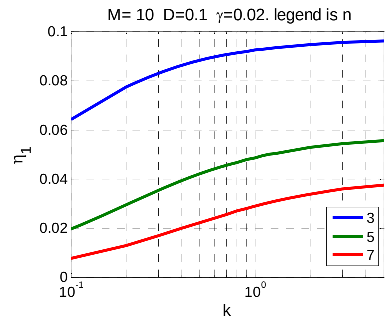

Fig. 4: The effect of k on the first natural frequency.

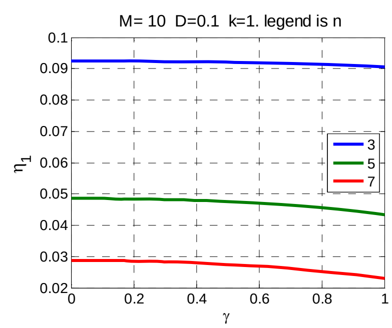

Fig. 5: The relationship between γ and the frequency.

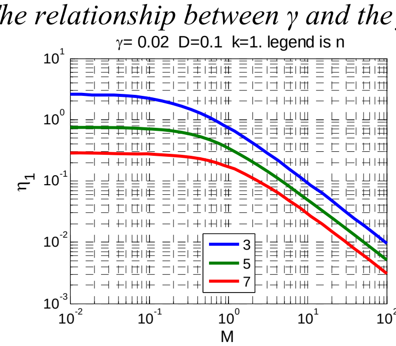

Fig. 6: The effect of $\widehat{\boldsymbol{M}}$ *on the first natural frequency.*

- **The amount of bending deformation**

Since there is only one pair of electrodes placed at the top and bottom surfaces of the Piezo-layer, energy can only be harvested from the bending vibrations of the Piezo structure. The bending index is defined as the ratio between the bending potential energy over the overall elastic potential energy:

$$BTR = \frac{\frac{1}{2}\int_{all\ members} EI\left(v''(y)\right)^2 dy}{\frac{1}{2}\int_{all\ members} EI\left(v''(y)\right)^2 dy + \frac{1}{2}\int_{all\ members} GJ\left(B'(y)\right)^2 dy}$$

The value of the Bending/Torsion Ratio (*BTR)* is one when the members are only bent and are not at all twisted and it will vanish when the deflection of the structure is solely due to torsion of the structure. Fig. 7 illustrates that the first and third modes become more torsional as the number of members increases. We are mostly interested in energy harvesting from the fundamental mode since it corresponds to the lowest natural frequency. Fig. 7 also illustrates that the tip mass can be utilized to improve the bending deformation of the first mode. The lateral distance between the members (characterized by *D*) also affects the available bending energy in the structure. As illustrated in Fig. 8 the lateral distance has a prominent effect on the *BTR*. The least the distance the more bending occurs in the fundamental mode of the structure. This suggests using narrow members to reduce the distance between the centerlines of two consequent members.

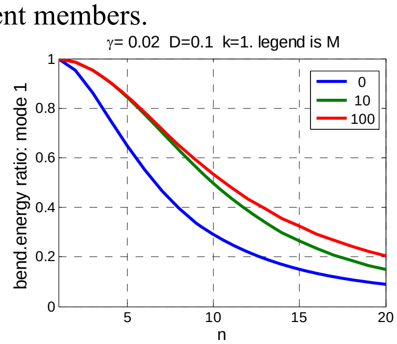

Fig. 7: Effect of tip mass on BTR

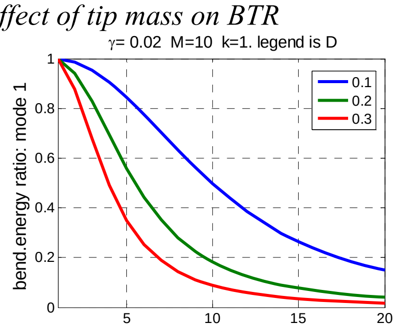

Fig. 8: Effect of members lateral distance on BTR

CONCLUSION

The dimensionless analysis of the zigzag structure was performed. The resulted dimensionless curves would facilitate design and performance analysis of zigzag MEMS energy harvesters. The dimensionless bending study of the structure helps making sure that the deflections can be easily converted to electric energy. It also identifies if there is a need for more complicated electrode configurations.

ACKNOWLEDGEMENT

This work was performed under the support of the US Department of Commerce, National Institute of Standards and Technology, Technology Innovation Program, Cooperative Agreement Number 70NANB9H9007".

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