

# NONLINEAR MAGNETIC COUPLING OF A PIEZOELECTRIC ENERGY HARVESTING CANTILEVER COMBINED WITH VELOCITY-CONTROLLED SYNCHRONIZED SWITCHING TECHNIQUE

Yu-Yin Chen<sup>1,2</sup>, Dejan Vasic<sup>1</sup>, François Costa<sup>1,3</sup>, Chih-Kung Lee<sup>2,4,5</sup>, Wen-Jong Wu<sup>4\*</sup>

<sup>1</sup>Système et Application des Technologies de l'Information et de l'Energie, UNIVERSud, Ecole Normale Supérieure de Cachan, France

<sup>2</sup>Institute of Applied Mechanics, National Taiwan University, Taiwan

<sup>3</sup>IUFM de Créteil, Université Paris 12, Place du 8 mai 1945, 93000 St Denis, France

<sup>4</sup>Department of Engineering Science and Ocean Engineering, National Taiwan University, Taiwan

<sup>5</sup>Institute for Information Industry, Taipei, Taiwan

\*Wen-Jong Wu: [wjwu@ntu.edu.tw](mailto:wjwu@ntu.edu.tw)

**Abstract:** This study reports the nonlinear magnetic coupling technique combined with the velocity-controlled synchronized switching technique to improve the output power of a piezoelectric energy harvesting cantilever beam. This is a new and interesting thinking to combine two nonlinear techniques. The magnetic coupling makes a nonlinear behavior of the cantilever and this behavior can broaden the available band-width. The nonlinear system combined with parallel SSHI can improve the output power at each frequency and from the simulation result; the synchronized switch technique will not influence the magnetic nonlinear effect. When these two nonlinear techniques are combined together, the advantage will also be combined. The theoretical analysis, equivalent circuit model and preliminary experimental results will be presented.

**Keywords:** energy harvesting. Magnetic coupling, synchronized switching technique

## INTRODUCTION

Scavenging ambient vibration energy with piezoelectric transducers is highly investigated by researchers in past few years. Besides the transducer optimization and design, the interfacing circuit to piezoelectric transducer which can highly improve the output power was also vastly studied [1]. The piezoelectric energy harvesting device has been proposed as a possible alternative energy source to supply energy to some low power consumption electronic devices. The most important application is to combine energy harvesting device with batteries to extend the life time and in a wireless sensor network node to transfer more data over a longer duration [2-4]. It has been known that when the mechanical system is excited at the resonant frequency there will be the largest strain and vibration displacement. and there will be the maximum output power than at non-resonant frequency. In fact, in the real applications, the exciting frequency of ambient vibration source is random and varied within some frequency range [3]. It is impossible to excite and design the energy harvester at only the resonant frequency and keep the system operating always on the maximum point. In the mechanical system, the quality factor is commonly very high and this causes the harvester only has high harvesting power output at single resonant frequency. In order to improve the power efficiency at the non-resonant frequency, a new design in mechanical system to work in wide frequency range is necessary.

It's a new design concept to use magnetic force to control the piezoelectric energy harvesting system. In the paper [5], the resonant frequency (26 Hz) of a

piezoelectric cantilever beam is successfully tuned by transverse magnetic force from 22-32 Hz and enables a continuous output power of 240- 280  $\mu$ W. By applying an axial force, the resonant frequency of a piezoelectric cantilever beam is also successfully tuned [6, 7]. But all above methods are active techniques to tune the resonant frequency of the structure and the mechanical system is still operated within the linear regime. However, the active tuning actuator will never increase the net power output [8]. The power consumed by active tuning is always higher than the harvesting power. The magnetic coupling of a piezoelectric cantilever beam is proposed to enhance harvesting efficiency in [9]. By using fixed magnets, this passive technique makes mechanical system operated within nonlinear regime without any external power to drive the system.

In the other hand, the interfacing circuit using non-linear technique called SSHI (synchronized switching harvesting on inductor) is very successful to boost the output power from piezoelectric energy harvester [10-15]. This approach is derived from a semi-passive technique, synchronized switch damping on inductor (SSDI) [16] technique. Comparing to the standard DC approach, the SSHI can increase around 400% power output from piezoelectric energy harvester in a specific design.

