GENETIC ALGORITHM OPTIMIZATION FOR MEMS CANTILEVERED PIEZOELECTRIC ENERGY HARVESTERS

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Abstract:
A design optimization based on genetic algorithms for increasing power output performance of MEMS piezoelectric cantilevered energy harvesters has been performed. By employing genetic algorithms, the piezoelectric energy scavenging system with optimized shape and lateral geometries of a unimorph cantilever beam and a proof mass is simulated to achieve a power response improvement of ~1.8× in comparison with a reference system composed of a rectangular cantilevered structure. In particular, by optimizing the lateral geometries of a trapezoidal cantilever with proof mass, the power scavenged from a unit volume of the piezoelectric material can be enhanced to 162%. In addition, the output power per surface area from an optimized structure with a reverse trapezoidal cantilever enhances 184% based on theoretical analysis that includes damping effects.

Keywords: energy harvester, unimorph cantilever, genetic algorithm, optimization, aluminum nitride

INTRODUCTION

As wireless sensor networks emerge onto commercial platforms, power generators based on vibration energy scavenging have attracted a great amount of attention for developing self-powered sensor spots for easier maintenance and higher cost efficiency. The technology based on scavenging ambient vibration energies enables a renewable energy source for low power electronic devices and sensors in autonomous miniature systems. Among all the existing technologies, a cantilevered piezoelectric energy harvester in flexural vibration utilizing d_{31} piezoelectric transduction have been widely applied for building macro- and micro-scale energy harvesters [1][2].

To improve the power efficiency, trapezoidal beam geometries have been proposed to create a uniform stress distribution along the beam length within a maximum tolerable amount [1][3]. Recently, numerical optimization on vertical [4] and lateral [1] geometries of the cantilever for macro-scale harvesters has been presented to increase power efficiency. However, for MEMS-based (micro-scale) energy harvesters, varying vertical (out of plane) geometry is difficult to realize via microfabrication processes. Typically, modification of the lateral (in plane) geometries of beams is more feasible and can be easily defined with 2-D layouts. In addition, instead of performing optimization only on the geometries of the beam, the geometries of the proof mass as a part of the vibrating system should also be considered to enable optimization of the entire system.

In this paper, genetic algorithms are employed for optimization, which allow a more complex system with more design variables to be considered. Both the lateral geometries (width and length) of a unimorph cantilever beam and those of a proof mass in an energy scavenging system are optimized to increase power output. The structure of the investigated energy harvester is shown in Fig. 1 with all the design parameters indicated. Aluminum nitride (AlN) is chosen as the active material for the beam layer due to its intrinsic piezoelectric characteristics even at elevated temperatures, CMOS-compatibility, bio-compatibility, and wafer-level process integrability. The piezoelectric AlN layer sits on top of a supporting layer of silicon carbide (SiC) to form a unimorph structure, and although not considered here, it is sandwiched by two thin layers of platinum (Pt) as top and bottom electrodes. The material of the proof mass is silicon (Si) and the properties for the main structural materials are listed in Table 1. Even though the specified materials and their properties are addressed, the optimization approach discussed here can be applied to general cases and other material sets.

Before introducing genetic algorithms and presenting the optimization procedure, formulation of the electromechanical model applied in this work for a cantilevered energy harvester is summarized below.

ELECTROMECHANICAL MODEL

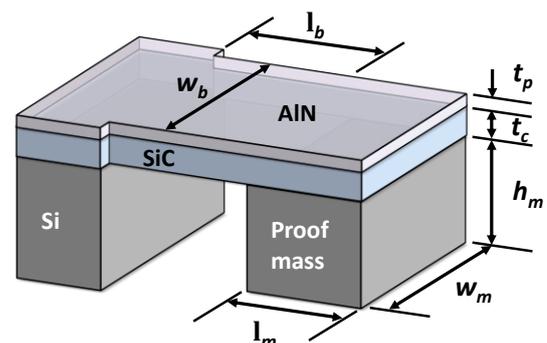A single-degree-of-freedom electromechanical model based on Euler–Bernoulli beam theory is

Fig. 1: Schematic of a unimorph cantilevered energy harvester structure with design parameters indicted.

