FREE PISTON STIRLING ENGINE DESIGN USING SIMILITUDE THEORY

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Abstract: The free piston Stirling engine shows some advantages for microminiature machine design: external combustion, closed cycle as well as the opportunity to use the converse cooling cycle. Based on non-dimensional analysis, the effect of the miniaturization for the design of such micro-heat engines can be studied. A similitude approach for Stirling engine is developed. Design charts are then set out so the engine parameters can be easily handled in relation with the working fluid, technological requirements and constraints. Some guidelines are then outlined for Stirling micro machine design.

Keywords: micro heat engine, Stirling, similitude theory, scaling

INTRODUCTION

The effect of the miniaturization of micro heat engines is crucial to asses the performance prospective of a given technology. Based on non-dimensional analysis, turbines have been shown to be the best promising technology for high specific power. Indeed, they scaled as 1/L if the maximal velocity is retained. Nevertheless, their high rotational speeds require specific developments to minimize wear and frictional losses. Moreover, the combustion residence time must be large enough to obtain high efficiency [1-4].

The specific power of the Stirling cycle scales as: $\omega \times p_{mean}$, in which p_{mean} is the mean charge pressure and ω the operating frequency. Based on this criterion, the miniaturization process brings no intrinsic benefit for the Stirling cycle. Yet, it is known for its high efficiency and reliability especially for free piston Stirling engines (FPSE) which appears to be suitable for miniaturization [5]. From a technological point of view the external combustion as well as the closed cycle are favourable advantages of a miniaturized heat engine.

Second order models are effective tools to study the global behaviour of classical Stirling engines [6]. In the case of the FPSE, the complexity takes things up a notch. Indeed, in this case no mechanical linkage fixes the strokes and phase angle for the piston and displacer (Fig. 1). Hence, a global complex dynamic analysis is required to predict the periodic steady operation. However, these approaches can not provide an easy understanding of the main phenomena which influence the global performances. In this frame, an analytical model that underlines the effect of the geometrical and thermal parameters on the performances has been developed [7], but this model relies on set of parameters defined beforehand. As a conclusion, there is a need for a simplified model to design a micro FPSE at a preliminary stage. The similitude theory is an efficient alternative approach to effective prior design and appraisal of the objective performances. It stems on the fundamental equations that describe the thermo-fluidic behaviour of the working fluid within the machine. Organ has proposed a set of similitude parameters dedicated to the Stirling engine [8]. We propose here to extend this work to the study of the FPSE.

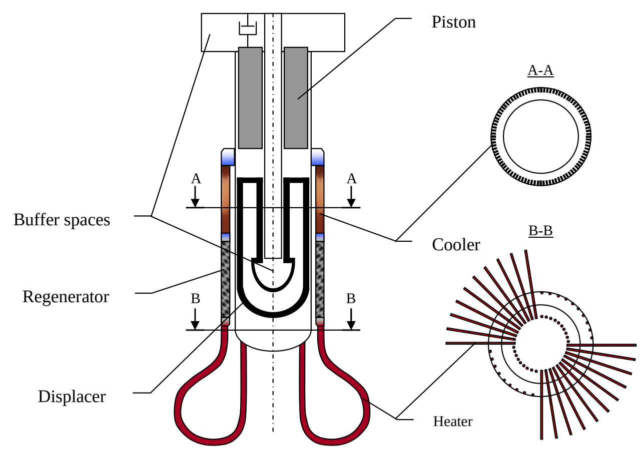

Fig. 1: FPSE schematic architecture.

First, scale analysis brings out the non-dimensional groups (NDG) related to the FPSE. The relevance of the NDG is then supported by a comparative study of ten Stirling engines, four of which are FPSE. Design charts are developed so the engine parameters (e.g. mean pressure, operating frequency, hydraulic radius, length, number of tubes for the heat exchangers, piston and displacer masses, buffer spaces volumes…) can be easily handled in relation with the working fluid. Finally, the effects of the scaling down process are analyzed.

