ANALYSIS OF VIBRATION ENERGY HARVESTERS UTILIZING A VARIETY OF NONLINEAR SPRINGS

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Abstract: This paper reports a numerical investigation of the potential benefits of utilizing softening springs in comparison to linear springs and hardening springs for vibration energy harvesters. Our results show that the energy harvester using softening springs is better than the energy harvester using linear springs or hardening springs for broadband random vibrations. This is due to its potential to give both wider bandwidth and larger harvested power.

Keywords: MEMS, energy harvester, nonlinear systems, softening springs, hardening springs.

INTRODUCTION

Energy harvesting from motion is promising as means to power wireless sensor nodes in constructions, machinery and on the human body [1].

Most vibration-based energy harvesters are spring-mass-damper systems which generate maximum power when the resonant frequency of the device matches the frequency of the ambient vibration. As many environmental vibration spectra exhibit a range of frequencies [2], resonant vibration energy harvesters will have restricted applicability in these environments. Nonlinear stiffness has been exploited to increase the bandwidth of the energy harvesters to overcome this limitation [3]. Nonlinear stiffness could be a hardening spring [4] and/or a softening spring [5-6]. Nevertheless, comparisons of the potential benefits of using linear springs, softening springs and hardening springs for vibration energy harvesters have not yet been reported, in particular when the devices are driven by broadband random vibrations. In this paper, we report a numerical investigation of potential benefits of utilizing softening springs in comparison to linear springs and hardening springs for vibration energy harvesters.

MODELING ANALYSIS

The equations of motion for a resonant energy harvester with a linear electromechanical transducer can be written:

$$m\ddot{x} = -F_{sp}(x) - \alpha q - b\dot{x} + ma \qquad (1)$$

$$-R\dot{q} = V = \alpha x + \frac{1}{C}q \qquad (2)$$

where m is the proof mass, x its displacement, $F_{sp}(x)$ the spring force, q the charge, b the damping coefficient, a the negative of the package acceleration, R the load resistance, C the clamped capacitance and α coefficient determining the linear electromechanical coupling.

We consider a phenomenological spring force on the form:

$$F_{sp}(x) = k_1 x + k_3 x^3 + k_5 x^5 \qquad (3)$$

The term $k_1 x$ is the linear part of the force and the term $(k_3 x^3 + k_5 x^5)$ models the nonlinear part. The linear stiffness k_1 is kept constant. The positive constant k_5 represents nonlinear spring stiffness at large deflections. The k_3 is changed to have different nonlinear stiffness for intermediate deflections: hardening springs or softening springs as shown in Figure 1.

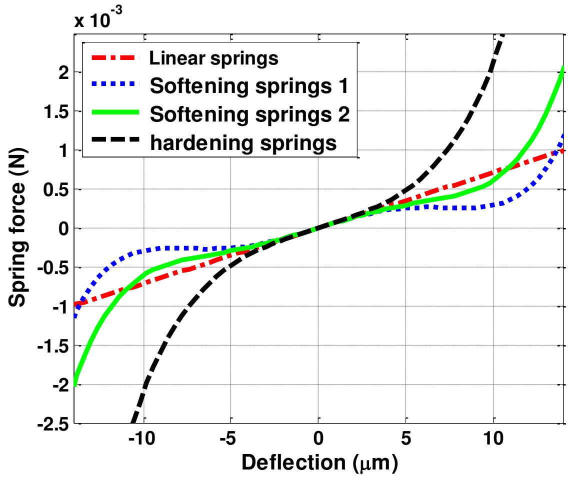

Figure 1: Spring force versus deflection for different springs: linear springs, softening springs 1, softening springs 2 (less soften than variety 1) and hardening springs.

Figure 2 shows the equivalent circuit for an energy harvester represented by the equations of motion (1) and (2). The selected parameters (table 1) are close to the dimension of the MEMS electrostatic energy harvester in [3].