Table 1: Material Properties of AlN, SiC and Si

	AlN	3C-SiC	Si
Young's Modulus [GPa]	283	450	150
Density [kg/m^3]	3260	3200	2330
Poisson Ratio	0.22	0.19	0.28
Dielectric Constant	10.73	6.52	11.7
Piezo. Coeff. e_{31} [C/m^2]	-0.58	--	--
Thermal Exp. Coeff. [K^{-1}]	4.6×10^{-6}	2.77×10^{-6}	2.6×10^{-6}

developed for predicting the performances of a unimorph cantilevered harvester under harmonic base excitation. The resonant frequency of a unimorph cantilever with a distributed rectangular proof mass at the free end can be approximated by

$$f_n = \frac{\lambda'^2_n}{2\pi}\sqrt{\frac{K_{eff}}{M_{eff}}} \qquad (1)$$

where $\lambda'^2_n = \lambda^2_n\sqrt{0.236/3}$ and λ_n is the eigenvalue for the nth mode resonance [2]. The effective spring constant and mass of the cantilever at the center of the proof mass can be expressed as

$$K_{eff} = \frac{3EI_{eff}}{(1-1_m/2)^3} \qquad (2)$$

$$M_{eff} = 0.236m_b + m \qquad (3)$$

where E is the Young's modulus of the piezoelectric material, l is the total length of the cantilever ($1 = l_b + l_m$), l_b is the active beam length equal to the length of electrodes, l_m is the length of the proof mass (as indicated in Fig. 1) and m_b and m are the mass of the beam and proof mass, respectively. The effective moment of inertia, I_{eff}, is given by Eqn. (4).

$$I_{eff} = \frac{1}{12}\left(w_b t_p^3 + nw_b t_c^3\right) + w_b\left(t_p b_p^2 + nt_c b_c^2\right) \qquad (4)$$

where b_p and b_c are the distance from the center plane of the piezoelectric layer and that of the supporting layer to the neutral plane, respectively, n is the Young's modulus ratio of the supporting layer to piezoelectric layer and the other variables are shown in Fig. 1.

Here, only the fundamental flexural mode of a unimorph cantilever under harmonic base excitation is considered. The total charge induced on the surface of the piezoelectric layer based on the linear constitutive equation of piezoelectricity as the driving frequency equals the natural frequency of the cantilever can be expressed as,

$$Q = \int_A D_3 dA = -e_{31}b_p w_{b,ave}\Phi - w_{b,ave}l_b\varepsilon_{33}\frac{v}{t_p} \qquad (5)$$

where D_3 is the electrical displacement in x_3 direction, A is the surface area of the active beam region, e_{31} is the piezoelectric coefficient, $w_{b,ave}$ is the average width of the active beam, ε_{33} is the dielectric constant, v is the potential difference across the piezoelectric layer,

and $\Phi = |\partial u(x,t)/\partial x| = mA_m(l_b^2 + l_m l_b)/4EI_{eff}\zeta_T$ is the amplitude of the flexibility of the beam where u(x,t) denotes the transverse displacement of the cantilever and ζ_T is the total damping ratio [5][6]. The steady state power response of the harvester can be expressed as a time-averaged output power, P, as described by Eqn. (6) when connecting to a pure resistive load.

$$P = \frac{\omega_n^2 e_{31}^2 w_{b,ave}^2 b_p^2 (\Phi)^2 R}{2\left(1 + \omega_n w_{b,ave}l_b\varepsilon_{33}R/t_p\right)^2} \qquad (6)$$

where ω_n is the angular resonant frequency and R is the load resistance.

GENETIC ALGORITHMS (GAs)

Genetic algorithms feature selection and variation of generations of solution populations based on natural evolutionary process and enable achieving global minimum in a large complex parameter space with small feasible region [7]. For conventional optimization techniques, the initial condition represents a key to the optimized solution but often difficult to estimate. However, for genetic algorithms, it is an initial set of randomly generated individuals called populations. Solutions are represented by each individual in the populations. A fitness function (objective function) provides the fitness measure for each individual in current population which determines the selection process in successive generations. Typically, two operators, crossover and mutation, are applied in the evolution process of individuals in each generation. Mutation introduces diversity and variation in the populations and offers the capability of preventing solutions contained by local minima.

