COMBINED OPTIMIZATION OF ELECTRICAL AND MECHANICAL PARAMETERS OF IN-PLANE AND OUT-OF-PLANE GAP-CLOSING ELECTROSTATIC VIBRATION ENERGY HARVESTERS (VEHS)

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Abstract: This paper presents a simple analytical method to optimize the efficiency of two types of electrostatic Vibration Energy Harvesters (VEH): the out-of-plane (OPGC) and in-plane (IPGC) gap-closing converters. The electrical and mechanical behaviours of the transducer are addressed simultaneously, while a voltage limitation on the transducer's terminals is set to prevent any damage in the conditioning electronic. The presented work allows to the designer to determine the best strategy depending on whereas the system is passive or able to be self-adapted to the external vibrations parameters. The calculations are validated by VHDL-AMS/ELDO simulations.

Keywords: Vibration energy harvesting (VEH), Energy scavenging, Electrostatic transduction, Power generation

INTRODUCTION

In order to improve the efficiency of an electrostatic Vibration Energy Harvester (VEH), it's common to design a transducer with the best maximal-to-minimal capacitance ratio and to initialize it with the highest voltage as possible. Indeed, the maximal power that can be harvested for a VEH working at constant charge is [1]:

$$P'_{h\_max} = \frac{U_0^{\ 2}}{2} C_{max} \left(\frac{C_{max}}{C_{min}} - 1 \right) f_{elec} \qquad (1)$$

where C_{max} and C_{min} are the maximal and the minimal values of the transducer's capacitance C_{tran}, U_0 is the initial voltage applied on C_{tran} at C_{max} and f_{elec} is the frequency of C_{tran}'s variations. However the designer has to consider that if a high initial voltage leads to a high electromechanical coupling, it also limits the capacitance variation due to the phenomenon of electrostatic instability (pull-in) [2]. Moreover, a too high capacitance variation could lead to voltages too high for the surrounding electronics. Therefore to maximize the efficiency of an electrostatic VEH, a compromise between the pre-charge voltage, the displacement range of the seismic mass and the maximal capacitance value of the transducer is required. In this work, we have performed an accurate analytical modelization of the Out-of-Plane-Gap-Closing (OPGC) and In-Plane-Gap-Closing (IPGC) architectures [3] in order to determine the best design in term of harvested power for practical implementation, i.e. we have taken into account a voltage limitation in the conditioning electronic and we have considered if the system is self-adapted or not with the vibration's amplitude changes.

DESCRIPTION ON THE TRANSDUCERS

The structures of the OPGC and IPGC VEHs are represented in Fig. 1. The transducer's capacitance C_{tran} is composed of a variable part C_{var} and a constant part C_{par} in parallel to C_{var}. d_0 is the gap between the electrodes when no voltage or acceleration is applied to the system. For our demonstration we will consider a mobile mass made of 400 µm-thick bulk silicon, having an area of 1 cm². In the IPGC architecture, 2 mm x 30 µm combs are etched on both sides of the mobile mass. The mechanical resonance is 200 Hz, C_{par} is equal to 10 pF and the thickness of stoppers t_s is 1 µm by default (= $t_{s\ min}$). The maximal voltage allowed in the system is 60 V.

When the pre-charge voltage is lower than the pull-in voltage U_{pi}, the system has two equilibrium positions: one stable and one unstable, situtated at $x_{eq\ stable}$ and $x_{eq\ unstable}$, so that $d_0 > x_{eq\ unstable} > x_{eq\ stable} \geq 0$. In addition, in a vibrating environment, when the mobile mass oscillates, it must never go out of the attraction zone of the stable equilibrium point. This attraction zone is delimited by $x_{eq\ unstable}$ which corresponds to the maximal allowed displacement x_{max} and to the maximal transducer capacitance C_{max} [4].

