PENDULUM-DRIVEN PASSIVE SELF-TUNING ENERGY HARVESTER FOR ROTATING APPLICATIONS

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Abstract: This paper presents a pendulum-driven passive self-tuning energy harvester for rotating applications. The compact harvester rotates in the vertical plane and is comprised of a steel ball on a cable (akin to a pendulum) and a relatively rigid piezoelectric generating beam. The ball impacts the beam periodically under the influence of gravity, driving generation in the beam. The centrifugal force from the rotation changes the resonant frequency of the cable with tip-mounted ball and of the harvester. With optimized design, the resonant frequency of the harvester can be passively tuned to match the frequency of the rotation. With a resonant frequency of 15 Hz, a prototype of the passive self-tuning energy harvester achieved more than 11 Hz bandwidth, which is much larger than the predicted 0.8 Hz bandwidth of an untuned harvester.

Keywords: passive tuning, energy harvester, centrifugal force

Vibrational energy harvesters such as piezo-electric, electrostatic and electromagnetic harvesters achieve their highest output power when they are working at resonance [1]. The resonant frequencies of conventional energy harvesters are fixed during the design process and cannot be changed after implementation. However, the frequencies of some vibration sources such as rotating vehicle wheels are time-dependent. Harvesters with tunable resonant frequencies are very desirable for such applications.

Both active and passive tuning schemes have recently been proposed. Examples of active tuning include varying the area moment of inertia [2], manually shifting the location of the tip mass [3], applying magnetic force to a cantilever beam [4], and by applying axial forces to cantilever beams to change their stiffness [5, 6]. Although these active tuning technologies successfully extend the harvesting bandwidths, they require the application of extra force or power, thereby lowering the generating efficiency.

Fewer examples of passive tuning have been reported. Kozinsky employed passive tuning by allowing a free ball in a vibrating hollow tube resonator to change its position [7]. This provides a promising theoretical prospect for passive self-tuning, but improved harvester performance was not reported using this approach.

We have previously reported a passively-tuned harvester for rotational applications in which the resonant frequency of a radially-oriented piezoelectric cantilever beam is passively tuned by beam tensioning due to centrifugal force [8]. Although the harvester successfully demonstrated self-tuning, its size was slightly large for practical applications such as tire pressure monitoring systems (TPMS) due to the need to minimize the beam's bending stiffness as compared with tensioning effects.

This paper presents a new and more compact type of self-tuning rotational harvester that demonstrates

improved tuning performance. The harvester comprises a relatively rigid piezoelectric generating beam that is driven by impact from a ball on a cable (akin to a pendulum) mounted adjacent to the generating beam on a vertically-oriented rotating platform such as a wheel (Figure 1).
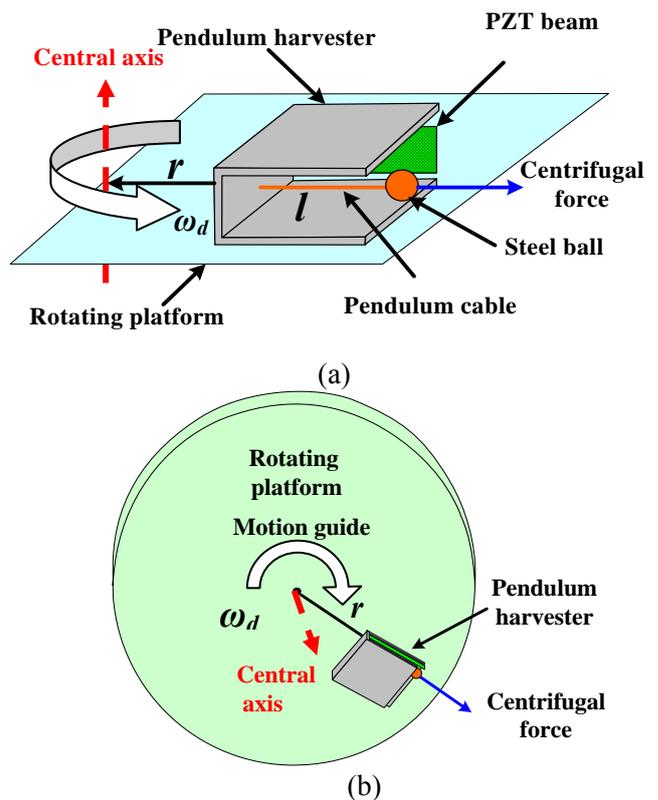

(a)

(b)

Figure 1. (a) Schematic view of the pendulum impact harvester with frequency self-tuning. Although in reality the system rotates in the vertical plane, here it is drawn in the horizontal plane for clarity. (b) Schematic diagram showing the harvester mounted on a vertically rotating wheel.