THEORY

Kinematic similarity

From the Schmidt analysis, it is possible to analytically express the instantaneous pressure and the power of a given Stirling machine as:

$$p = \frac{M\,r}{s}\,\frac{1}{1+\beta\cos(\phi-\theta)} \tag{1}$$

$$Power = p_{mean}\,\omega\,\frac{V_E}{2\,\beta}\,\frac{\kappa\,(\tau-1)\sin(\alpha)}{\sqrt{\tau^2+\kappa^2+2\,\kappa\,\tau\cos(\alpha)}} \tag{2}$$

In which β is related to geometric and kinematic parameters. Thus, the similitude process begins with the conservation of the kinematic parameters some of them defined by Fig. 2:

$\kappa = V_{swc} / V_{swe}$ the swept volume ratio

$\tau = T_c / T_e$ the temperature ratio

α the volume phase angle

Moreover, the dead volume ratio with respect to the swept volume is kept constant during the similitude process. Consequently for each heat exchanger and regenerator, the following non dimensional parameter is defined:

$$DG_{\delta x} = \frac{4\,r_{hx}^2\,L_x\,n_{Tx}}{V_{sw}} \tag{3}$$

In which, n_{Tx} is the number of tubes, r_h the hydraulic radius, L_x the length of the considered heat exchanger or regenerator. x→e, x→k and x→r for the heater, cooler and regenerator respectively.

From the expression of the power, it is possible to define the Beale number:

$$N_{Be} = \frac{Power}{\omega\,p_{mean}\,V_{sw}} \tag{4}$$

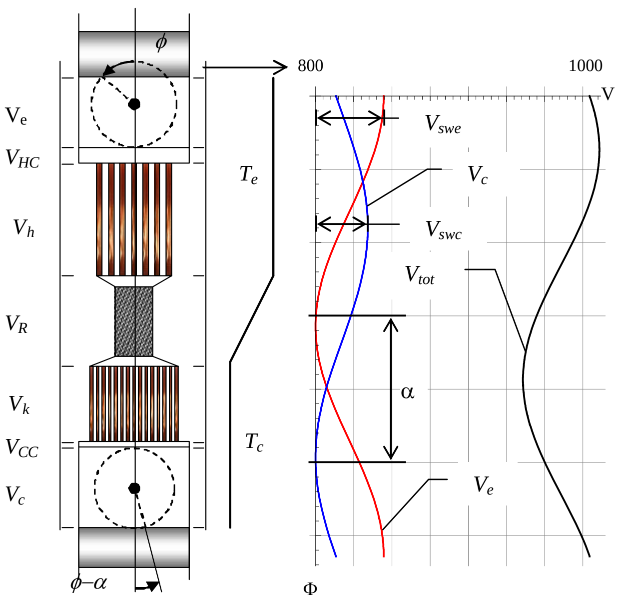

Fig. 2: Generic scheme for the Schmidt analysis.

Energetic similarity

We assume a 1D behaviour of the gas within the heat exchangers which is the usual assumption for Stirling engine modelling. Fundamental equations related to the fluid behaviour are the following:

Continuity equation

$$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial z}\left(\frac{\dot m}{A_{ff}}\right)=0 \tag{5}$$

Momentum equation

$$\frac{\partial}{\partial t}\left(\frac{\dot m}{A_{ff}}\right)+\frac{\partial}{\partial z}\left(\frac{\dot m^2}{\rho A_{ff}^2}\right)=-\frac{\partial p}{\partial z}-\frac{2}{d_h}\frac{f_F\,\dot m\,|\dot m|}{\rho A_{ff}^2} \tag{6}$$

Energy equation for gas

$$A_{ff}\frac{\partial}{\partial t}\left[C_v\,\rho\,T+1/2\left(\frac{\dot m}{\rho A_{ff}}\right)^2\right]+h\,A_L\,(T-T_s) + \frac{\partial}{\partial z}\left[C_p\,\dot m\,T+1/2\left(\frac{\dot m}{\rho A_{ff}}\right)^2\right]=0 \tag{7}$$

The previous equations are then modified in order to get NDG. The reference parameters are T_c, ω, p_{mean} and $L=\sqrt[3]{V_{sw}}$ which are significant for a given engine [8]. From the result of the process, two new groups are defined for each heat exchanger and regenerator:

$$N_{Mx}=\frac{\omega\,L_x}{\sqrt{r\,T_c}} \tag{8}$$

$$DG_{Re}=\frac{p_{mean}\,r_h}{\omega\,\mu\,L_x} \tag{9}$$

Thermal similarity

Another important parameter is the thermal transfer coefficient of the exchangers. Based on the NTU experimental correlations new groups are defined therefore:

$$N_{Tx} == \left(\frac{L_x}{r_{hx}}\right)^{1.2}\left(\frac{p_{mean}}{\omega\,\mu}\right)^{-0.2} \tag{10}$$

For a tubes heat exchangers

$$N_{Tx} == \left(\frac{L_x}{r_{hx}}\right)^{1.5}\left(\frac{p_{mean}}{\omega\,\mu}\right)^{-0.5} \tag{11}$$

