

THE IMPORTANCE OF COUPLING STRENGTH FOR MAXIMIZING THE OUTPUT POWER OF ELECTRODYNAMIC VIBRATIONAL ENERGY HARVESTERS

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Abstract: For electrodynamic (magnetic/inductive) vibrational energy harvesters, this paper introduces and explores the impact of a key design variable—“coupling strength”—that governs the energy harvesting effectiveness. First, the importance of magnetic coupling strength is explained analytically using a linear vibrational harvester model. Second, from Faraday’s law of induction it is shown that a radial magnetic field pattern is the most practical means for maximizing the coupling strength for oscillating rectilinear motions. Third, a harvester with tunable coupling strength is designed, built, and tested to demonstrate and validate the theory. By increasing the coupling strength, the harvester is shown to capture >90% of the theoretical available power, a significant improvement over most prior works (<<50%).

Keywords: magnetic energy harvester, vibrational energy harvester, electrodynamic transduction, modeling

INTRODUCTION

Electrodynamic (magnetic/inductive) vibrational energy harvesters have received much attention, in part because of the relatively high power density afforded by electrodynamic transduction [1-5]. These devices function by converting relative motion between coils and magnets into electrical energy. However, it has been well documented that most vibrational energy harvesters, including electrodynamic harvesters, fall far short of their theoretical energy reclamation limits [1, 4, 5]. This is partially due to the lack of consideration in the electrodynamic coupling structure design, which involves the design of the shape and dimensions of the magnet(s) and the coil, as well as the motion path.

The degree to which an electrodynamic energy harvester can effectively convert mechanical to electrical energy is highly dependent on the “strength” of the electrodynamic interactions. An important parameter is the electrodynamic transduction coefficient, K (with a unit of $V \cdot s/m$), which represents the voltage generated per unit of relative velocity (m/s). Sometimes, this term is called “coupling coefficient” [6] or, errantly, “coupling factor” [5]. The transduction coefficient is proportional to the length of the coil and the magnetic flux density acting upon the wire. Thus, one way to increase K is to add more coil turns. However, this will either increase the size or the resistance of the coil, neither of which are desired in most applications. As a second method, by carefully designing the magnetic field pattern, and the relative motion between the magnetic and the coil, the magnetic flux density can be increased.

For example, Spreeman et al. have performed a comparison of different electrodynamic transducer architectures using numerical simulations [7]. The study shows that the opposing magnet architecture yields the highest transduction coefficient. The

structure has been used by Waters et al. to construct a highly sensitive energy harvester for low-level vibrations. The harvester generates a voltage of 2 V amplitude at vibration level as low as 0.0027g (the power output of this specific device is not mentioned) [8].

These prior efforts have focused attention on the importance of the transduction factor. However, as will be shown in this paper, realizing the theoretical performance limits for an electrodynamic harvester is actually dependent on maximizing the coupling strength, a function of K , coil resistance and damping coefficient that will be defined later.

THEORY

Electrodynamic Coupling Strength

The linearized model of a mass-spring-damper-based electrodynamic energy harvester has been used widely in the literature [5, 6]. The power delivered to a resistive (non-reactive) load is maximized when (a) the device works at the mechanical natural frequency and (b) the load resistance is equal to the magnitude of the harvester output impedance (which at the natural frequency is the sum of the coil resistance and the mechanical damping equivalent resistance) [5, 6]. At this optimum condition, the load power is given by

$$P_{\text{load_max}} = \frac{m^2 a^2}{8b} \frac{1}{\frac{R_{\text{coil}} b}{K^2} + 1} = P_{\text{theory_max}} \eta, \quad (1)$$

where m is the proof mass (in kg), a is the excitation acceleration amplitude (in m/s^2), b is the mechanical damping coefficient (in $N \cdot s/m$), R_{coil} is the coil resistance (in Ω) and K is the electrodynamic transduction coefficient (in $V \cdot s/m$). Since the term $\frac{R_{\text{coil}} b}{K^2}$ is always greater than 0, it can be seen that

$P_{\text{theory_max}} = \frac{m^2 a^2}{8b}$ is the theoretical maximum power for a given mass, excitation acceleration and damping coefficient. The non-dimensional term η is defined as

the "power ratio" and represents the percentage of theoretical maximum power actually delivered to the load.