In this study, the nonlinear magnetic coupling technique combined with the velocity-controlled synchronized switching technique is presented to improve the output power of a piezoelectric energy harvesting cantilever beam. Fig 1 shows the schematic structure diagram of this newly designed device. The magnetic coupling makes the behavior of the

cantilever beam works within nonlinear regime. The nonlinear magnetic coupling technique is composed of two magnets. One is mounted on the tip of the beam and the other one is fixed on a stage. Because the two magnets repulse to each other, the system will be a bi-stable system and there will be two possible stable positions shown as Fig 1. When the distance between these two magnets is designed properly, the nonlinear behavior can broaden the available band-width. Piezoelectric patches are designed to be separated into two parts. One is connected to synchronized switching technique circuit and this circuit design can boost the power output of piezoelectric patch to boost power output in broader frequency range. In order to archive synchronized switching at the proper time, the other piezoelectric patch is connected to velocity control circuit. The theoretical analysis, equivalent circuit model, simulation and experimental results will be presented. The simulation and experimental results will show when piezoelectric energy harvesting cantilever beam combined with these two nonlinear techniques (magnetic coupling + SSHI), the available band width can be broadened and the power output of each frequency can be effectively improved.

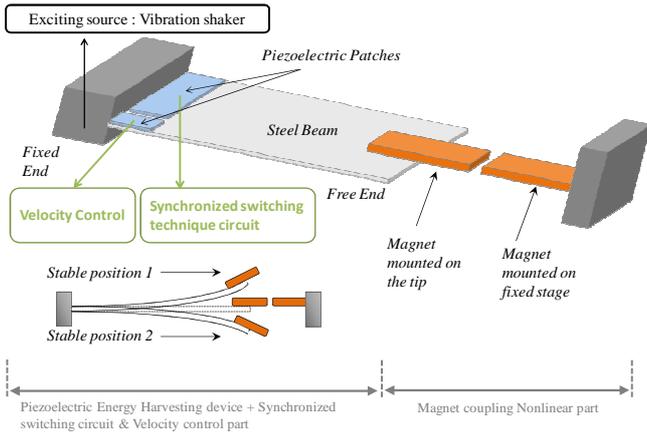


Fig 1: The schematic circuit diagram of magnetic coupling of a piezoelectric cantilever combined with the synchronized switching technique

## THEORETICAL ANALYSIS

The model of the nonlinear magnetic coupling energy harvesting device is composed of two parts. One is for the piezoelectric-element and the other one is for the magnetic force. Fig 2 shows the schematic diagram of nonlinear magnetic coupling of the piezoelectric energy harvesting cantilever beam. The dimensions and axes are also shown in Fig 2. The piezoelectric layer bounded on the steel cantilever beam can be regarded as an energy harvesting device. The analysis of piezoelectric-element part is the same as the classic model analysis [12]. Considering piezoelectric cantilever beam vibrating at the first mode, the piezoelectric patches can be modeled as a simple one dimension model. The governing equations of the piezoelectric can be expressed as the following (1).

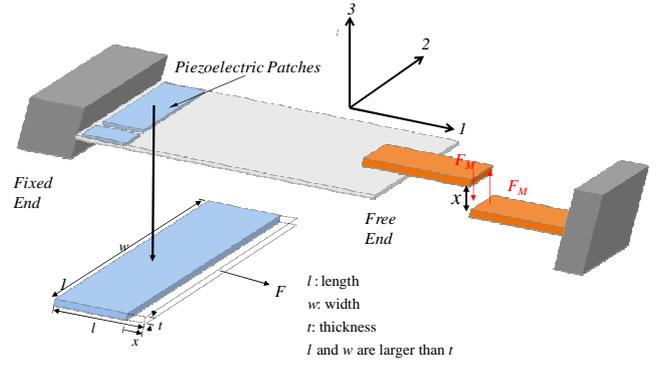


Fig. 2: Schematic of nonlinear magnetic coupling of piezoelectric harvesting cantilever beam

$$\begin{cases} F = K_p^E x + \alpha V \\ I = \alpha \dot{x} - C_0 \dot{V} \end{cases} \quad (1)$$

