

STUDY OF PASSIVE SELF-TUNING RESONATOR FOR BROADBAND POWER HARVESTING

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Abstract: The concept of passive self-tuning resonator for broadband power harvesting is described. A self-tuning resonant power harvester adapts its internal degrees of freedom to stay in resonance with the drive in a wide range of frequencies. This will enable energy harvesting from a variety of vibration sources that have different and sometimes variable resonant frequency. A mechanical resonator with a mass that is free to move along its length is analyzed, and the potential energy analysis of the system confirms that it adapts its resonant frequency to the drive as the mass moves to find the potential minimum. The parameter requirements for the existence of the self-tuning behavior and the expressions for the tuning range are derived. The experiments designed to demonstrate this effect are described and experimental challenges are discussed.

Keywords: adaptive energy harvesting, passive tuning, resonant power harvesting, vibration scavenging

INTRODUCTION

Most vibration power harvesters rely on piezoelectric, electrostatic, or electromagnetic vibration-to-electricity conversion techniques [1]. All of these deliver maximum power when operating at resonance with the environmental vibrations. However, real-life vibration sources have different and sometimes variable resonant frequency, so it is essential to be able to tune the resonant frequency of the power harvester in order to harvest maximum possible power.

Several tuning schemes (see Table 1) have been proposed that rely on lowering of the quality factor of the resonator, collective mode operation, active tuning, or frequency up-conversion [2]. Most of these techniques result in less harvested power or require larger device volume. Frequency up-conversion does not have these disadvantages, but no reliable implementations have been demonstrated so far.

This paper explores an alternative approach to overcome the limitation of single-resonance harvesters by adding to the system one or more degrees of freedom that self-adjust so that the entire system becomes resonant with the drive. This self-adaptive property of resonant systems with adjustable degrees of freedom allows for resonant response and thus efficient energy harvesting for a broad range of excitation frequencies without sacrificing the system's quality factor. This article describes the concept of a passive self-tuning resonator for broadband power harvesting and reports on the theory, design, and initial testing of several model implementations of this concept.

The proposed self-tuning structure can serve as a main vibrating element in a variety of vibration energy harvesters. Piezoelectric, magnetic, or other materials necessary for mechanical-to-electrical transduction can be used as a structural framework or added on as thin films.

Table 1: Comparison of known tuning techniques for resonant vibration power harvesting.

Tuning method	Advantages	Disadvantages	References
Increase the damping to broaden the frequency band at resonance	Simple implementation	Reduces harvested power	
Wideband response from an array of elements with different resonant frequencies	Effective over a large frequency range	Large percentage of elements are related	Shahruz 2006, Sari 2007
Active tuning of the spring constant by adaptive electrical control schemes	Operation at resonance ensured by adaptive circuit	Power for active tuning exceeded the harvested power	Ottman 2002, Roundy 2005, Chen 2007
Initial active tuning of the spring constant by mechanical means	No power consumption after the initial tuning	Initial active tuning has to be performed on every device	Leland 2006, Challa 2008, Morris 2008
Up-conversion of low vibration frequency to high resonant frequency of the resonant harvester	Small devices used to harvest low-freq environmental vibrations	Mechanical up-conversion schemes suffer from wear	Rastegar 2006, Tieck 2006, Kulah 2008

THEORY

Self-tuning behavior of nonlinear systems has been observed in vibrating soap films [3] and vibrating smectic liquid crystal films [4]. The study of the self-tuning dynamics in a string loaded with a bead [5] showed that the mass m at the position x_m on the string feels an effective potential

$$V[x_m] = -\pi\omega_d y(x_m)^2 / 4 \quad (1)$$

as the string is driven by a periodic force at the driving frequency ω_d . Since the mass is confined to the string, its transverse displacement $y(x_m)$ is the same as that of the string at the longitudinal position x_m . The equilibrium positions of the bead calculated in [5] agreed with the experimental observations.

Here a generalized analysis that takes into account mechanical dissipation in the system is presented. Let's consider a resonator that is loaded with mass m , which is free to move along its length. For simplicity and in contrast to [5], no change in the spatial mode shapes due to the mass loading is assumed. The amplitude of a damped resonator in response to a periodic drive at frequency ω_d is

$$y(x_m) = \frac{A(x_m)\omega(x_m)^2 / Q}{\sqrt{[\omega(x_m)^2 - \omega_d^2]^2 + [\omega(x_m)\omega_d / Q]^2}} \quad (2)$$

$A(x_m)$ is the amplitude of the response normalized to the maximum mode displacement, Q is the quality factor characterizing the mechanical dissipation in the system, and $\omega(x_m)$ is the resonant frequency of the system.

In this case, however, $\omega(x_m)$ depends on the position of the sliding mass. As the maximum potential energy and the maximum kinetic energy of the resonator have to be the same, the kinetic energy of the resonator without the mass must be equal to the combined kinetic energy of the resonator and the mass. Therefore, as the mass is added to the system, the resonant frequency of the system decreases. This gives the resonant frequency ω of the loaded resonator as a function of the unloaded resonant frequency ω_0 and of the ratio of the moveable mass m to the resonator mass M , $\mu = m/M$:

$$\omega(x_m)^2 = \frac{\omega_0^2}{1 + 2\mu A(x_m)^2} \quad (3)$$

This expression for $\omega(x_m)$ is then used in (2) to account for the changing resonance frequency of the

loaded system.