It is clear that the term $\frac{R_{\text{coil}}b}{K^2}$ needs to be minimized to make the actual load power close to $P_{\text{theory_max}}$. Because of the importance of this dimensionless ratio, a parameter denoted as the "coupling strength" is defined as

$$\gamma = \frac{K^2}{R_{\text{coil}}b}. \quad (2)$$

The relationship between coupling strength γ and power ratio η is plotted in fig. 1. It can be seen that the power ratio increases monotonically with the coupling strength. Therefore, the magnetic coupling architecture should maximize the coupling strength. A coupling strength of 1 yields a power ratio of 50%. In order to achieve a power ratio of >90%, the coupling strength needs to be greater than 10.

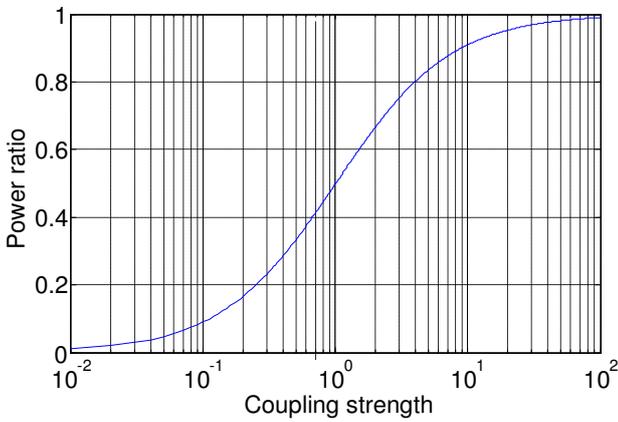


Fig. 1: Theoretical relationship between the power ratio and the coupling strength.

Optimum Field Pattern

From Faraday's law of induction, the induced voltage in an electrodynamic transducer is given by

$$\mathbf{V} = \int_{l_{\text{coil}}} \vec{v} \times \vec{B} \cdot d\vec{l}, \quad (3)$$

where \vec{v} is the velocity of the coil relative to the magnet, \vec{B} is the magnetic flux density at the coil segment, $d\vec{l}$ is the vector length of the coil segment, and l_{coil} is the total length of the coil. The scalar form of (3) is given by

$$V = \int_{l_{\text{coil}}} v B \sin \theta \cos \phi dl = K v, \quad (4)$$

where θ is the angle between the directions of velocity and the flux density, and ϕ is the angle between the directions of the coil segment and the vector perpendicular to velocity and flux density. Since the motion is assumed to be rectilinear, the velocity is the same along the coil, v can be taken out of the integral, and $K = \int_{l_{\text{coil}}} B \sin \theta \cos \phi dl$ is the

transduction coefficient. For a given magnitude of flux density and coil length, the transduction coefficient is maximized when $\theta = 90^\circ$ and $\phi = 0^\circ$, which means that the three vectors are perpendicular to each other.

Therefore, for rectilinear motion, both the magnetic field and the coil need to be in the plane perpendicular to the velocity. Two simple configurations are possible: first, a straight line coil in a parallel field perpendicular to the coil (fig. 2(a)); second, a circular coil in a radial field (fig. 2(b)). The second configuration is more useful, because it enables a multi-turn coil.

