INCORPORATION OF COLLECTOR EFFECTS IN AN OPTIMAL DESIGN PROCEDURE FOR MICRO HEAT EXCHANGER PLATES: A CASE STUDY

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Abstract:
This paper presents a procedure for micro heat exchanger design that incorporates hydraulic as well as thermal effects of flow non-uniformity induced by collectors. The procedure is based on a hierarchy of two models: a one-dimensional model and a CFD model. The one-dimensional model is used to adjust the geometry according to the design objectives. The CFD model is used to fit the parameters of the one-dimensional model to bring both models in accordance. The present study provides quantitative data for the optimal plate dimensions and resulting maximal power density of a gas-gas micro heat exchanger with 80% effectiveness. The results show the necessity of modeling flow distribution effects in the device.

Keywords: Thermal design, Flow configuration, Parallel plates, CFD-assisted design

INTRODUCTION

Small-sized heat exchangers are envisaged for thermal management and efficiency improvement of portable equipment such as electronic devices, fuel cells, or ultra-micro gas turbines [1-4]. Due to smaller boundary layer thicknesses and an increased heat exchanger surface area-to-volume ratio large power densities can be attained. However, the thermal effectiveness of micro heat exchangers can suffer from the decreasing stream-wise dimensions due to axial conduction [5]. Rogiers and Baelmans [6] were the first to propose a design method that incorporates this limiting effect by fixing the effectiveness as a constraint in addition to the pressure drop. The original study considered a parallel plate geometry with uniform flow distribution. In practice however, the hot and cold flows are distributed from and collected by the manifolds. This leads to flow non-uniformities and results in a lower mass flow rate and decreased utilization of available heat transfer surface.

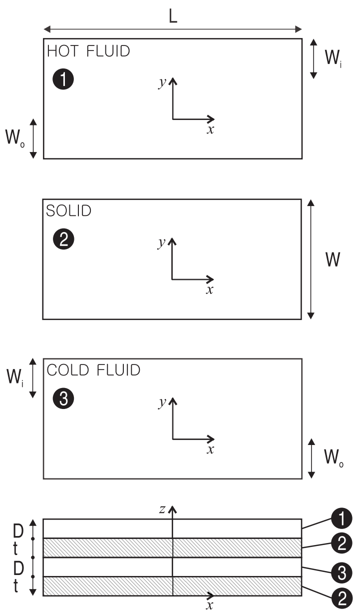

Fig. 1: Constructal unit of the heat exchanger

In this paper we propose a design procedure that incorporates flow distribution effects. The procedure is essentially a hierarchical method that iteratively meets the design goal by combining high fidelity simulations with an optimization based on a lower order model. In this case, the high fidelity model is a CFD model, whereas the low order model is a one-dimensional model with coefficients that are fitted to the numerical data [6].

This paper is organized as follows. First the problem setting is given with a description of the geometry, the design goal and the design parameters. Second, the CFD model is proposed. Then, the model fitting and optimization strategies are discussed. Finally, a case study is presented and the resulting designs are discussed.

PROBLEM SETTING

The heat exchanger we consider is a stack of alternating hot and cold fluid layers separated by solid plate layers (see Fig. 1). Each fluid layer has an inlet and outlet connection. The hot and cold layers are symmetrical in order to allow for four fluid connections. This type of heat exchanger can be produced by metal etching [2-3]. The dimensions are: the total length L and width W, the inlet/oulet width W_{io} of the flow connections, the channel depth D, and the thickness t of the plate material separating the fluid streams.

The design goal is to obtain maximal thermal power density for a given pressure drop and a given effectiveness (heat recuperation efficiency). The capacity rates of hot and cold flows are presumed to be equal. Further, the plate thickness and all material properties are given. The proposed method uses the length and plate spacing as primary design variables. The effect of width and inlet/outlet width can then be studied parametrically.