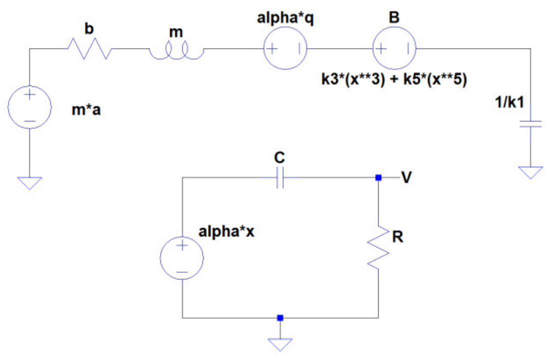

Figure 2: The equivalent circuit for the energy harvesters

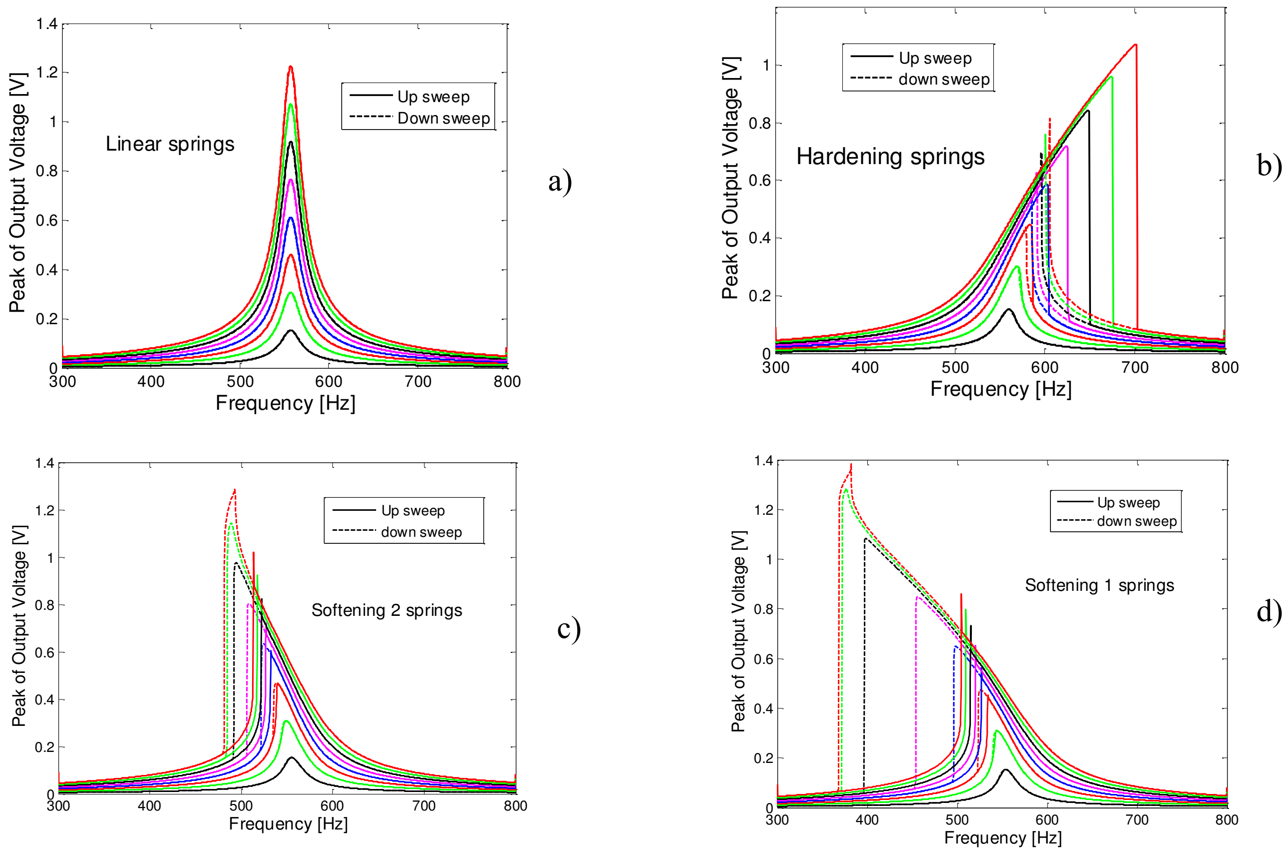

Figure 3: The frequency response for frequency sweeps at different peak excitation amplitudes of 0.05 g, 0.10 g, 0.15g, 0.20 g, 0.25 g, 0.30 g, 0.35 g and 0.40 g. a) linear springs b) hardening springs c) softening spring 2 d) softening springs 1