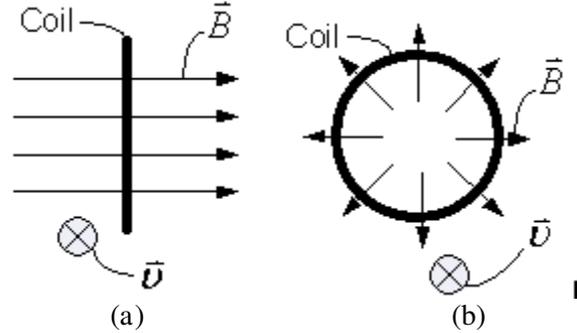


Fig. 2: Two perpendicular configurations: (a) straight coil with parallel magnetic field; (b) circular coil with radial magnetic field.

For the radial field configuration in fig. 2(b), the coupling strength is given by

$$\gamma = \frac{K^2}{R_{\text{coil}}b} = \frac{\left(\int_{l_{\text{coil}}} B_{\text{radial}} dl \right)^2}{\rho \frac{l_{\text{coil}}}{A_{\text{coil}}}}, \quad (5)$$

where B_{radial} is the radial component of the flux density (assuming circular coil moving perpendicular to the axis), l_{coil} is the total length of the coil, ρ is the resistivity of the conductor, and A_{c} is the cross section area of the conductor. The right hand side of (5) can be further expressed as

$$\frac{\left(\int_{l_{\text{coil}}} B_{\text{radial}} dl \right)^2}{\rho \frac{l_{\text{coil}}}{A_{\text{coil}}}} = \frac{\left(\overline{B_{\text{radial}}} l_{\text{coil}} \right)^2}{\rho \frac{l_{\text{coil}}}{A_{\text{coil}}}} = \frac{\overline{B_{\text{radial}}}^2 V_{\text{coil}}}{\rho b}, \quad (6)$$

where $V_{\text{coil}} = l_{\text{coil}} A_{\text{coil}}$ is the conductor volume, $\overline{B_{\text{radial}}}$ is the average of the radial flux density over the conductor volume. Equation (6) assumes that the coil is thin enough that the flux density variation over the cross section of the coil, as well as the skin effect, are negligible. Since the resistivity of the conductor ρ is usually predetermined and the damping coefficient b is determined by the mechanical design, (6) indicates that the product of the coil volume and square of the average radial flux density are the primary design factors that determine the coupling strength. In other words, a good electrodynamic configuration should

have a high radial flux density acting over a large-volume coil.

One of the easiest ways to construct a strong radial field is to use two opposing disc-shaped permanent magnets, as illustrated in fig. 3. A strong radial magnetic field is generated around the periphery of the air gap between the magnets, making this region the preferred location for the coil.

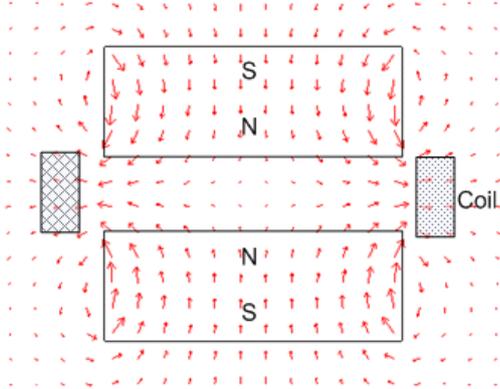


Fig. 3: Field pattern of opposing disc magnets.

EXPERIMENTAL

Device Design

An electrodynamic energy harvester based on the opposing magnets structure is built to verify the coupling strength theory and to demonstrate an example of strongly coupled device. The schematic and picture of the device is shown in fig. 4.

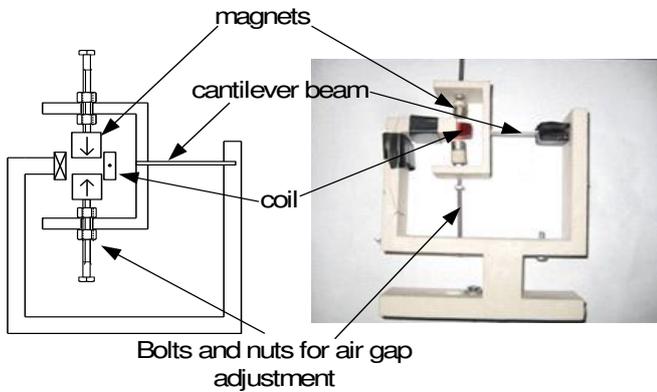


Fig. 4: Schematic and picture of the energy harvester prototype for experimental verification.