For a mesh grid regenerator

Dynamic similarity

Dynamical equilibrium equations for the moving parts are obtained from the Newton's second law. Thus, four more non dimensional groups are derived. They account for the stiffness behaviour of the buffer spring chambers as well as the balance between pressure and inertia effects:

$$N_{bu}=\frac{\gamma A_b}{V_{b0}} \tag{12}$$

$$NF_{pd}=\frac{(A_{dcs}-A_{dhc})\,p_{mean}}{m_d\,\omega^2\,L} \tag{13}$$

$$NF_{pp} = \frac{A_p \, p_{mean}}{m_p \, \omega^2 \, L} \tag{14}$$

It is worthy of note that the essential role of the dissipative effect from the pressure drop is handled by the DG_{Re} group. Indeed, the friction factor is related to the Reynolds number and the geometrical aspect of the exchangers both being kept constant by previous requirements.

DISCUSSION

Analysis of documented engines

In order to support the approach, the NDG have been evaluated for fully documented real engines namely: GPU-3, PD46, V160, MP1002CA, USSP40, 400HP, RE-1000, CTPC. The last two of them are FPSE.

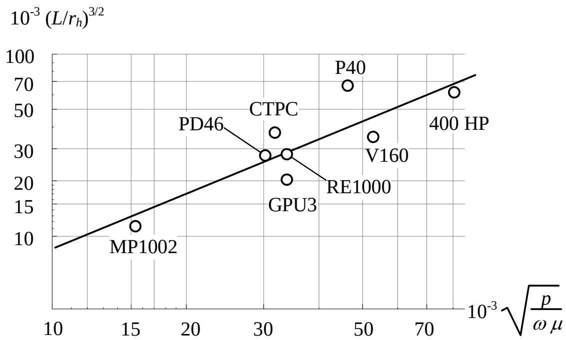

Fig. 3: NTU group for the regenerator.

Figure 3 shows that the chosen NDG *NTx* (see Eq 10) is indicative. For the particular case of FPSE, the dynamic groups NF_{pd} and NF_{pp} (Eq. 13-14) are evaluated with two more FPSE engines: B10 [9] and DFPSE [10].

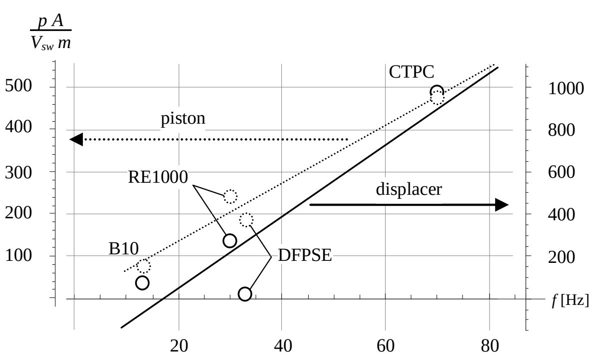

Fig. 4: Dynamic groups for piston and displacer..

Again, it appears that the dynamic characteristics of FPSE can defined with respect to a given non dimensional parameter.

Design charts for scaled engines

We postulate that the swept volume V_{sw} is representative of the Stirling engine size. With the additional previous choices of the extreme temperatures T_e and T_c and required power, it is possible to define the remaining parameters. Based on the set of NDG for the RE-1000 engine, designs charts are drawn.

The charge pressure and operating frequency can thus be easily handled in relation with the working fluid (Fig. 5-6).

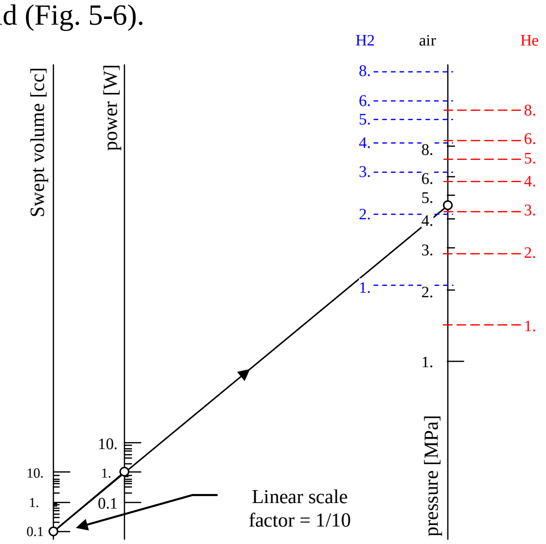

Fig. 5: Example data plot.

Figure 5 shows the evaluation of the mean pressure of the engine with given swept volume and power. As the pressure increases higher power can be produced by the engine. Moreover, the same power can be reached using a lower pressure and a suitable working fluid. These results are in agreement with the classical literature of Stirling engines.