In order to prove the existence of the adaptive behavior, it is necessary to show that the response amplitude, A , is indeed dependent on the drive frequency, ω_d . Since the mass is free to adjust its position, it will do so to find the minimum of the effective potential (1). Eq. (2) can, therefore, be substituted into (1) with $\omega(x_m)$ given by (3). The condition $dV/dx_m = 0$ is solved for A to obtain the expression that gives the response amplitude A_m minimizing the effective potential (1):

$$A_m^2 = \frac{1}{2\mu} \sqrt{\left(\frac{\omega_0}{\omega_d}\right)^2 \frac{1}{Q^2} + \left(\left(\frac{\omega_0}{\omega_d}\right)^2 - 1\right)^2} \quad (4)$$

The power harvested by a resonant power harvester at constant acceleration a is [1]

$$P = \frac{a^2 (m + M) \omega_d^2 \omega(x_m)}{2Q([\omega(x_m)^2 - \omega_d^2]^2 + [\omega(x_m)\omega_d / Q]^2)} \quad (5)$$

Here, a common damping matching condition is assumed, $Q_e = Q_m = Q$, i.e. the electrical damping is matched to mechanical damping to maximize the harvested power output at resonance.

Fig. 1(b) shows the power generated by a harvester based on a resonator with a free mass ($Q=20$, $\mu=2$), which adjusts its position to minimize the effective potential, as a function of the drive frequency, ω_d . For comparison, the dotted curve is the power harvested from an unloaded resonator. As the free mass seeks to minimize its energy, the resonator system tunes itself to resonance, which allows maximum power to be harvested in a broad band of frequencies. The plot of calculated resonant frequency of the free mass resonator system in Fig. 1(b) confirms that the system indeed adjusts its resonant frequency, ω , to be equal to that of the drive.

For the string mode shape, $y = \sin[\pi(x_m + L/2)/L]$, the longitudinal position of the moving mass as a function of the drive frequency is shown in Fig. 1(c). The mass can move to the center of the string to achieve lowest possible frequency or to the ends to ensure the highest possible frequency.

In general, the tuning range τ for a mass-resonator system described by (3) is given by

$$\tau = \frac{\omega_0 - \omega_{\min}}{\omega_0} = 1 - \frac{1}{\sqrt{1 + 2\mu}} \quad (6)$$

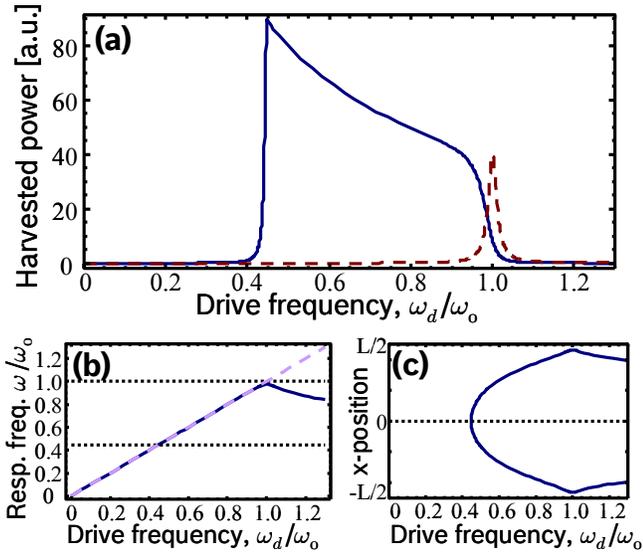


Fig. 1: (a) Calculated harvested power from a harvester based on a resonator with adjustable mass (solid blue) vs. a resonator without a mass (dashed). (b) Calculated resonant frequency of the resonator-mass system (solid blue). Dashed purple line marks $\omega = \omega_d$. (c) Mass position along a string as the drive frequency is swept.

The effective quality factor of the self-tuning system combines the mechanical quality factor Q and the tuning range:

$$Q_{eff} = \left(\frac{1}{Q} + 1 - \frac{1}{\sqrt{1+2\mu}} \right)^{-1} \quad (7)$$

Without any mass, $Q_{eff} = Q$. For large added mass, $\mu \gg 1$, Q_{eff} is dominated by the tuning behavior and converges to 1. Because the increase in the response range does not require the sacrifice in the peak amplitude, this effect differs significantly from increasing dissipation in order to gain a broader range of response frequencies (first row of Table 1).

Eq. (4) indicates that the tuning effect either does not exist or has a limited range if the quality factor of the system is too low or the moving mass is too small. The minimum mechanical Q required for the tuning effect depends on ω_d/ω_0 and μ :

$$Q > \left[4\mu^2 \left(\frac{\omega_d}{\omega_0} \right)^2 - \left(\frac{\omega_0}{\omega_d} - 1 \right)^2 \right]^{-1/2} \quad (8)$$

Alternatively, the minimum mass ratio required for tunability of the system can be defined that depends on ω_d/ω_0 and Q .