In the device, each disc magnet (thickness x diameter: 6.4 mm x 9.5 mm, NdFeB Grade N50) is attached to a bolt, which screws into a nut on the magnet holder. Such arrangement enables adjustment of the air gap between the magnets, without substantially altering any of the other electro-mechanical parameters. Since the average radial magnetic flux density changes with the magnet position, this structure enables manual tuning of the coupling strength. The magnet holder is connected to the base frame via an aluminum cantilever beam (thickness x width x length: 0.8 mm x 12.7 mm x 33.1 mm) that behaves as a spring. A circular coil (outer diameter x inner diameter x height: 19.1 mm x 11.9

mm x 8.9 mm, ~3,000 turn AWG 39 copper magnet wire) is attached to the frame in a position surrounding the air gap between the magnets, resembling the configuration in fig. 3.

Measurement Procedures and Results

The goal of the experiment is to experimentally verify the theoretical relationship between the power ratio and the coupling strength (Fig. 1), and to demonstrate that the opposing magnet harvester architecture can achieve a high power ratio. The coupling strength variation is implemented by changing the air gap between the magnets. In the experiment reported here, four different air gaps (6.25 mm, 8.15 mm, 10.6 mm and 20 mm) are used.

For each air gap, a series of experiments are performed (details below) to extract model parameters and overall energy harvesting performance. These measurements enable estimation of the coupling strength and power ratio. In the experiments the maximum load power is inferred based on the measured open-circuit voltage and harvester output impedance. The power ratio can be expressed as

$$\eta = \frac{P_{\text{load_max}}}{P_{\text{theory_max}}} = \frac{8|Z_{\text{out}}|}{\frac{V_{\text{oc}}^2}{m^2 a^2}} = \frac{V_{\text{oc}}^2 b}{m^2 a^2 |Z_{\text{out}}|}, \quad (7)$$

where V_{oc} is the measured open circuit voltage amplitude at the natural frequency and Z_{out} is the measured output impedance at the natural frequency.

For each air gap, the fundamental transducer model parameters are extracted using the electrical and mechanical tests described in [5]. The harvester performance is then measured immediately after the parameter extraction. For this, the harvester is mounted on a shaker, and the open-circuit voltage is measured using a signal analyzer (Stanford Research Systems, SR785) as the sinusoidal shaker excitation frequency is swept around the harvester natural frequency (~10 Hz). A constant, relatively low acceleration amplitude of 0.003 g (0.0294 m/s²) is used to restrict the magnet motion to within the mechanical stops. For the different air gaps, the peak open-circuit voltage always occurs at frequency close to the natural frequency (± 0.1 Hz), as predicted by the model [5, 6].

The harvester output impedance is then measured with the harvester attached to a static mechanical reference (a sturdy table) to block the frame motion while the magnet holder oscillates. A frequency sweeping sinusoidal voltage signal generated by the signal analyzer is applied to the coil, and the current flowing on the coil is measured with a current probe (Tektronix, TCP312). The amplitude ratio and phase difference between the voltage and current is measured by the signal analyzer, and the magnitude of the impedance at the natural frequency is recorded.

All of the measured and calculated parameters are listed in table 1. For example, at the 8.2 mm air gap, a maximum coupling strength of 13.6 is achieved, and

Table 1: List of measured and calculated (in bold) parameters for each air gap (numbers in the parentheses are 95% confidence bounds).