Where

$$K_p^E = c_{11}^E \frac{w \cdot t}{l}, \quad \alpha = e_{31} w, \quad C_0 = \epsilon_{33}^S \frac{w \cdot l}{t} \quad (2)$$

$K_p^E$  is short circuit stiffness,  $c_{11}^E$  is elastic constant in '1' direction,  $\alpha$  is force-voltage coupling factor,  $e_{31}$  is piezoelectric constant,  $C_0$  is clamped capacitance of piezoelectric and  $\epsilon_{33}^S$  is permittivity constant.

The magnetic force  $F_M$  can be simplified to one dimensional model and it is acting only in '3' direction [9]. By using the curve fitting method, the magnetic force  $F_M$  can be express as equation (3).

$$F_M(x, \eta) = \frac{ax}{bx + cx^4} \quad (3)$$

$x$  is the distance between two magnets,  $a$   $b$  and  $c$  are the fitting parameters. Fig 3 shows the spring force of the beam, the vertical magnetic force and potential energy. There are two minima points in the potential energy curve, and this means that there are two possible stable positions of the system. When the structure vibrates at non-resonant frequency, if the potential energy is enough to drive the structure from one stable position to another stable position, the deflection of the structure will be larger than linear system case and piezoelectric energy harvester can output more energy. By combining the piezoelectric-element model and the magnetic model, the schematic of nonlinear magnetic coupling of piezoelectric harvesting cantilever beam in Fig. 2 can be modeled as equivalent mechanical model by mass, damper, spring, piezoelectric system and magnetic force shown as Fig 4. The governing equations can be expressed as equation (4) shown.

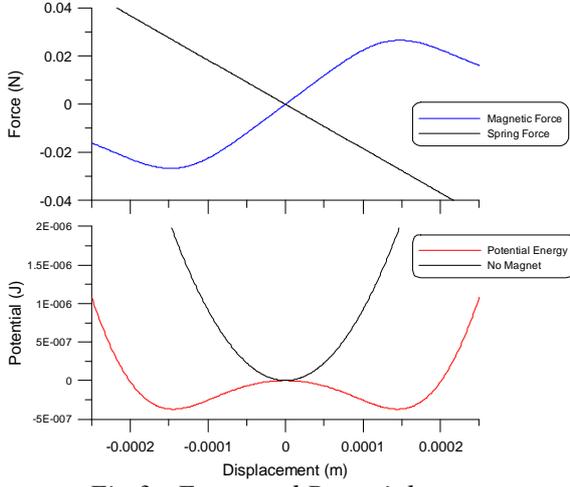


Fig 3: Force and Potential curve.

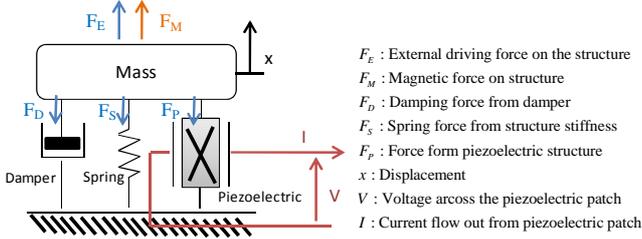


Fig. 4: Equivalent mechanical model of nonlinear magnetic coupling of the piezoelectric-element and the structure..

$$\begin{cases} m\ddot{x} + D\dot{x} + K^E x = F_E + F_M - \alpha V \\ F_M = \frac{ax}{bx + cx^4} \\ I = \alpha\dot{x} - C_0\dot{V} \end{cases} \quad (4)$$

$D$  is the damping ratio of the structure;  $K^E$  is the equivalent stiffness of the structure and piezoelectric materials. According to the equation (4), the equivalent circuit model can be represented as Fig 5. The mechanical part and electrical part are the classical equivalent circuit models of piezoelectric-element. The magnetic force is taken into account by adding a nonlinear magnetic feedback loop.