TABLE I. MODEL PARAMETERS

Symbol	Description	Value
m	Proof mass	5.76 mg
b	Mechanical damping	$7{\cdot}10^{-4}$ Ns/m
k_1	Linear stiffness	71 N/m
	Softening nonlinear stiffness 1	$-0.92{\cdot}10^{12}$ N/m^3
k_3	Softening nonlinear stiffness 2	$-0.6{\cdot}10^{12}$ N/m^3
	Hardening nonlinear stiffness	$0.9{\cdot}10^{12}$ N/m^3
k_5	Nonlinear stiffness	$5{\cdot}10^{21}$ N/m^5
C	Transducer capacitance	9.8 pF
α	A linear electromechanical coupling constant	$-1.84{\cdot}10^{5}$ V/m

RESULTS AND DISCUSION

Sinusoidal excitations

Figure 3 shows the output voltage of the energy harvester as a function of frequency for frequency up-sweeps and down-sweeps at different excitation amplitudes. For the linear springs, the resonance frequency does not change with increasing excitation amplitudes. Moreover, the output voltages of frequency up-sweeps and down-sweeps are identical (Fig. 3-a). For the nonlinear springs, the output voltage curves are the same as for the linear springs when the excitation amplitude is small, e.g. 0.05 g. When excitation amplitudes get larger so that the spring nonlinearity is pronounced, the resonant frequency is shifted toward higher frequency for hardening springs (Fig. 3-b) and to lower frequency for softening springs (Fig. 3-c, d). Consequently, the bandwidth of hardening springs is wider for frequency up-sweeps while the bandwidth of softening springs is wider for frequency down-sweeps. In addition, in comparing the two softening springs, the softening springs 1 obtain wider bandwidth than the softening springs 2 do.

White noise excitations

Figure 4 shows the average output power as a function of load resistance under broadband excitation at level of $3.0{\times}10^{-4}$ g^2/Hz. When the load resistance is very small $(R \to 0)$, the current i through it will be approximately the short circuit current, $i = \alpha C\dot{x}$, the system can be described by a nonlinear second order model whose reduced probability distribution of the velocity is independent of the mechanical nonlinearities and therefore gives the same output power for all types of springs [7]. The equations of motion (1) and (2) will then agree with [8] and it is understandable that no benefit of nonlinearities was found in that work. In our simulations, we see that the average output power is almost the same for different springs for small load resistances. When increasing the load resistance, the nonlinearity in the stiffness clearly affects the average output power.

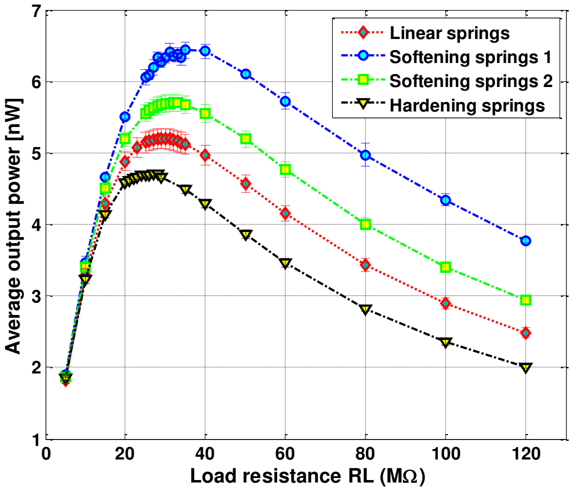

Figure 4: Average output power as a function of load resistance under broadband excitation at level of 3.0×10⁻⁴ g²/Hz. The optimal loads are 25 MΩ, 29 MΩ, 32 MΩ and 35 MΩ for hardening springs, linear springs, softening springs 2 and softening springs 1, respectively.

Firstly, the optimal load resistance is slightly different for different springs. From [7], we know that the optimal load resistance for the linear spring harvester under the white noise excitation is given by ($R_{opt} = 1/\omega_0 C$) which is the same for all cases considered here. In Figure 4, we see that the most compliant springs, e.g. softening springs 1, have the largest optimal load resistance value (*35 M*Ω) while the hardening springs have the lowest optimal load resistance value (*25 M*Ω).

Secondly, the softening springs 1 obtain the highest average output power at the optimal load resistance while the hardening springs give the lowest average output power. That is still true if we compare the two cases for any choice of the same load resistance.