DESIGN OPTIMIZATION

Two design optimization cases based on lateral geometries of the cantilever and/or proof mass with global constraints on resonant frequency, maximum allowable stress, and maximum width of the beam are investigated. Case I considers a trapezoidal cantilever structure which has been proposed in [1][3]. In Case II, a reverse trapezoidal shape is adopted and in addition, the air damping effects are considered for better power
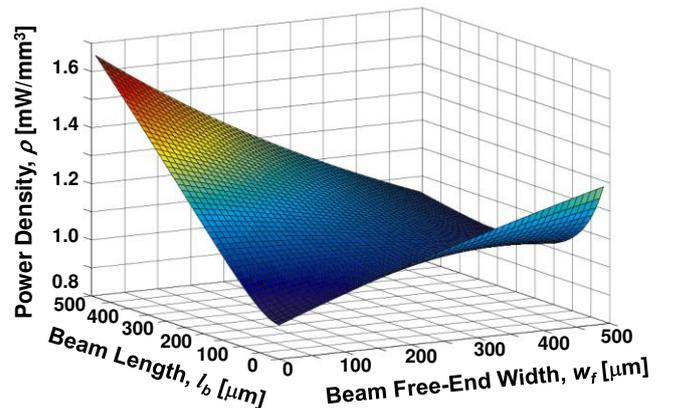

Fig. 2: Surface plot of power density as a function of w_f and l_b of a trapezoidal beam.
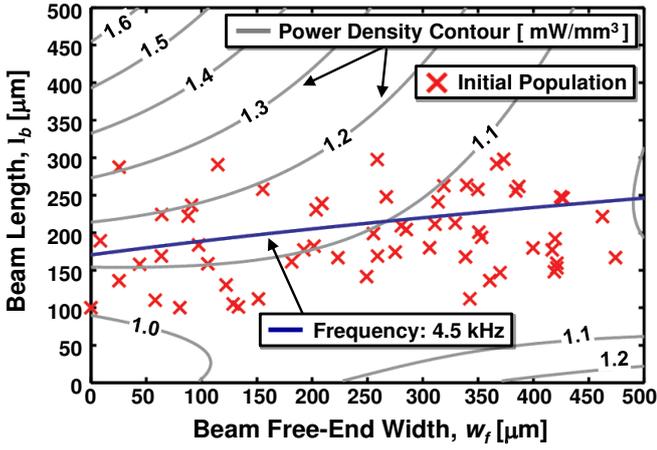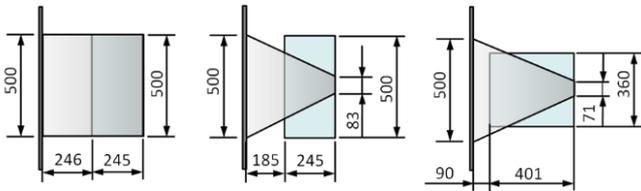
Fig. 3: Initial population distribution in the contour plot of power density (gray) and frequency (blue) in terms of w_f and l_b at the beginning of optimization.

output prediction. For both cases, the amplitude of the input acceleration is set to 1.5 g and the targeted frequency is 4.5 kHz with a maximum beam width of 500 μm. The maximum allowable stress is chosen to be 120 MPa which is the ultimate tensile strength of the Pt electrodes whose geometries are not considered in the optimization. The geometries of the reference structure with a rectangular cantilever and proof mass that satisfies all the global constraints are chosen for maximum power output. The optimization analysis based on genetic algorithms is performed with MATLAB®.

Optimization Case I

In Case I, the ability of power scavenging per unit

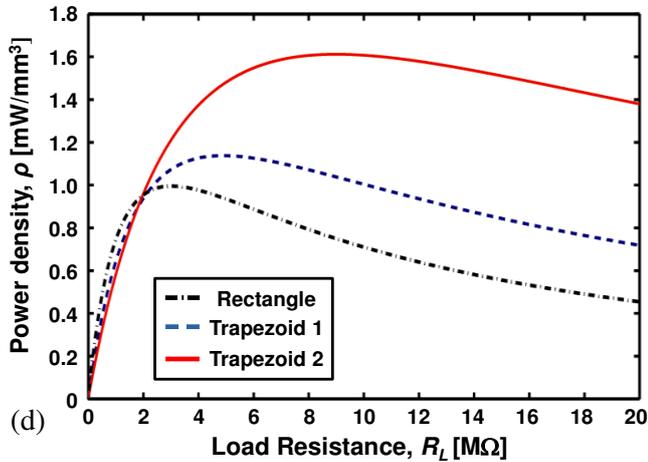
(a) Rectangle   (b) Trapezoid 1   (c) Trapezoid 2