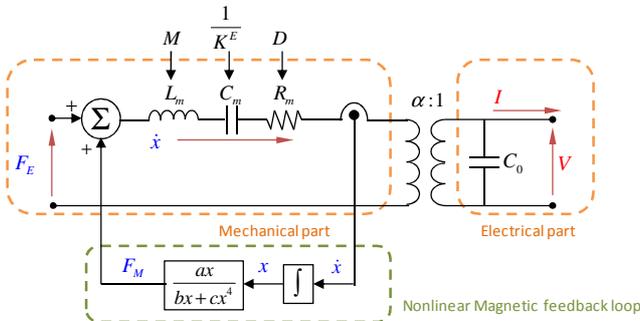


Fig. 5: Equivalent circuit model of nonlinear magnetic coupling of piezoelectric-element and the structure.

### SSHI-P (Synchronized Switch Harvesting on Inductor – Parallel)

In order to improve the power output of piezo-

element at each frequency, synchronized switch harvesting on inductor is used here. The analysis and model are the same as classic ones [12]. By using an inductor in parallel with the piezoelectric-element to resonate with the static capacitor and through the switches to confine the current flow, the output power can be improved. The schematic diagram of the SSHI-parallel configuration is shown in Fig 6 and the waveforms are shown in Fig 7.

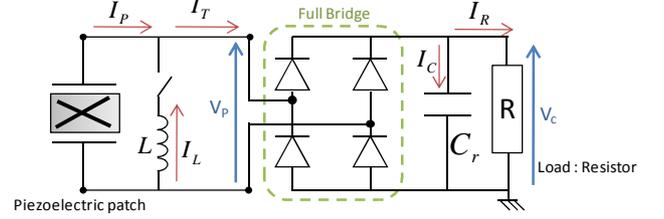


Figure 6: Schematic diagram of the SSHI-parallel piezoelectric energy harvesting device with full bridge rectifier before a resistor load

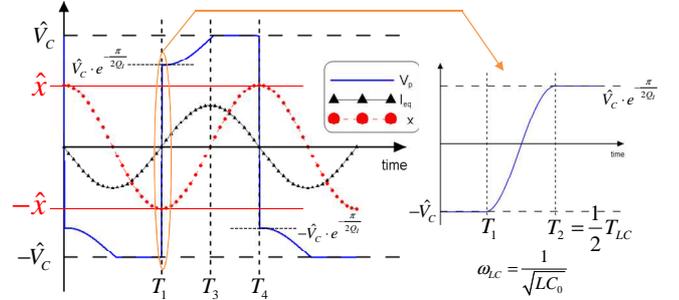


Figure 7: Waveform of the SSHI-parallel piezoelectric energy harvesting device.

Considering the time period from  $T_1$  to  $T_4$  (half cycle), the current flow  $I_R$  from  $T_1$  to  $T_4$  equals to the resonance current  $I_L$  from  $T_1$  to  $T_2$  plus the piezoelectric current  $I_P$  from  $T_1$  to  $T_4$ . Integrating the current from  $T_1$  to  $T_4$ ,  $V_C$  can be obtained as shown in equation (5). The power output from the piezoelectric-element can be simply obtained by using  $P = V^2/R$  as shown in equation (6).

$$\hat{V}_C = \frac{2\alpha R}{\pi + \left(1 - e^{-\frac{\pi}{2Q_l}}\right) C_0 R \omega_0} \frac{\hat{F}_E}{D} \quad (5)$$

$$P = \frac{\hat{V}_C^2}{R} = \frac{4\alpha^2 R}{\left[\pi + \left(1 - e^{-\frac{\pi}{2Q_l}}\right) C_0 R \omega_0\right]^2} \frac{\hat{F}_E^2}{D^2} \quad (6)$$

### Velocity Control

One small piezoelectric patch is designed for generating the velocity control signal and making SSHI-P technique switching at the proper time. The equivalent circuit and the waveform are shown in Fig 8.