As the wheel rotates in the vertical plane, gravitational effects are superimposed on the ball's

circular motion and drive the ball to impact the generating beam. The generating beam then vibrates at its resonant frequency, capturing energy from the ambient low frequency vibration. The impact force is maximized when the natural frequency of the cable-ball element is equal to the rotational frequency of the wheel. Passive tuning arises because the cable-ball element's natural frequency depends on the centrifugal force and hence on rotational speed. For an appropriate design of the harvester and the radius r at which the harvester is mounted, the resonant frequency of the harvester will track the driving frequency over a large range of rotational speeds.

DESIGN AND SIMULATION

If the cable and ball were mounted on a frictionless pivot, the system would be best described by the conventional equations of a pendulum. In contrast, when the root of the cable is instead supported by a fixed or cantilevered support, the cable behaves more like a beam, and the displacement of the mass is best described by beam bending. From classical beam theory, the stiffness of a cantilever beam can be tuned by applying an axial force [9]. As was shown in [8], it is possible to exploit this frequency-dependent beam tensioning to ensure that the resonant frequency tracks the driving frequency. A similar approach is used here to ensure that the more compact, pendulum-driven harvester remains on resonance over a wide frequency range.

For small deflections, the equation of motion for a beam (such as the tensioned cable) with an applied centrifugal force f_c is given as [9]

$$v''''(x) + \frac{f_c + \rho I \omega_r^2}{EI} v''(x) - \frac{\rho A \omega_r^2}{EI} v(x) = 0 \quad (1)$$

where $v(x)$ is the oscillation amplitude of the cable at position x along the cable, E is the cable's Young's modulus, I is its moment of inertia, ρ is its mass density, A is its cross sectional area, and ω_r is the resonant frequency. The moment of inertia is given by

$$I = \frac{\pi R^4}{64} , \quad (2)$$

where R is the diameter of the cable.

A detailed analysis can be found in [8, 9]. Solving Eq. 1, the relation between the resonant frequency ω_r and the applied force (centrifugal force) f_c is given as

$$2\beta^6 - k^2\beta^4 \sinh \lambda_1 l \sin \lambda_2 l + \beta^2(k^4 + 2\beta^4)\cosh \lambda_1 l \cos \lambda_2 l \quad (3)$$
$$-k_2(\lambda_1^2 + \lambda_2^2)(\lambda_1 \cosh \lambda_1 l \sin \lambda_2 l - \lambda_2 \sinh \lambda_1 l \cos \lambda_2 l) = 0$$

in which

$$\lambda_1 = \sqrt{\frac{-k^2 + \sqrt{k^4 + 4\beta^4}}{2}}, \lambda_2 = \sqrt{\frac{k^2 + \sqrt{k^4 + 4\beta^4}}{2}},$$
$$\beta^4 = \frac{\rho A \omega_r^2}{EI}, k^2 = \frac{f_c + \rho I \omega_r^2}{EI}. \quad (4)$$

The variable k_2 in Eq. 3 is given as

$$k_2 = \frac{m\omega_r^2 - f_c/(r+l)}{EI} , \quad (5)$$

where m is the mass of the steel ball, l is the length of the cable. The centrifugal force is also related to the rotational speed or driving frequency ω_d by

$$f_c = m(r+l)\omega_d^2 , \quad (6)$$

thereby linking the driving frequency to the resonant frequency.

The above models have previously been used to describe the tensioning and resulting resonant frequency shift of a piezoelectric cantilever beam harvester [8]. For the impact vibration harvester in which the driving beam and generating beam are separate, the resonant frequency of the impact harvester can be estimated as in [10] (as long as the stiffness of the cable is much less than that of the piezoelectric beam),

$$\omega_{im} = 2\omega_r , \quad (7)$$

where ω_r is the resonant frequency of the single cable-ball vibration. Together, these equations yield a relationship between the resonant frequency of the harvester and the rotational speed. The resulting relationship can be used to design self-tuning rotational energy harvesters.

Once the harvester is built, the only parameter that can be changed is the radius at which it is mounted. By varying the radial distance from the root of the harvester to the center of a rotating plate, the radius at which the resonant frequency and the driving frequency are optimally matched can be obtained. For an impact harvester using the design parameters shown in table 1 and neglecting damping, the predicted driving frequency ω_d and the resonant frequency ω_{im} of the harvester are plotted versus centrifugal force in figure 2 with an optimal mounted radius r of 5 mm.