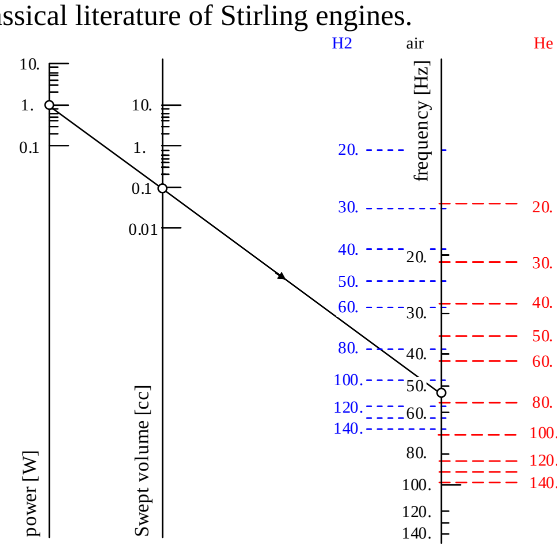

Fig. 6: Example data plot.

Figure 6 shows the evaluation of the operating frequency. Once again, the classical result that a smaller size is balanced by a higher frequency is obtained. The numerical evaluation of the relation can be useful for preliminary design regarding technological constraints. Contrary to a geometrical scaling, it can be seen in Fig. 7 that the smaller the size of the engine, the higher are the relative moving parts masses with respect to the whole mass of the engine.

CONCLUSION

A simple preliminary FPSE design method has been developed. Scaling effect on the Stirling engine parameters can be studied. Moreover, additional constraints such as maximum pressure or power requirements can be added to easily validate a suggested design. The preliminary defined engine parameters can then be used in a refined model to optimize the design.

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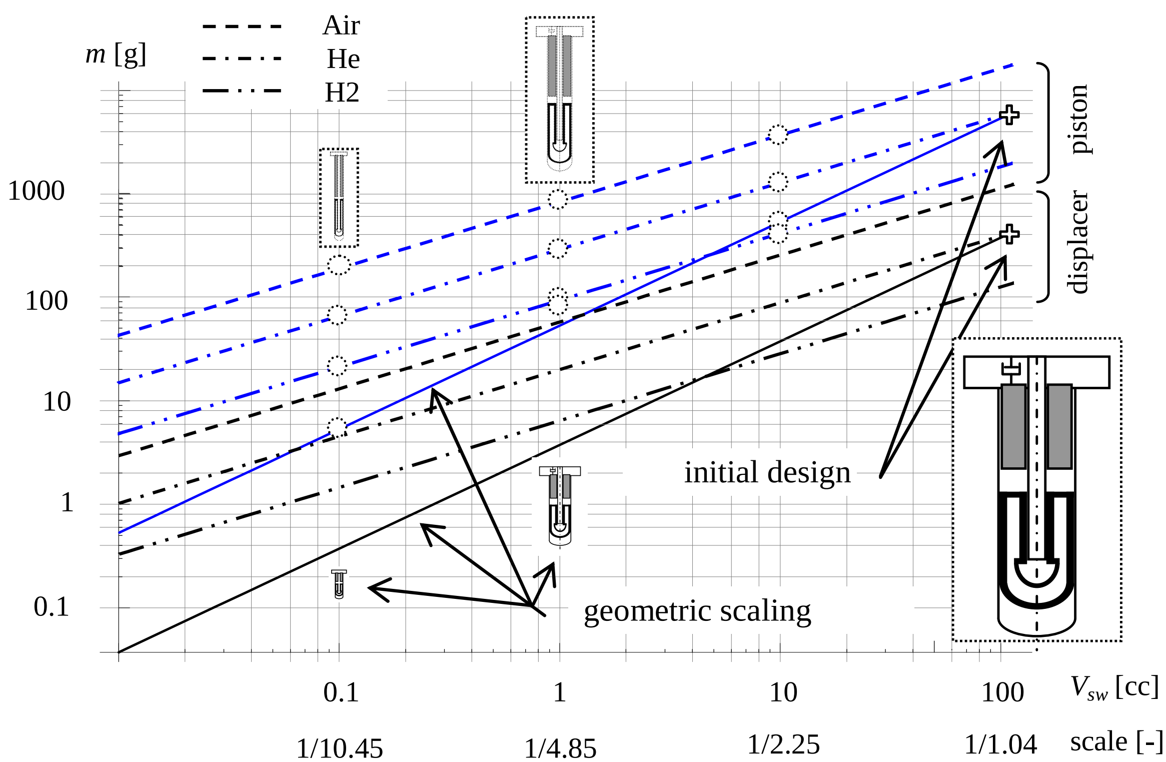

Fig. 7: Example data plot.