The stable and unstable positions of the mobile electrode in OPGC device are given by resolving the equation:

$$x(d_0 - x)^2 = \frac{\varepsilon S U_0^{\ 2}}{2k} \qquad (2)$$

For the IPGC architecture, the stable position is x = 0 and the unstable position of the mobile mass is given by:

$$x_{unstable} = \pm \sqrt{d_0^{\ 2} - U_0 \sqrt{\frac{2\varepsilon N S d_0}{k}}} \qquad (3)$$

where N is the number of fingers attached to the mobile mass and S the overlapped surface between two fingers. The variable capacitance is at its minimal value when the mobile mass is at its rest position.

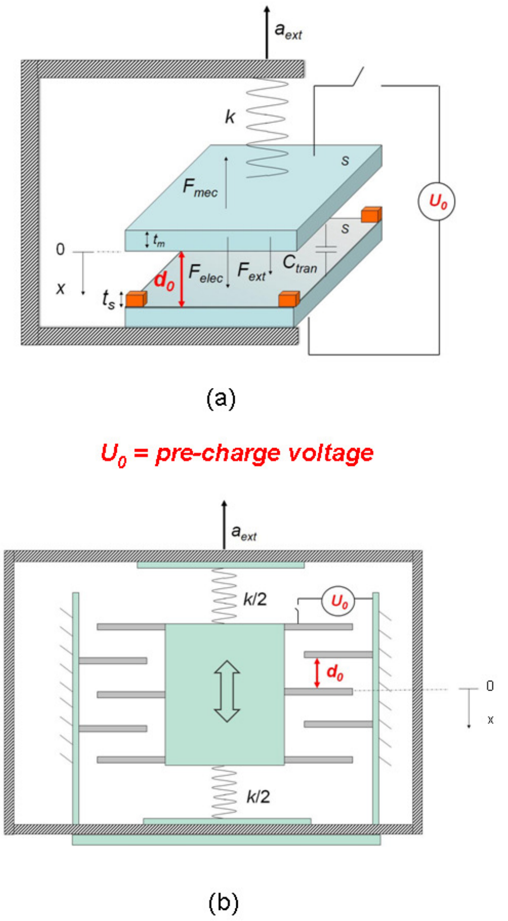

Fig. 1. Architectures of the OPGC (a) and IPGC (b) Vibration Energy Harvesters.

In [4] we assumed, in order to determine C_{min}, that for OPGC VEH the mass oscillates symmetrically around the static stable equilibrium point. However for a more accurate value of C_{min} we have to take into account the influence of the electrostatic force when the electrodes are close. A first approximation consists in assuming a symmetrical displacement around the position x_{med} corresponding to the half of the maximal electrostatic force:

$$x_{med} = \frac{F_{elec\_max}}{2k} = \frac{1}{4} U_0^{\ 2} \frac{\varepsilon S}{(d_0 - x_{unstable})^2} \qquad (4)$$

The position of the mobile electrode corresponding to the minimal capacitance C_{min} is then given by:

$$x_{min} = 2x_{med} - x_{max} \qquad (5)$$

OPTIMIZATION OF THE DESIGN

The design optimization consists in calculating $\{d_0, U_0\}$ such as the harvested power $P'_{h\ max}$ is maximised and the voltage across the transducer U_{Ctran} is lower than the maximal voltage allowed by the system. In [4], we showed that in order to get a maximum of converted power, it is better to work with a low pre-charge voltage allowing a large C_{max}/C_{min} ratio. Then the main limitation comes from the voltage $U_{Ctran\ max}$ allowed by the conditioning electronic, which is equal to C_{max}/C_{min} times U_0.

Consequence of a voltage limitation on C_{tran}

In Fig. 2, we trace the evolution of $P'_{h\ max}$ as a function of $U_{Ctran\ max}$ for various values of d_0. The first part of each curve (in doted line) is associated to the lower values of U_0 and corresponds to $x_{eq\ unstable}$ beyond the stoppers, which is not possible. C_{max}/C_{min} remains constant at its maximum value corresponding to $x_{max} = d_0 - t_{s\ min}$ and $P'_{h\ max}$ increases proportionally to $U_0^{\ 2}$. The top of curves corresponds to the optimum pre-charge when the mobile electrode is just in contact with the stoppers. From this point, any increase of U_0 decreases $P'_{h\ max}$ since $x_{eq\ unstable}$ and then C_{max}/C_{min} decrease. If a voltage limitation is set, this extremum has to fit with it in order to maximize the harvested power.