Table 1. Optimized parameters of the harvester

ρ	Density of the cable	1150 kg/m^3
E	Young's modulus of the cable	3×10^9 Pa
l	Length of the cable	20 mm
R	Diameter of the cable	75 µm
m	Tip mass	0.4 g
r	Root to center distance	5 mm

Figure 2(a) shows that the centrifugal force increases with the driving frequency and in turn tensions the cable to tune the system's resonant frequency. For the optimal radius r, the predicted resonant and driving frequencies match well over a wide frequency range from 6 Hz to 19.5 Hz with a 0.5 Hz maximum mismatch. The resonant frequency is exactly equal to the driving frequency at an intersection frequency of 15 Hz, which can be considered to be the resonant frequency of the self-tuning harvester. Figure 2(b) plots the predicted resonant and driving frequencies versus vehicle speed, assuming a tire with 58.4 cm outer diameter.
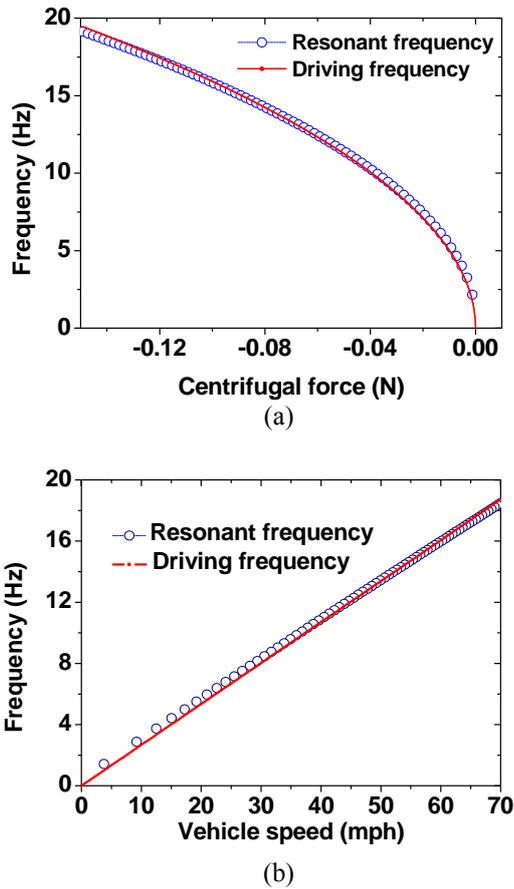
Figure 2. Predicted resonant and driving frequencies plotted vs. (a) centrifugal force and (b) vehicle speed.

In addition to the advantages of self-tuning, the vibration of the piezoelectric generating beam upon impact up-converts low ambient frequencies (1-20 Hz) to high generating frequencies (more than 100 Hz). An analysis of impact vibration can be found in [10, 11]. The impact vibration process can be divided into two stages. The first stage begins when the driven ball impacts the generating beam and continues while they vibrate together. If the mass of the generating beam is negligible compared with the mass of the ball, this stage can be approximately considered as an inelastic impact [11]. The second stage begins when the ball and the generating beam separate. During this stage, the generating beam vibrates alone with exponentially-decayed amplitude at its self-resonant frequency, while the ball is driven and oscillates at the ambient low frequency. The stages repeat under a periodic driving vibration.

EXPERIMENT

The pendulum-driven self-tuning energy harvester concept was demonstrated with a 0.4 g steel ball on a 2 cm long, 75 μm-diameter nylon cable epoxied to a frame that also held a separate PZT generating beam as shown in figure 3. The harvester was epoxied to an electrical fan with the cable oriented parallel to the fan blade and along the radial direction. The root of the cable was offset from the center of the fan. The fan was refitted to enable continuously-varying speed and

simulate the time-varying rotational motion of a wheel. The wires of the rotating harvester were connected to an optimal static load resistance through a slip-ring. A frequency range from 6.2 Hz to 16.2 Hz was considered to correspond to the specific case of rotating tires in a speed range from 25 miles per hour (mph) to 65 mph.
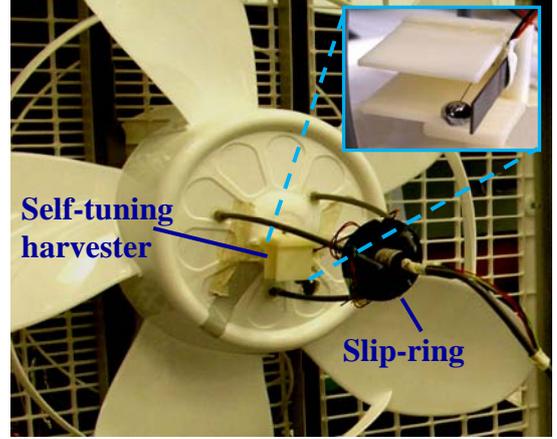

Figure 3. Test set-up with an electrical fan and a harvester prototype mounted on it.