NUMERICAL MODEL

Momentum and heat transfer are modeled under the assumptions of fully developed flow and heat transfer between parallel plates. This allows for integration of the momentum and thermal energy equations along the channel depth (z-direction) leading to a set of two-dimensional equations. The momentum equations become:

$$\vec{u}_{1,2} = -\frac{D^2}{12\mu}\vec{\nabla}p_{1,2} = -\frac{2D_h^{\,2}}{Po\mu}\vec{\nabla}p_{1,2}, \qquad (1)$$

with $\vec{u} = \frac{1}{D}\int_0^D \vec{u}dz$, hydraulic diameter D_h=2D, and Po=96 the Poiseuille number for fully developed flow between parallel plates. Subscripts "1" and "2" refer to the hot and cold side respectively. Inserting the velocity into the continuity equation $\vec{\nabla}\cdot\vec{u} = 0$, yields an equation for the pressure:

$$\vec{\nabla}\cdot\left(-\frac{2D_h^{\,2}}{Po\mu}\vec{\nabla}p_{1,2}\right) = 0. \qquad (2)$$

At the inlet and outlet of each flow region, the pressure is imposed. Elsewhere, the normal pressure gradient is set to zero in order to model impermeable walls. Thermal energy conservation in the flow regions is modeled as purely convective in the plane of the flow, and purely conductive in the z-direction. Integrating the flow energy equation along the z- direction yields an equation in the bulk temperature $T_{1,2}$:

$$\vec{\nabla}\cdot\left(\rho c_p u_{1,2}T_{1,2}\right) = \frac{1}{D}\left[2h\left(T_{1,2} - T_w\right)\right], \qquad (3)$$

with the convective heat transfer coefficient $h=kNu/D_h$, and Nu=8.235 the Nusselt number for fully developed heat transfer between parallel plates with constant heat flux boundary conditions. At the inlets, the temperature is imposed. Elsewhere, the normal temperature gradient is set to zero. Heat transfer in the plate is modeled as purely conductive with convective boundary conditions lumped as source terms into the two-dimensional equations. Integrating the wall energy equation over the thickness t yields:

$$\vec{\nabla}\cdot\left(-k_w\vec{\nabla}T_w\right) = \frac{1}{t}\left[h\left(T_w - T_1\right) + h\left(T_w - T_2\right)\right]. \qquad (4)$$

Note that the lateral conduction resistance of the plate is not taken into account since it can be neglected for air with steel as plate material [6].

The previous equations are discretized and solved by an in-house CFD package that employs the Finite Volume Method. Convective fluxes are approximated by upwind interpolation, and diffusive fluxes are approximated by central differencing. The pressure and velocity fields are solved first. Subsequently the thermal equations are solved iteratively by performing a minimum of 100 multiplicative Schwarz iterations with exact sub-solves. From the result, the plate mass flow and outlet temperature are computed as:

$$\dot{m}_{1,2} = \int_{out1,2} \rho\vec{n}\cdot\vec{u}_{1,2}dA, \qquad (5)$$

$$T_{1,2,o} = \frac{1}{\dot{m}_{1,2}c_p}\int_{out1,2} \rho c_p\vec{n}\cdot\vec{u}_{1,2}T_{1,2}dA. \qquad (6)$$

Since all simulations in this paper are for balanced heat exchangers, the effectiveness is computed as:

$$\varepsilon = \frac{\left(T_{2,o} - T_{2,i}\right)}{\left(T_{1,i} - T_{2,i}\right)}. \qquad (7)$$

REDUCED MODEL AND DESIGN PROCEDURE

The goal of this study is to provide the design engineer with a simple algorithm for selecting dimensions of a micro heat exchanger plate that yield a given pressure drop and effectiveness while maximizing the thermal power density. The main idea is to perform the optimization based on a reduced physical model, and to use the CFD model to fit the coefficients of this reduced model. This leads to a simple iterative scheme. We first present the reduced model. Then we describe how its coefficients are fitted, and how it is used to adapt the dimensions according to the design goals. Finally we describe the whole design procedure.

Reduced model

The reduced model is an equivalent representation of the heat exchanger as a counter-flow parallel plate heat exchanger. The length, width, channel size and plate thickness are the same as the corresponding dimensions of the actual heat exchanger. Also the transport coefficients Po and Nu are taken equal. In order to bring the performance characteristics of the reduced model into accordance with the numerical model, empirical configuration factors F_{hy}, F_{th} and F_{ax} are introduced into the former.

The mass flow rate per channel of the reduced model is given by (see [6]):

$$\dot{m} = \frac{1}{F_{hy}}\left(\rho\frac{2D_h^{\,2}}{Po\mu}\frac{\Delta p}{L}WD\right). \qquad (8)$$

The effectiveness is described by the analytical model of Kroeger [6] that incorporates axial wall conduction:

$$\varepsilon = 1 - \frac{1}{1 + NTU'\left(1 + M'\cdot\Phi\right)/\left(1 + M'\cdot NTU'\right)}, \qquad (9)$$

$$\Phi = \sqrt{\frac{M'\cdot NTU'}{1+M'\cdot NTU'}}\tanh\left[\frac{NTU'}{\sqrt{\frac{M'\cdot NTU'}{1+M'\cdot NTU'}}}\right], \quad (10)$$