By using a current sensing resistor  $R_C$  in parallel with piezoelectric-element, the velocity control signal can be obtained. In order to reduce the high frequency noise and make sure that the velocity signal is clean enough to drive the switches, a passive low pass filter is applied here. The optimal time to switch is when the voltage of the piezoelectric-element reaches peak voltage and it is the same moment as the current of the piezoelectric-element crossing the zero point. Traditional technique is to use peak detector to drive the switch but velocity control is used here to drive the switch and thus the switching time is more accurate. As Fig 8 shows, the blue line  $V_C$  is the open circuit waveform of the piezoelectric-element and red line  $V_S$  is the velocity control signal which is in phase with  $I_P$ . When using velocity signal to drive the switch, it always drives the switch on the peak voltage of the piezoelectric-element.

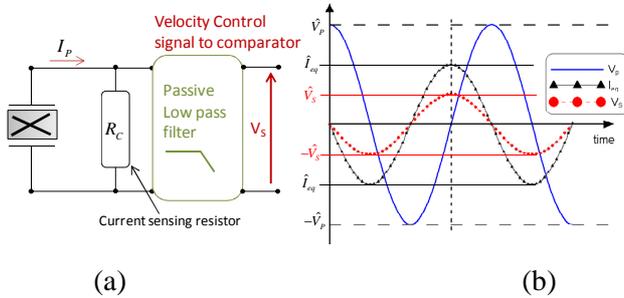


Fig. 8: (a) The equivalent circuit diagram of the velocity control circuit. (b) Waveform of the velocity control circuit.

## SIMULATION, EXPERIMENTAL RESULTS AND DISCUSSION

In Table 1, the dimensions of piezoelectric patches and cantilever beam are tabulated listed. Fig 9 shows the experimental setup and circuit schematic diagram of the nonlinear magnetic coupling design of the piezoelectric energy harvester combined with SSHI-P technique. Fig 10 shows the pictures of the experimental setup. The two patches are marked as  $P_1$  and  $P_2$ .  $P_1$  is connected to the SSHI-P circuit and  $P_2$  is connected to the velocity control circuit. The piezoelectric cantilever beam is excited by a shaker (Brüel & Kjær 4809); the vibration signal generated by a data acquisition card (NI-DAQ USB-6259) and the acceleration is  $2 \text{ m/s}^2$ . To realize the SSHI-P circuit, the two diodes and the two MOSFET (metal oxide field-effect transistor) switches are used as shown in Fig 9. When the velocity signal goes zero crossing from negative to positive, the NMOS (N-Channel MOSFET) switch (IRFU210) is switched on and when the signal goes zero-crossing from positive to negative, the PMOS (P-channel MOSFET) switch (IRF9640) is switched on. The two diodes confine the current flow and the inductor  $L$  will resonate with the static capacitor of the piezoelectric-element. The voltage of the piezoelectric -element and the output power are increased. The two switches used here are high voltage

MOSFET because the voltage of piezoelectric-element will reach around 170V. The diodes used are also high voltage Schottky- diode to ensure the voltage does not go into the breakdown region.

Fig 11 shows the experimental results without SSHI-P technique. The driving signal is chirping with frequency range from 5Hz to 30Hz in 250 seconds. The driving frequency cannot start from 1Hz because the limitation of the vibration shaker. The distance between the two magnets is 3.5mm. The black curve is the output voltage for the linear system without magnetic nonlinear force and red curve is output voltage with the magnetic nonlinear force added in. From the results, the magnetic nonlinear coupling can effectively extend the available band-width, unlike in the linear system, where the energy is only available at the resonant frequency. Comparing nonlinear to linear results, the voltage close to the resonance frequency is almost the same as nonlinear system, but more energy can be harvested in nonlinear system at non-resonant frequency when the displacement is large enough to drive the beam from one stable position to the other stable position. In the nonlinear magnetic coupling system the voltage can be increased around two or three times than the linear system. The amount of increased voltage is depended on how large the deflection in between the two bi-stable position caused by the magnetic coupling. There is a critical frequency in this nonlinear magnetic coupling system. When the potential energy is not enough to drive the system from one stable to another, the voltage in the nonlinear system will be the same with the linear system.

Fig 12 shows the simulation results of the output voltage of the magnetic nonlinear harvester and Table 2 shows the results of measurements and simulation parameters. The simulation model is based on Fig 5. The nonlinear system simulation model is built in Matlab and PSIM software packages. The simulation results are driven by chirping frequency. The chirping frequency is ranging from 1Hz to 30Hz in 300 seconds and its voltage swing rate is the same as the experimental conditions. Comparing to the experimental results, the simulation results show good agreement with the experimental data.