For instance, for the OPGC device, the best design is with a gap d_0 = 26.5 µm and a pre-charge of 2.91 V, leading to a maximal harvested power of 14.9 µW. In these conditions, the C_{max}/C_{min} ratio is 895 pF / 43 pF ~ 20.

For the IPGC architecture, the best design is with a gap d_0 = 46.7 µm and a pre-charge of 3.15 V, leading to a maximal harvested power of 33.6 µW. The C_{max}/C_{min} ratio is 940 pF / 50 pF ~19

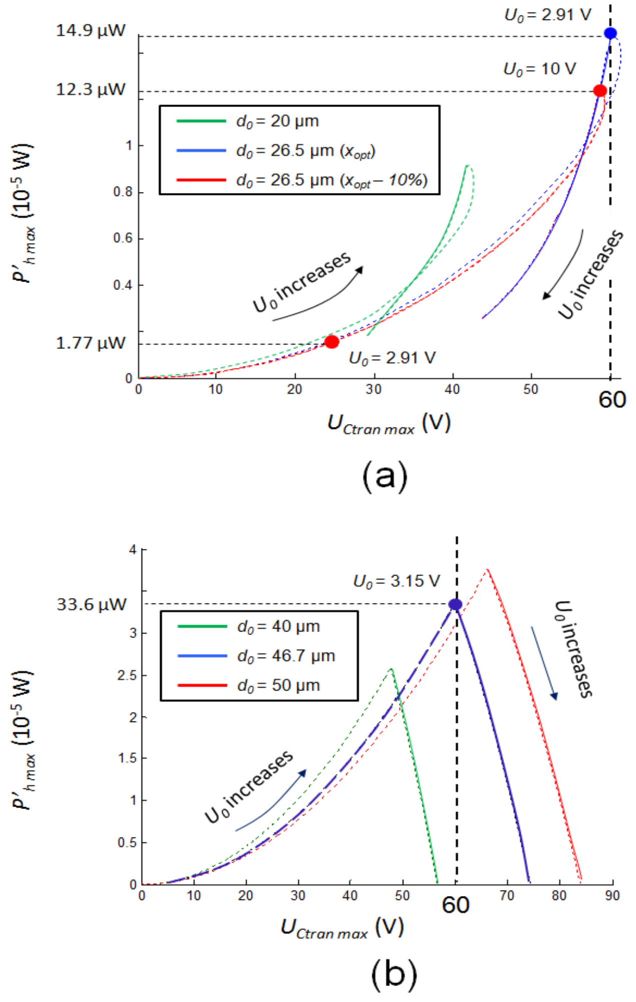

Fig. 2: Evolution of the maximal harvested power $P'_{h\ max}$ as a function of the maximal voltage across the transducer $U_{Ctran\ max}$ for the OPGC (a) and the IPGC (b) architectures.

Consequence of a variation of the external acceleration

In this paragraph we study the impact of a 10 % reduction of the mobile electrode displacement due to a decrease in the external acceleration for a given design of the OPGC transducer. In the case where the optimal x_{max} is very close to d_0, an amplitude decrease (i.e., x_{max} decrease) implies a dramatic decrease of C_{max}/C_{min}:

$$\frac{C_{max}}{C_{min}} = \frac{d_0 - x_{min}}{d_0 - x_{max}} \qquad (6)$$

This leads to a smaller C_{max}/C_{min} ratio for which the device's design is no more optimal. From the harvested power optimization point of view, it is equivalent to say that the stopper's thickness t_s is now 3.5 µm instead of 1 µm. Compared to figures obtained in the preceding section, the highest C_{max}/C_{min} ratio becomes 259 pF / 29 pF ~ 9. The impact of such a vibration change is illustrated in Fig. 2a with the red dotted curve. The second half of the curve fits with the previous case since d_0 is the same and the mobile electrode doesn't reach the stopper's position. If U_0 is maintained at the same level, the new maximal harvested power dramatically decreased to $P'_{h\,max} = 1.77$ µW, namely a decrease of almost 90 %, which is disastrous for the energy harvesting process. However if after this amplitude change the pre-charge can be adjusted to its new optimal value of 10 V, the C_{max}/C_{min} ratio becomes 6, and the harvested power is 12.3 µW for a maximum output voltage of 58.5 V.