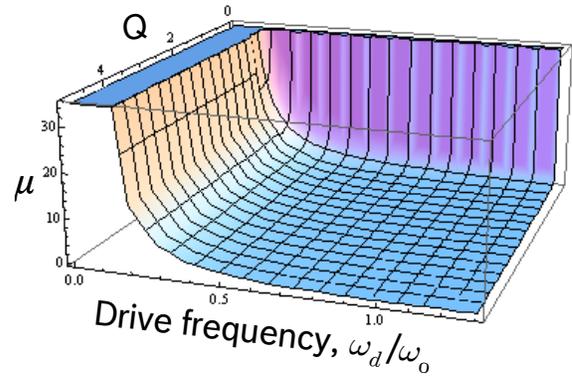


Fig. 2: Threshold surface for realization of self-tuning behavior in $(Q, \mu, \omega_d/\omega_0)$ -parameter space.

Visually, self-tuning behavior occurs if the operating parameters of the system $(Q, \mu, \omega_d/\omega_0)$ lie above the surface shown in Fig. 2.

The resonator structure with free masses can be engineered to have higher-order vibrational modes spaced close enough together so that the tuning range covers the entire frequency band between them. A combination of self-tuning ability and mode engineering should make it possible to design a power harvester with a continuous resonance spectrum over a very large range of frequencies.

EXPERIMENTAL

Several experiments were performed to study self-tuning effect in the model systems that could later be used as a basis for functional power harvesters. Free mass is incorporated into the harvester structure as a slider confined to the resonator. Systems composed of a bead on a string [5] and of a pellet in a hollow channel (Fig. 3(a)) were investigated.

A sliding mass inside a hollow channel promises to be a more practical implementation for a self-tuning energy harvester as it can be built using microfluidic fabrication technology and the channels can be filled with free particles and lubricating fluid to reduce friction. Fig. 3(a) illustrates a model setup to test the behavior of a stainless steel pellet free to move inside a hollow plastic tube cantilever. One end of the cantilever is clamped to the shaker that provides the drive. The motion of the cantilever is detected using a laser vibrometer (Polytec OFV 302/3001) and a silicon mirror attached to the unclamped end of the tube. Resonant frequency of the unloaded resonator is 87.7 Hz with $Q=6.5$ and its response is shown as a thin gray curve in Fig. 3(b)). As the drive frequency was swept up slowly towards the resonant frequency of the cantilever, the pellet started to move towards the unclamped end of the

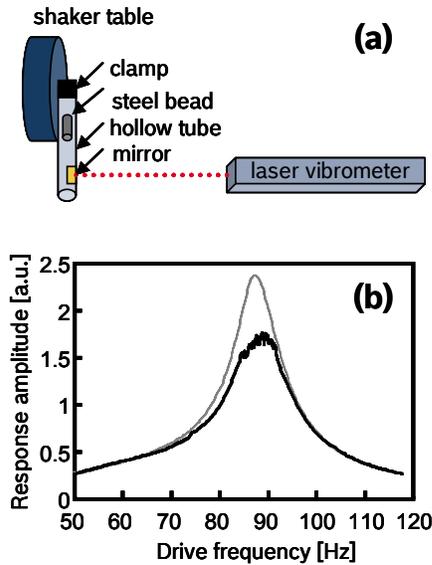


Fig. 3: (a) Schematic of an experimental setup to study self-tuning behavior. (b) Measured resonance curves for a hollow cantilever resonator with (thick black curve) and without (thin gray curve) a moveable mass inside.

tube, where the resonant frequency of the system would be the lowest. However, before the tuning behavior could occur, the pellet found a minimum of the gravitational potential in the tube and started to spin in place increasing the dissipation in the system. The measured cantilever response is shown in Fig. 3(b) as a solid black curve. The energy dissipated by the spinning pellet is clearly seen in the difference between the gray and black curves.

Similar experiments were performed on a magnetomotively driven and detected metal string with a tungsten bead whose shape was modified to prevent spinning. However, similar response was observed as the bead was wobbling near and at resonance.

The experiments elucidate several challenges to the practical device design that need to be overcome in order to take advantage of self-tuning behavior: influence of gravity and coupling of the predicted linear motion of the mass to other degrees of freedom, such as spinning and wobbling. These effects modify the effective potential (1) that the free mass experiences and change the self-tuning behavior of the system. These non-idealities require careful study and need to be addressed in future experiments aimed at reducing the self-tuning power harvester to practice.

CONCLUSION

In summary, it has been shown theoretically that a resonator loaded with a free-moving mass adapts its resonant frequency to that of the drive. This effect provides for the resonant response in a broad range of frequencies. A power harvester based on a self-tuning resonator will have a maximum power output over a wide frequency band. If the experimental challenges are overcome and a reliable design is achieved, the self-tuning power harvester will enable energy harvesting for a variety of applications where vibrations occur at variable frequencies or change their frequency during operation due to environmental conditions.

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