Fig. 5: (a)-(c) Lateral geometries of reference rectangular and optimized trapezoidal devices; (d) power density versus load resistance for the three structures in Case I.
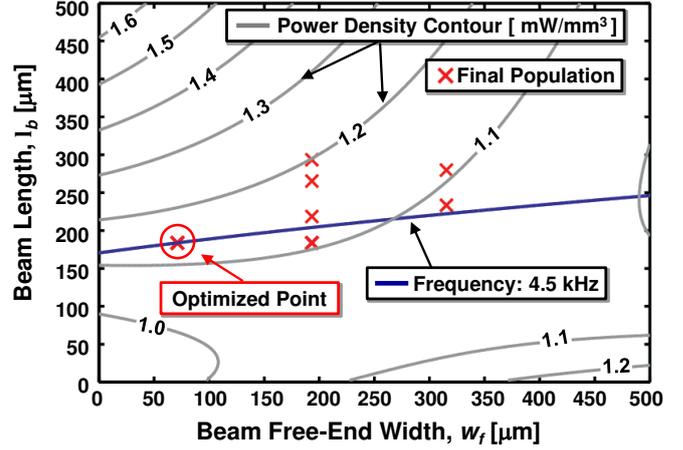
volume of the piezoelectric material serves as the objective function. In Case I(a), for which only the geometries of the cantilever beam are considered, the optimization problem has only two variables and the response of the power density can be shown in the surface plot in terms of w_f and l_b (Fig. 2). The nonlinear constraint on frequency which is not shown in the figure represents a single line on the surface.

The behavior of the power density in terms of w_f and l_b can be better illustrated on the 2-D contour plot (Fig. 3). For this optimization problem, a population size of 60 is chosen with a randomly uniformly generated initial population shown as red crosses in Fig. 3. Fig. 4 shows the population distribution when the optimization is terminated after seven generations and most of the individuals converge to the optimized point. It can be verified from the contour plot that a global maximum point is found after the optimization. The rest of the red crosses represent the individuals selected and produced for increasing diversity in the population which do not converge to the optimal point. The optimized solution shows similar results as in [1].

In Case I(b), same optimization is performed where both the geometries of cantilever and proof mass are considered. As the simulated results shown in Fig. 5, by changing the cantilever into a trapezoidal
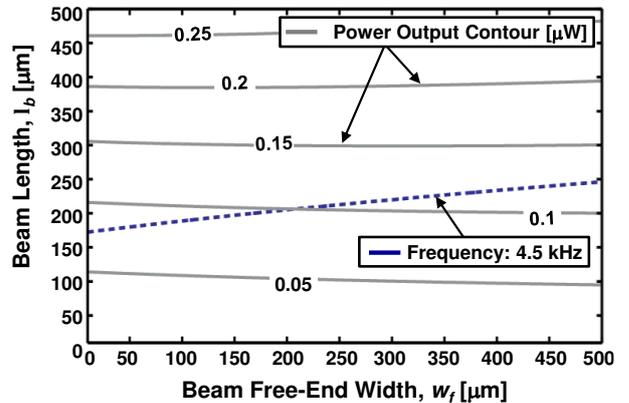

Fig. 4: Population distribution in the contour plot at the end of optimization.

Fig. 6: Contour plot of total output power (gray solid lines) and frequency (blue dashed line) in terms of w_f and l_b.

shape (with optimized geometries shown in Fig. 5(b)), more power can be extracted from a certain volume of the active piezoelectric material. In addition, by optimizing both the geometries of the beam and proof mass (Fig. 5(c)), the power density can be improved by 162% with respect to the reference rectangular cantilever, which is much larger than that when considering only the beam geometries, showing a 114% improvement on power density.

Although altering the shape of a cantilever in to a trapezoid shows an improvement on the power scavenging ability from a unit volume of the piezoelectric material, it can be observed from Fig. 6, the total power scavenged from the device reduces. In fact, the rectangular beam is shown to provide maximum total power among the three structures in Case I, which is assumed to be due to the sacrifice of the active beam surface area covered by electrodes. In the above analysis, a constant Q of 500 is assumed for both shapes of the cantilever structure. However, the beam geometries should have some influences on the energy loss mechanisms, which determine the Q. In fact, the geometries of the cantilever would primarily affect the air damping coefficients [8]. Therefore, in Case II, an optimization of the cantilever geometries for maximizing total power scavenged per footprint of the device will be conducted with the considerations of air damping effects.