The optimal load resistance R_L was simply chosen to match the source resistance R_S, which is given as

$$R_S = R_L = \frac{1}{\omega_g C_g} , \qquad (8)$$

where ω_g and C_g are the self-resonant frequency and the capacitance of the generating piezoelectric beam, respectively. Although the piezoelectric beam is also tensioned, its bending stiffness is high enough to dominate over the tensioning effects. The vibrational frequency of the generating beam therefore remains essentially constant, ensuring that the matched load resistance of 220 kΩ remains constant as well.

Before measuring the performance of the harvester with self-tuning, the harvester's optimal mounting radius r was experimentally confirmed and fine-tuned. Since the stiffness of the cable changes synchronously with rotational speed, the resonant frequency of the harvester could not be measured by sweeping the driving frequency as in a conventional measurement. Instead, an impulse force (acceleration) was applied to the rotating fan, and the exponential decay signal was measured. To cancel the influence of gravity, the rotating fan was placed horizontally, and an impulse of acceleration was applied by shaking the rotating fan. The resonant frequency at that fan speed was measured from the decaying output waveform, and the measurement was repeated for a range of fan speeds. The optimal radius r is measured to be 7.5 mm, which is slightly larger than the predicted optimal radius due to the effects of damping in the real test. Figure 4 shows that at the optimal radius, the measured resonant frequency matches the driving frequency very well over a wide frequency range from 6 Hz to 16.2 Hz. The maximum mismatch between the resonant and driving frequencies is less than 0.2 Hz over this range.
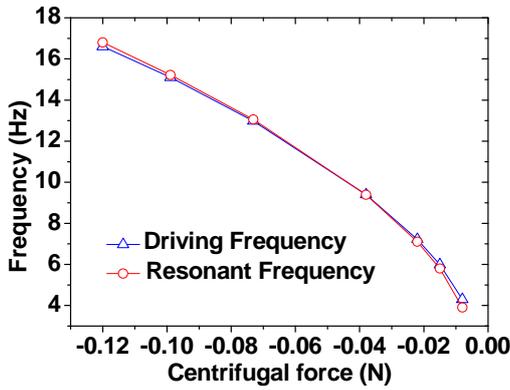
Figure 4. The measured driving frequency and resonant frequency plotted versus calculated centrifugal force with an optimal mounting radius of 7.5 mm.

The output power and voltage of the pendulum-driven harvester were measured and compared with semi-empirical values for an untuned version (Figure 5). The peak-to-peak voltage remained nearly constant from 4 Hz to 15 Hz, reflecting that the harvester remains nearly on resonance. Moreover, the output power increases with impact frequency as expected, then drops off at high frequency as increased cable tensioning limits impact. The maximum and minimum output powers were 123 µW at 16.2 Hz and 60 µW at 6.2 Hz, corresponding to a maximum power density of 30.8 µW/cm^3. The self-tuning harvester showed a more than 11 Hz bandwidth compared with a 0.8 Hz bandwidth predicted for an untuned harvester. The untuned power was calculated analytically based on the measured peak output power at the resonant frequency and the measured damping ratio, which was calculated from the exponentially-decayed waveform obtained from the above measurement of the resonant frequency according to equation (9),

$$\xi = \frac{1}{2\pi} \ln\left(\frac{p_1}{p_2}\right), \qquad (9)$$

where p_1 and p_2 are consecutive peak amplitudes. The peak output power of the frequency-matched but untuned harvester can be considered to be the same as that of the tuned harvester at the resonant frequency.
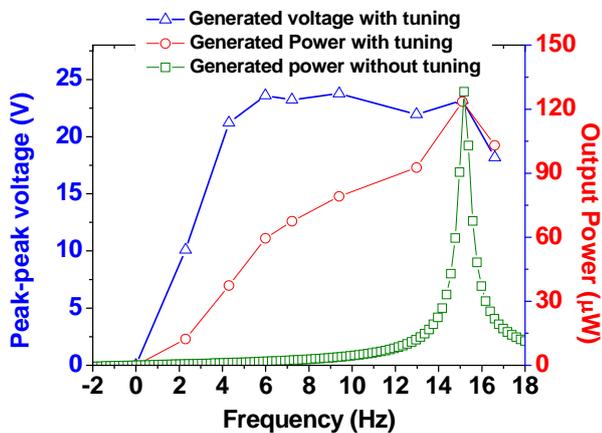

Figure 5. Measured output power and generated voltage of the self-tuning energy harvester plotted vs. frequency, along with output power of a comparable untuned harvester.

CONCLUSION

This paper presents a compact self-tuning energy harvester in which the resonant frequency is passively tuned to match the driving frequency. The experiment verifies the analytical model and successfully demonstrates passive self-tuning technology over a wide bandwidth. This self-tuning technology is potentially applicable not only to piezoelectric harvesters but also to self-tuning electrostatic and electromagnetic systems.

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