Here, *NTU´* and *M´* are modified versions of the number of transfer units and the axial conduction parameter for a counter-flow heat exchanger:

$$NTU' \equiv F_{th} NTU = F_{th}\frac{UA}{\dot{m}c_p} = \frac{F_{th}}{\dot{m}c_p}2k\frac{WL}{D}\frac{Nu}{4} \quad (11)$$

$$M' \equiv \frac{1}{F_{ax}}M = \frac{1}{F_{ax}}\frac{UA_{ax}}{\dot{m}c_p} = \frac{1}{F_{ax}}\frac{1}{\dot{m}c_p}\frac{2k_w Wt}{L} \quad (12)$$

Expressions for the last equalities can be found in [6].

Fitting strategy

The general expression for thermal power density per unit of inlet temperature difference is:

$$PD \equiv \frac{\dot{Q}'''}{(T_{1,i}-T_{2,i})} = MDc_p\varepsilon\,, \quad (11)$$

with the mass flow rate density given by:

$$MD \equiv \frac{\dot{m}}{WL2(D+t)}\,. \quad (12)$$

Since our objective is to maximize power density for a given effectiveness, one can see from Eq. (11) that the reduced model must yield the same mass flow rate and the same effectiveness for a given set of geometrical parameters. The former is realized by fitting F_{hy} to numerical data, while the latter is realized by determining F_{th} and F_{ax}.

By performing a numerical simulation, the mass flow rate can be computed with Eq. (5). Setting now $\dot{m}_{CFD} = \dot{m}$, with the right hand side given by Eq. (8), the hydraulic configuration factor can be easily solved.

For fitting the effectiveness, there is some freedom on how to compute the configuration factors. One should however keep in mind that the nature of the optimization problem is such that the size of the heat exchanger is reduced until axial conduction deteriorates the performance [6]. Therefore the effectiveness without axial conduction (*M*=0), as well as the effectiveness with axial conduction taken into account (*M*≠0) should be fitted. These two requirements allows to compute F_{th} and F_{ax} uniquely. One can show that F_{th} is the classical F-factor appearing in heat exchanger design theory [7].

The configuration factor for surface area utilization F_{th} can now be computed as:

$$F_{th} = \frac{NTU'(M'=0,\varepsilon_{CFD,noax})}{NTU} = \frac{1}{NTU}\frac{\varepsilon_{CFD,noax}}{1-\varepsilon_{CFD,noax}}, \quad (13)$$

where $\varepsilon_{CFD,noax}$ the effectiveness (Eq. (7)) obtained from a CFD computation with axial plate conduction neglected. The configuration factor for thermal deterioration due to axial conduction F_{ax} is computed as:

$$F_{ax} = \frac{M}{M'(NTU',\varepsilon_{CFD})} \quad (14)$$

where ε_{CFD} is the effectiveness (Eq. (7)) obtained from a CFD computation with axial plate conduction taken into account. Computation of *M´* involves the numerical solution of Eqs. (9)-(10) for given ε and *NTU´*.

Optimal dimensions

The fitted reduced model is used to determine the optimal *D* and *L* that maximize power density for a given effectiveness and pressure drop. Rogiers and Baelmans [6] derived a closed form solution to this problem under the assumptions that, (1) the configuration is purely counter-flow, and; (2) the relative plate thickness *t*/*D* is constant. Based on modified expressions for the mass flow and effectiveness that take into account flow non-uniformity, a similar derivation as the one presented in [6] can be carried out. This leads to following expressions for the optimal dimensions:

$$D = \sqrt{\frac{\mu\alpha}{\Delta p}\frac{F_{hy}Po}{4}\frac{1}{F_{ax}}\frac{k_w}{k}\frac{t}{D}\Psi\Omega\Theta^{-1}}\,, \quad (15)$$

$$L = \sqrt{\frac{\mu\alpha}{\Delta p}\frac{F_{hy}Po}{F_{th}Nu}\left(\frac{1}{F_{ax}}\frac{k_w}{k}\frac{t}{D}\right)^2\Psi\Omega^3\Theta^{-3}}\,, \quad (16)$$

where the Ψ , Ω and Θ functions are defined and correlated in [6] as function of the imposed design effectiveness. The constraint of a constant plate thickness *t* is attained by iterating over these expressions, and updating *t*/*D*. It should be noted that the resulting geometry is optimal for the final *t*/*D*, and not necessarily optimal for the given *t*. Nevertheless, we adopt this design approach for its simplicity. Further notice that the one-dimensional theory does not give an answer on how to choose the plate width. It should be taken into account in a parameter study (see the section “CASE STUDY”).