Figure 5 shows the output power spectral density (PSD) of different springs for various white noise excitation levels. When the white noise excitation level is small, e.g. 0.46×10^{-4} g^2/Hz, the output PSD is almost the same for different springs. Nevertheless, when excitations get larger, the nonlinearities in stiffness provide an enhanced bandwidth of the harvester. The hardening springs increase the bandwidth towards higher frequencies (Fig. 5-b) while the softening springs increase the bandwidth towards lower frequencies (Fig 5-c, d).

To compare the bandwidth for different springs, the output PSDs for different springs at the white noise excitation level of 2.8×10^{-4} g^2/Hz are shown together in Figure 6. At this excitation level, the 3-dB bandwidth of the softening springs 1 increase by about 2.5 and 10 times compared to the bandwidth of the hardening

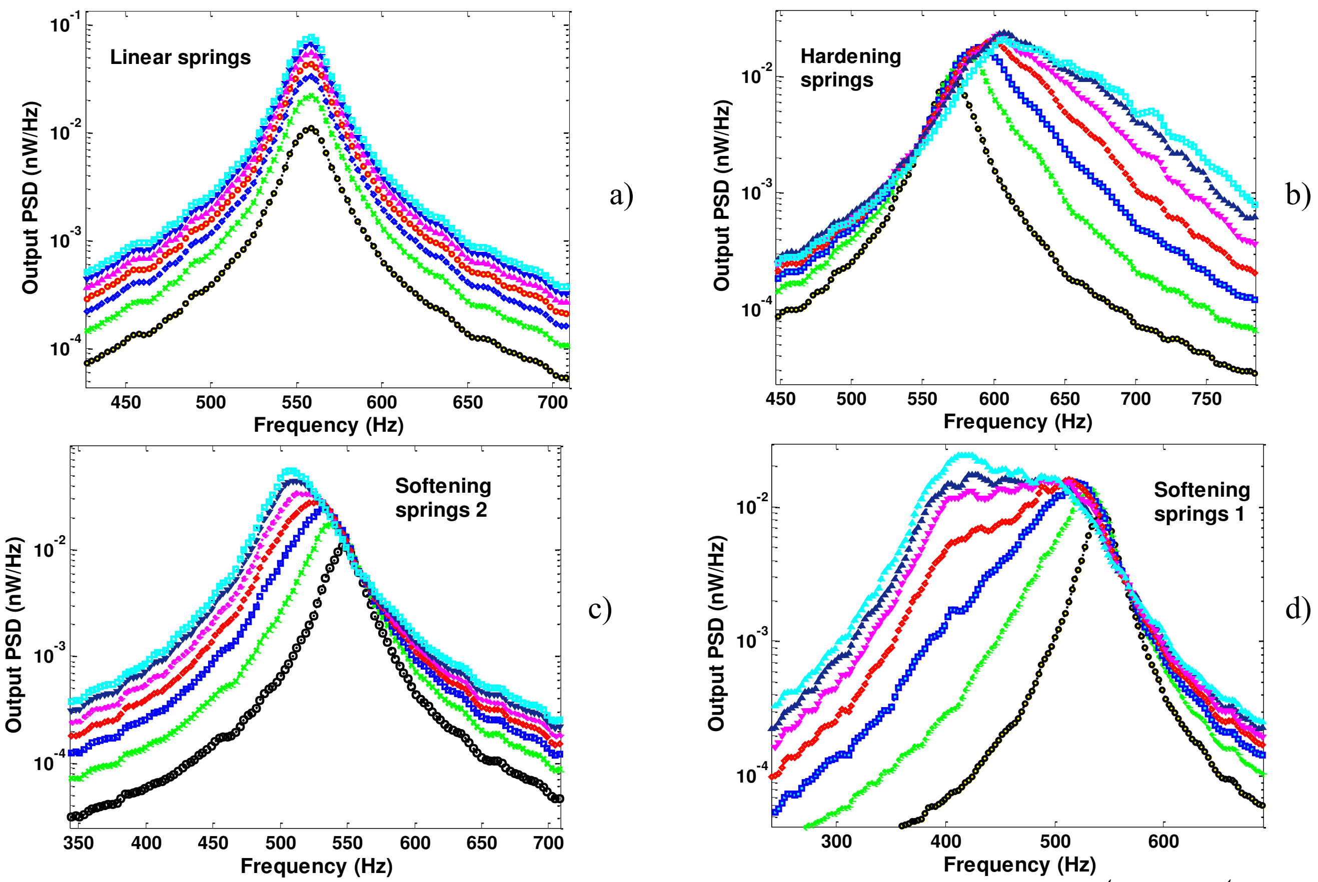

Figure 5: Output PSD as a function of frequency for many broadband excitation levels: 0.46×10⁻⁴, 0.92×10⁻⁴, 1.4×10⁻⁴, 1.8 ×10⁻⁴, 2.3×10⁻⁴, 2.8×10⁻⁴, 3.2×10⁻⁴ g²/Hz. a) linear springs b) hardening springs c) softening spring 2 d) softening springs 1.