Consequently there are two approaches to limit the impact of the vibration's amplitude variations:

- to have a smart system where the pre-charge can be adjusted to the fluctuation of the external acceleration during the conversion process. Such architecture has been proposed by A. Dudka et al in [5].
- to limit the transducer's sensitivity. This can be obtained with thicker stoppers, leading to a smaller value of C_{max}, and then to a smaller C_{max}/C_{min} ratio.

For instance we can choose to design a transducer with $C_{max}/C_{min} = 4$. The design optimisation gives the following parameters: $d_0 = 27.4$ µm and $U_0 = 15$ V and the stoppers thickness is $t_s = 5.6$ µm. The maximal harvested power $P'_{h\,max}$ is almost 11.4 µW for $U_{Ctran\,max} = 60$ V. If the mobile electrode displacement is reduced of 10 % due to a lower external acceleration, $P'_{h\,max}$ decreases to 8 µW for a voltage $U_{Ctran\,max} = 54$ V, which is much less critical than in the previous case (Fig. 3).

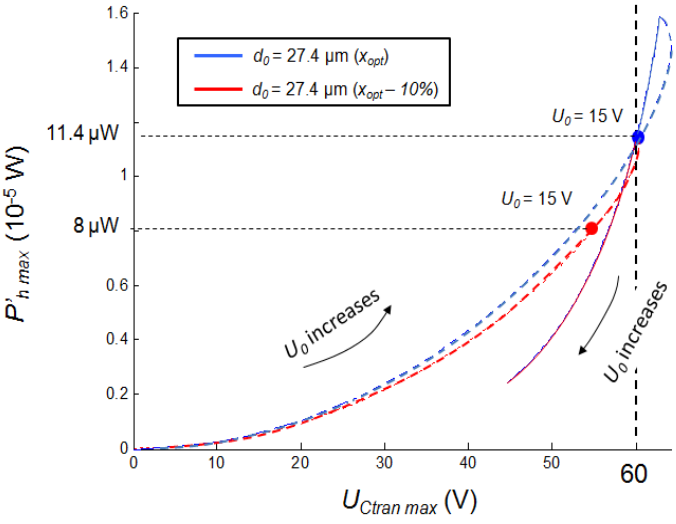

Fig. 3: Evolution of $P'_{h\,max}$ as a function of $U_{Ctran\,max}$ for an OPGC design optimized with $C_{max}/C_{min} = 4$, with a maximal external acceleration and with an acceleration inducing 10 % loss of the mobile *electrode's displacement*

VHDL/AMS MODELING

We have validated our results with a VHDL-AMS/ELDO modeling and performed a simulation of the OPGC transducer implemented in the circuit proposed by Miranda et al. [1]

The Fig. 4 presents the simulated curves highlighting behavior of the optimized transducer for a C_{max}/C_{min} ratio of 4. The graph shows the displacement x of the mobile electrode when it is submitted to an external sinusoidal acceleration a_{ext} at the device mechanical resonance. The graph shows also the variations of C_{tran}, $U_{Ctran\,max}$ and the energy E harvested by the transducer per capacitance variation cycle. On the energy plot, only the curve's envelop is meaningful; it represents the energy harvested after each conversion cycle.