Fig 13 shows the experimental result of the magnetic nonlinear system and linear system combined with parallel SSHI technique. The velocity control signal is used to control the switching time and through the comparator to drive two MOSFETs. The power of comparator is supplied with an external power supply though. From the results and comparing with Fig 11, the SSHI-P can boost the output voltage (from 110V to 170V) whatever in linear or nonlinear system, also improve the power output around the resonant frequency and the synchronized switch technique will not influence the magnetic nonlinear effect. When these two nonlinear techniques are combined together, the advantage will also be combined. The available band-width is broadened and

output power at the broadened frequency is also improved. Although the experimental results show significant improvement of output power, theoretically the voltage of magnetic nonlinear system combined with SSHI-P should be larger than the experimental results. There are two reasons to explain the loss. Firstly, there are switching losses in the switches (MOSFETs). Secondly, the voltage is too high so as the high frequency switch noise is presented and hardly to keep it clean enough. When the velocity control signal is not clean enough, the switch will on and off many times with the LC resonance. When LC resonance cannot work perfectly, the effect of SSHI-P will not be good enough.

Table 1: Dimension of the piezoelectric patches and cantilever beam

Steel Beam	Length × Width × Thickness	170 mm × 35 mm × 0.5 mm
	First Bending Mode	10.4 Hz
Piezoelectric patches	P1	28 mm × 16.5 mm × 0.5 mm
	P2	6 mm × 16.5 mm × 0.5 mm

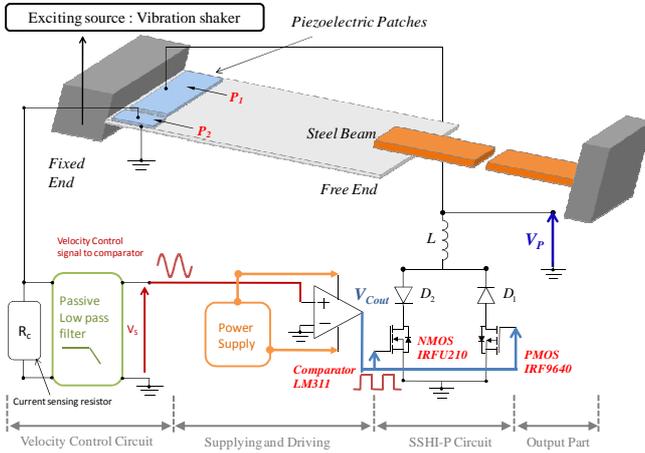


Fig. 9: Experimental setup and circuit schematic diagram of the nonlinear magnetic coupling of the piezoelectric energy harvester combined with SSHI-P.

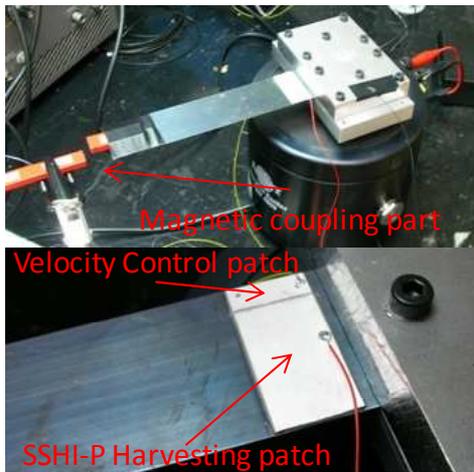


Fig. 10: Photos of the experimental setup.

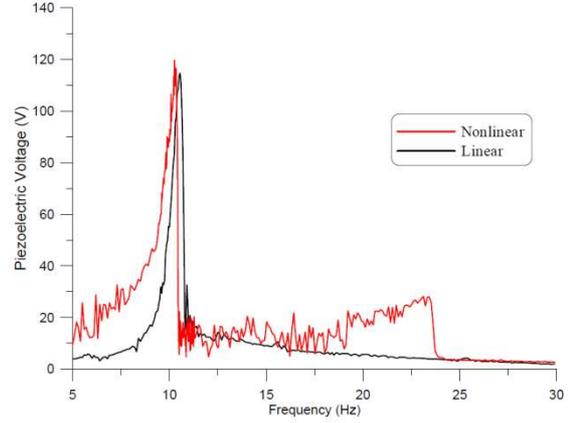


Fig. 11: Experimental results of the nonlinear magnetic coupling of piezoelectric energy harvester without SSHI-P.