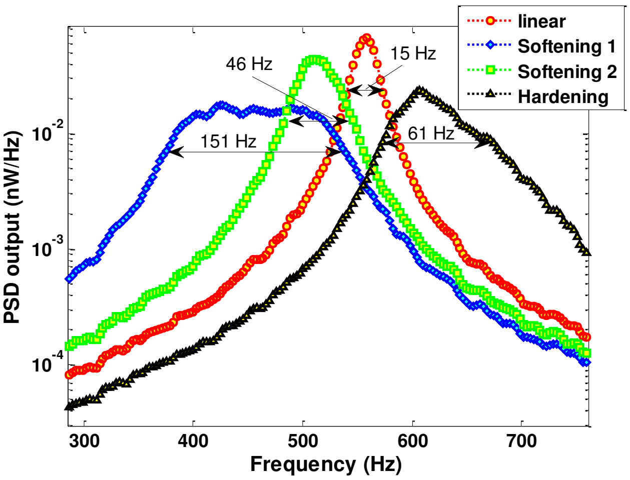

Figure 6: Output spectral density as a function of frequency under broadband excitation at level of 2.8×10^{-4} g^2/Hz. The bandwidth is calculated at 3-dB.

springs and linear springs respectively.

The most important criterion to evaluate the vibration energy harvester under white noise excitation must be the average harvested power. To compare the harvested power under white noise excitation for different springs, the average output power as a function of the average PSD of the excitation is depicted in Figure 7. For sufficiently intense white noise, the softening springs achieve higher average output power than the linear springs and the hardening springs do. Additionally, in Figure 7 we also observe that the softening springs 1 harvest more output power than the softening springs 2. So, both softening springs and hardening springs can enhance the bandwidth of the energy harvester, but softening springs have the advantage that they give more output power.

CONCLUSION

We have presented the potential benefit of using softening springs in comparison to linear springs and hardening springs for vibration energy harvesters. Through numerical calculations, we showed that the nonlinearities in the stiffness can provide an enhancement in the performance of the energy harvester.

For sinusoidal excitation, the hardening springs increase the bandwidth for frequency up-sweeps. In contrast, the softening springs increase the bandwidth for frequency down-sweeps.

For white noise excitation, we found that the nonlinearities in the stiffness can increase the bandwidth of the energy harvester. However, softening springs obtain more harvested power than either linear springs or hardening springs do. Furthermore, increased softening behavior in the stiffness will increase the bandwidth and the harvested power.

In conclusion, the softening springs are better than linear springs or hardening springs in designing energy harvester for random vibration environments. This is due to its potential to give both wider bandwidth and larger harvested power. Experimental results on a benefit of

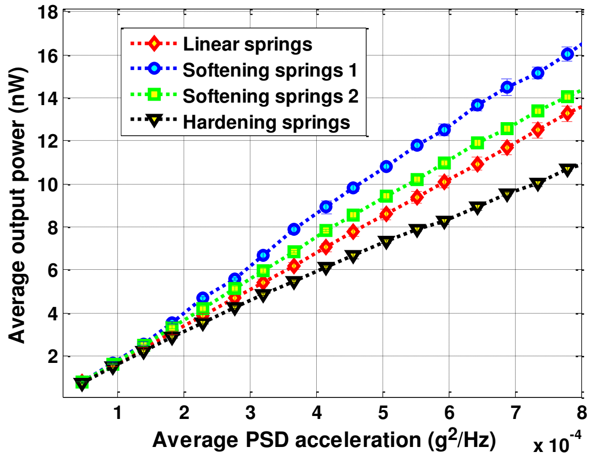

Figure 7: Average output power as a function of the average PSD of the excitation. The load resistance is the optimal load in Figure 4.

using softening springs for vibration energy harvester will be reported elsewhere [9].

ACKNOWLEDGEMENTS

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