We observe a shift of the median position x_{med} which is due to the shift of the resonance frequency of the resonator when high electrostatic coupling with the spring-mass system occurs [6]. As expected, $U_{Ctran\,max}$ and the harvested energy increase with the external acceleration. However, the amplitude of the mobile mass vibration first increases and then, for higher values of a_{ext}, starts to decrease. This is related to the nonlinearity of the electrostatic transducer and to the change of the transducer's impedance when the external acceleration changes. Although this phenomenon was not explicitely taken into account in our analysis, it doesn't come to contradiction with it. It can be seen that for any value of a_{ext}, the mobile mass vibration amplitude and average position are related to the observed x_{max} in accordance with [Eq. 4] and [Eq. 5]. The used model of transducer is not provided with stoppers, so the pull-in occurs at the end of the simulation. It can also be observed an irregular (nonsinusoidal) motion of the resonator for a_{ext} amplitudes close to the pull-in value. This is related with the increased non-linearity of the system in this zone, which unvalidates the model of zero and first harmonic used in this analysis.When a_{ext} is large enough to allow a maximal displacement of the electrode, $U_{Cvar\ max}$ is saturating at ~ 60 V. The harvested energy is about 57 nJ so the maximal harvested power is 57 nJ × 200 Hz = 11.4 µW as predicted in our calculations. If the external acceleration is reduced of 10 %, the new maximal harvested power is about 8.3 µW. This result matches our calculations.

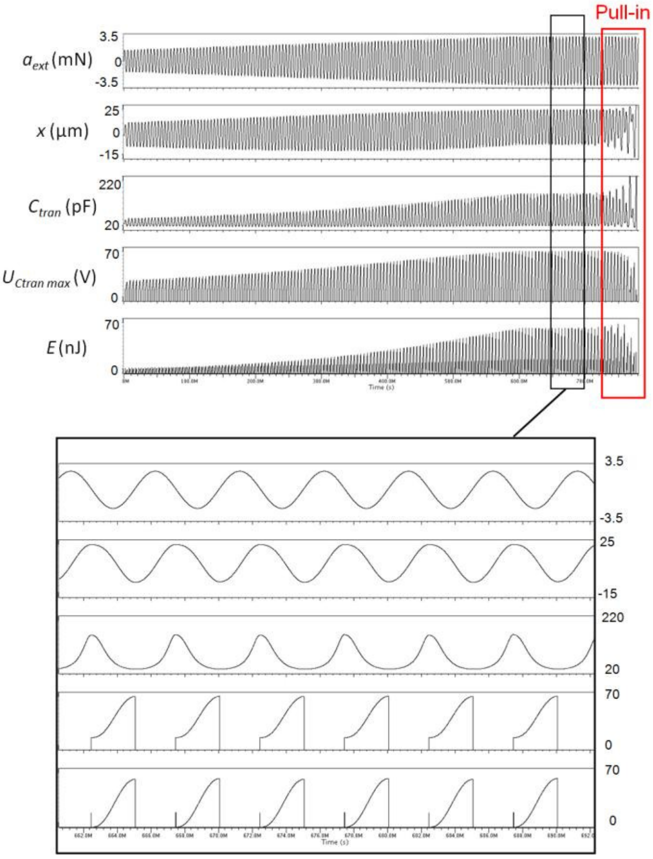

Fig. 4: VHDL-AMS modeling of the optimized OPGC device with a C_{max}/C_{min} ratio of 4 and pre-charged with U_0 = 15 V

CONCLUSION

We have detailed how to design the OPGC and IPGC VEHs in order to harvest the maximum of power. The originality of this work consists in taking into account both electrical and mechanical aspects for the transducer's design optimisation and a constraint on the maximal voltage allowed across the transducer's terminal. We studied the case where the vibration's characteristics are known and also the influence of 10 % decrease of the mobile electrode's displacement. It appears that the decrease of the external acceleration provides a dramatically fall about 90 % of the maximal harvested power, for high C_{max} / C_{min} ratio. If the voltage U_0 is adjusted, the decrease is only about 20 %. So, it would be very useful to have an adaptive system as the one described in [5]. For a passive device, a much less sensitive system has to be designed, i.e. with a smallest C_{max}/C_{min} ratio, for example with C_{max} /C_{min} = 4. Then, the same decrease of the external acceleration will induce a power loss of only 30 %. Our results have been validated with a behavioral VHDL-AMS modeling of the OPGC transducer implemented in the conditioning circuit of Miranda [1].

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