Parameter (unit)	Air gap (mm)			
	6.3	8.2	11	20
Coil resistance (Ω)	89.9 (± 0.1)			
Coil inductance (H)	0.019 (± 0.002)			
Mass (kg)	0.0550 (± 0.0001)			
Spring constant (N/m)	245 (± 17)			
Damping coefficient (N*s/m)	0.0319 (± 0.0007)	0.0317 (± 0.0011)	0.0279 (± 0.0008)	0.0339 (± 0.0009)
Transduction coefficient (V*s/m)	5.92 (± 0.48)	6.24 (± 0.58)	3.18 (± 0.30)	2.26 (± 0.44)
Coupling strength	12.2 (± 2.0)	13.6 (± 2.6)	4.02 (± 0.70)	1.68 (± 0.65)
Open circuit voltage (V)	0.375 (± 0.019)	0.336 (± 0.001)	0.221 (± 0.002)	0.0811 (± 0.0095)
Output real impedance (Ω)	2050 (± 1)	1440 (± 12)	593 (± 1)	206 (± 1)
Estimated maximum load power (μ W)	8.58 (± 0.87)	9.81 (± 0.078)	10.2 (± 0.1)	3.99 (± 0.93)
Theoretical maximum load power (μ W)	10.3 (± 0.2)	10.3 (± 0.4)	11.8 (± 0.4)	9.68 (± 0.25)
Power ratio	0.834 (± 0.086)	0.948 (± 0.034)	0.872 (± 0.028)	0.412 (± 0.097)

95% of the theoretical maximum power is available for harvesting. At the smaller air gap of 6.25 mm, the radial magnetic flux density is “intensified,” but the spatial “span” of the strong radial field is reduced, leading to a lower transduction coefficient and coupling strength.

The overlaid plots of the theoretical and experimental percent maximum power vs. coupling strength are shown in fig. 5. Considering the error bars (95% confidence intervals) specified for each data point, the experimental results reasonably match the theory. The uncertainties arise from measurement limitations as well as the variation of parameters between repeated experiments for the same air gap. The latter is attributed to the non-robust materials used for the frame and the magnet holder (bonded powder rapid prototype material), as well as the non-perfectness of the screw engagement.

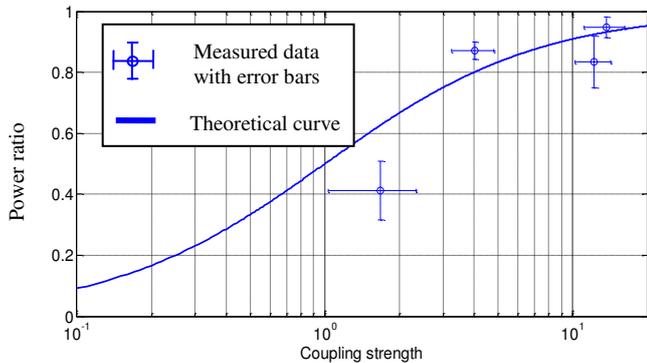


Fig. 5: Theoretical and experimental relationship between the power ratio and the coupling strength.

CONCLUSION

The study shows that the coupling strength is one of the key factors for electrodynamic energy harvester design. With a strongly coupled energy harvester, the load power can be very close (~95% is demonstrated) to the absolute maximum power set by the mechanical system. To achieve a strongly coupled harvester, a well-design magnetic structure is important. One of the most practical topologies is a circular coil moving perpendicular to a radial magnetic field pattern. As has been demonstrated in this paper as well as several previous works, opposing magnets are one of the

simplest and most effective structures for creating a strong radial magnetic field.

In summary, it seems that the opposing magnet structure provides a simple and effective means for achieving a high power ratio. Further optimization of this basic architecture will likely not lead to significant improvements in the power generation. However, it is noted that the coupling strength is proportional to the system volume, following from (6). Thus, in order to maintain a high coupling strength at smaller length scales a more sophisticated device architecture or magnetic structure may be desired.

ACKNOWLEDGEMENT

This work is supported in part by NASA Grant NNX07AD94A and ARO Grant W911NF-09-1-0511.

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