Design procedure

The design procedure can now be sketched as follows:

(1) Set constraints: ε_d, Δp, *W*, *t*

(2) Initialize: $Po \leftarrow 96$; $Nu \leftarrow 8.235$; materials; $F_{hy} \leftarrow 1$; $F_{th} \leftarrow 1$; $F_{ax} \leftarrow 1$;

Table 1: Dimensions and other characteristics of the optimized designs

W [mm]	W_{io} [mm]	L [mm]	D [μm]	MD [kg/(s m^3)]	PD [kW/(m^{3}K)]	F_{hy} [-]	F_{th} [-]	F_{ax} [-]
15	15	18.0	161	275	111	1.00	0.95	1.00
	7.5	24.7	178	145	58.3	1.27	0.59	0.95
	5	28.9	194	112	45.0	1.47	0.53	0.83
30	30	18.0	161	275	111	1.00	0.95	1.00
	15	31.1	192	96	38.6	1.46	0.44	0.86
	10	39.6	216	67	26.8	1.69	0.38	0.70
60	60	18.0	161	275	111	1.00	0.95	1.00
	30	37.1	211	65	26.2	1.88	0.35	0.83
	20	52.6	247	41	16.4	2.13	0.29	0.59
90	90	18.0	161	275	111	1.00	0.95	1.00
	45	40.3	224	50	20.1	2.36	0.30	0.89
	30	59.6	272	32	12.8	2.65	0.26	0.55

(3) Loop n=1, 2, 3 …
- (a) $D \leftarrow$ RHS(Eq. (15))
  $L \leftarrow$ RHS(Eq. (16))
- (b) $F_{hy}{}^* \leftarrow F_{hy}$; $F_{th}{}^* \leftarrow F_{th}$; $F_{ax}{}^* \leftarrow F_{ax}$;
- (c) Compute F_{hy}, F_{th}, F_{ax}, ε_{CFD}
- (d) IF ($\left|\varepsilon_{CFD} - \varepsilon_d\right| \le 0.1\%$,
  AND $\left|F_{hy} - F_{hy}^*\right| / F_{hy} < 0.01$,
  AND $\left|F_{th} - F_{th}^*\right| / F_{th} < 0.01$,
  AND $\left|F_{ax} - F_{ax}^*\right| / F_{ax} < 0.01$
)
  Exit Loop

CASE STUDY

We investigate the optimal design of an air-air heat exchanger with ε_d=0.8, Δp=1 kPa, t=100 μm, k=0.025 W/(m K), ρ=1.205 kg/m^3, μ=1.81·10^{-5} Pa s, c_p=1006 J/(kg K), k_w=19 W/(m K). The design algorithm is executed for W = {15; 30; 60; 90}mm, and for W_{io}/W = {1/1; 1/2; 1/3}. This provides a good quantitative understanding of the influence of flow distribution on the power density performance of parallel plate heat exchangers. The mesh employed has 180 by 180 cells.

The numerical results of the case study are presented in Table 1. For every plate width, there is a reference case with W_{io}=W which yields one-dimensional velocity and temperature fields. In this case the optimal designs are all the same. One can see that due to discretization errors F_{th} is 0.95 whereas a value of 1.00 is expected from theory. The effect on the optimal dimensions is however mitigated by the square root in Eq. (16).

For every given width, the power density and mass flow density decrease drastically with decreasing W_{io}. This is due to both increased friction and reduced thermal usage of the surface area. This flow collection effect increases with increasing width. For W=15mm the deterioration is at least a factor of 2 compared to the uniform case. For W=90mm the deterioration surpasses a factor of 5. The differences in plate spacing D due to flow collection are relatively modest: at most 70% increase with respect to the one-dimensional case. The differences in L are much larger: up to 230% increase compared to the uniform flow case.

CONCLUSION

In this paper a procedure is proposed to design micro heat exchangers with flows arranged predominantly in counter-flow. The procedure makes use of a small number of CFD computations to repeatedly fit a reduced model and uses the latter for adjusting the design to meet the effectiveness constraint and to achieve a maximal power density. The method was illustrated for different plate and inlet widths. This study reveals that additional friction losses and decrease in thermal utilization due to flow distribution are highly detrimental to the performance of a micro plate heat exchanger.

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