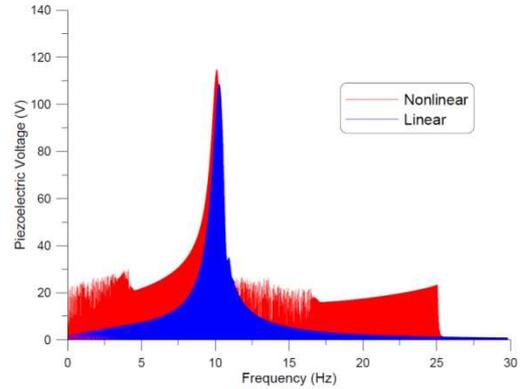


Fig. 12: Simulation results of nonlinear magnetic coupling of piezoelectric energy harvester without SSHI-P.

Table 2: Measurements and model parameters.

$f_1$	Open circuit resonance frequency	10.4 Hz
$C_0$	Clamped capacitance of the piezoelectric element	$P_1$ 15.57 nF $P_2$ 2.45 nF
	$\alpha$	Force-voltage coupling factor
$M$	Mass	49 g
$K^E$	Equivalent stiffness of the structure when piezoelectric is short-circuited	209.229 Nm <sup>-1</sup>
$D$	Damping ratio of the structure	0.15 Nm <sup>-1</sup> s <sup>-1</sup>

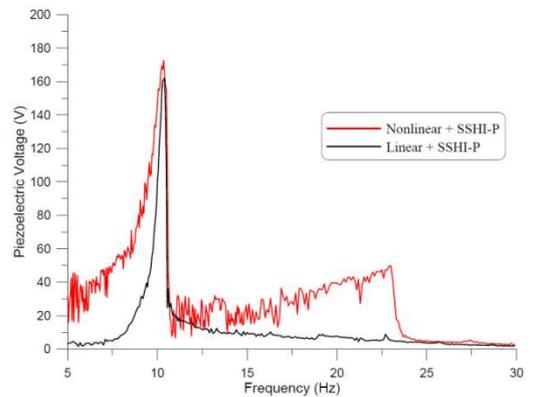


Fig. 13: Simulation results of nonlinear magnetic coupling of piezoelectric energy harvester combined with SSHI-P.

## CONCLUSION

Combining magnetic nonlinear coupling technique with the synchronized switch technique on piezoelectric energy harvester proposed in this study is a new and interesting design. When these two nonlinear techniques working together, the available band-width is broadened and the power output within the broadened frequency range is improved. These two techniques can work at the same time and will not influence each other, and the benefits of these two nonlinear techniques are also combined together. Comparing the nonlinear experimental results with linear results, the effect of nonlinear magnetic coupling technique is verified. The voltage gain is almost the same at resonant frequency; however, at the non-resonant frequency, the output voltage of the piezoelectric-element can be improved around two or three times. Synchronized switch harvesting on inductor technique is used here to improve output power at each frequency. The voltage gain can be improved around 1.5 times than the system without using SSHI-P technique. The distance between two magnets decides the potential energy gap between two stable positions. When the potential energy gap is large, it means that there will be more deflection and more energy boosted, but it will be hard to drive to such position though. When the frequency is higher than the critical frequency, the nonlinear system will work at the same as the linear system. The SSHI-P technique can increase output power at each frequency, but when the frequency is close to the resonant frequency, the boosting effect is more effective. This is because that at non-resonant frequency, the loss in circuit is larger and is less effective. The nonlinear magnetic coupling combined with the SSHI technique proposed in this study is new techniques for piezoelectric energy harvester. Through these two nonlinear techniques, the piezoelectric harvester can work more efficiently at non-resonant frequency and more output power at a broadened frequency range can be obtained.

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