Optimization Case II

Based on the results of Case I, here, the optimization analysis is performed on a cantilever with a reverse trapezoidal shape, and damping effects at atmospheric pressure is introduced. The mechanical

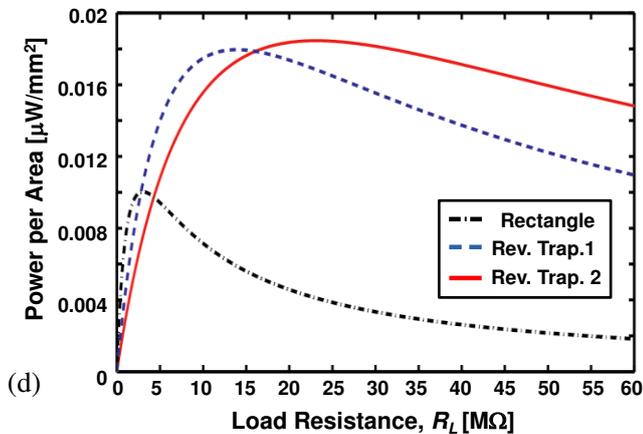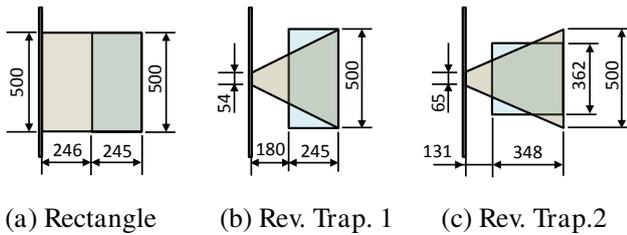(a) Rectangle   (b) Rev. Trap. 1   (c) Rev. Trap.2

(d)

Fig. 7: (a)-(c) Lateral geometries of reference rectangular and optimized reverse trapezoidal devices; (d) power per footprint versus load resistance for the three structures in Case II.

damping ratio is assumed to be dominated and determined by the air damping coefficients. The air damping coefficients for the beam and proof mass can be expressed as Eqn. (7) and (8), respectively.

$$\zeta_{air,beam} = \frac{3\pi\mu w_b + 0.75\pi w_{b,ave}^2 \sqrt{2\rho_a \mu \omega_n}}{2\rho_b t_b w_{b,ave}^2 \omega_n} \quad (7)$$

$$\zeta_{air,mass} = \frac{3\pi\mu w_m + 0.75\pi w_m^2 \sqrt{2\rho_a \mu \omega_n}}{2m\omega_n} \quad (8)$$

where μ is the air viscosity, ρ_a is the air density, ρ_b and t_b are the density and thickness of the beam, respectively [9].

The optimized geometries and the results of Case II are presented in Fig. 7. The total power per footprint enhances to 179% with optimized reverse trapezoidal beam geometries (Fig. 7(b)). On the other hand, a 184% improvement on total power per footprint is simulated with optimized beam and proof mass (Fig. 7(c)). The air damping coefficients of the beam corresponding to the referenced and optimized geometries show that a slightly higher Q is achieved with the reference rectangular structure due to the larger average beam width.

CONCLUSION

A power efficiency optimization for a piezoelectric cantilevered energy harvester on lateral geometries has been demonstrated. Genetic algorithms are applied for the nonlinearly-constrained optimization of an energy scavenging system with multiple design variables. The design case of a trapezoidal shape cantilever shows that the power scavenged from unit volume of the piezoelectric material can be improved to 1.6× with optimized geometries of beam and proof mass in comparison with a reference rectangular device. Moreover, the other design case on reverse trapezoidal shape cantilever with damping effects introduced into the system has been optimized and the simulated output power per footprint shows a 1.8× enhancement over the reference device.

REFERENCES
[1] Benasciutti, D. et al. 2009 Microsys. Tech. **15** 657-668
[2] Shen, D. et al. 2008 J. Micromech. Microeng **18** 055017
[3] Roundy, S. et al. 2005 IEEE Pervas. Comput.**4** 28-36
[4] Mehraeen, S. et al. 2008 IEEE Ind.Technol.pp 1-8, 21-24
[5] Lu, F. et al. 2004 Smart Mater. Struct. **13** 57-63
[6] Erturk, A. and Inman, D.J. 2008 J. Intell. Mater. Syst. Struct. **19** 1311-132
[7] Gen, M. and Cheng, R. 1997 Genetic Algorithms and Engineering Design (New York: John Wiley & Sons, Inc.)
[8] Choi, W.J. et al. 2006 J. Electroceram **17** 543-548
[9] Hosaka, H. et al. 1994 *IEEE MEMS '94* 193-198