

# REALIZATION OF NONLINEAR HARDENING SPRINGS WITH PREDEFINED CHARACTERISTIC FOR VIBRATION TRANSDUCERS BASED ON BEAM STRUCTURES

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**Abstract:** This paper presents a realization approach of nonlinear hardening springs with predefined characteristic. In the first part of the paper transient simulations, based on measured vibration profiles of a car engine, are used to show that the output power can be increased by about 20% using nonlinear hardening springs. Afterwards the realization approach based on a cantilever beam clamped between two shaped blocks is introduced. With the shape of the blocks the nonlinearity can be defined. The approach can be applied to piezoelectric as well as electromagnetic vibration transducers. Static mechanic finite element analyses (FEA) calculations have been used to verify the results. Finally a spring element has successfully been fabricated.

**Keywords:** Nonlinear hardening spring, Stochastic vibration, Cantilever beam, Optimal spring characteristic

## INTRODUCTION

The conversion of vibrational energy into electrical energy is in the most of all cases based on passive resonant vibration transducers. A basic characteristic of this kind of vibration transducers is the small bandwidth. Beside the resonance frequency of the transducer the conversion efficiency drops significantly. Due to this the vibration in the application (amplitude and frequency) should preferably be well known and should not change over time. In this ideal case a maximum output performance can be achieved. However in the real world the vibration source is often stochastic in nature which evokes some serious challenges for the implementation of vibration transducers because the output power is possibly not sufficient for the underlying application. Recently it has been shown that especially for stochastic vibrations nonlinear hardening springs have the potential to maximize the output power of vibration transducers [1, 2]. On the example of a measured stochastic car engine vibration this paper shows how the spring characteristic which yields a maximum output power can be defined. Consequently the next step considers the realization of a spring element with a predefined nonlinear characteristic. The approach is based on a cantilever beam clamped between two shaped blocks (Fig. 1). With the shape of the blocks the nonlinearity can be defined. The calculation of the block shape is an approximation procedure based on the differential equation of a cantilever beam with a fixed end and a single suspension point with an offset in the  $y$ -direction. Because both piezoelectric- as well as electromagnetic vibration transducer can be build up based on cantilever beams [3, 4] the approach can directly be applied to both conversion mechanisms.

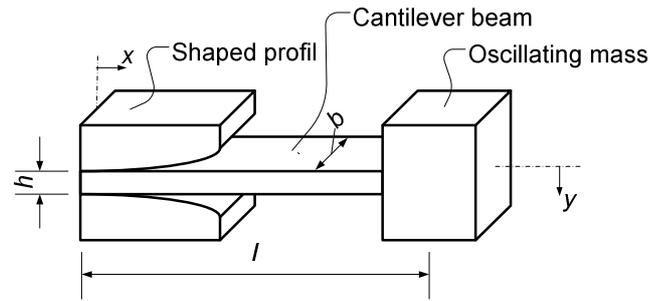


Fig. 1: Nonlinear hardening spring realized with a cantilever beam clamped between two shaped blocks. With the shape of the block the spring characteristic can be defined.

## FINDING THE OPTIMAL SPRING

The considered stochastic vibration environment for the spring development is in the engine compartment of a four cylinder in-line diesel car engine. The vibration characteristic of car engines depends on the load condition and changes over time. The most energetic vibration frequencies can be defined using the order domain analyses [5]. However this approach cannot be used to investigate the potential of nonlinear springs. For this purpose a transient numerical model as shown in Fig. 2 has been implemented in a Matlab/Simulink<sup>®</sup> environment. Basic features of the model are that the measured acceleration profile is used as the excitation, the nonlinearity of the spring can be adjusted and the oscillation range is limited by inelastic collision at mechanical stoppers (as it is for any real vibration transducer). The considered nonlinearity of the spring can be defined by:

$$f(x) = kx(1 + \mu_{nl}x^2), \quad (1)$$

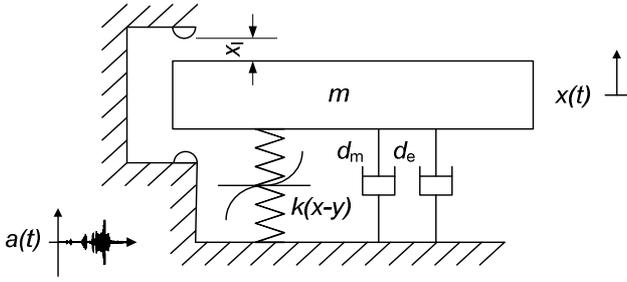


Fig. 2: Schematic diagram of the numerical model used for transient simulations with the measured vibration profile.

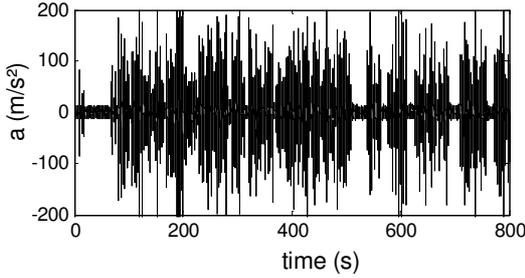


Fig. 3: Acceleration profile of a country driving route used for the calculation example.

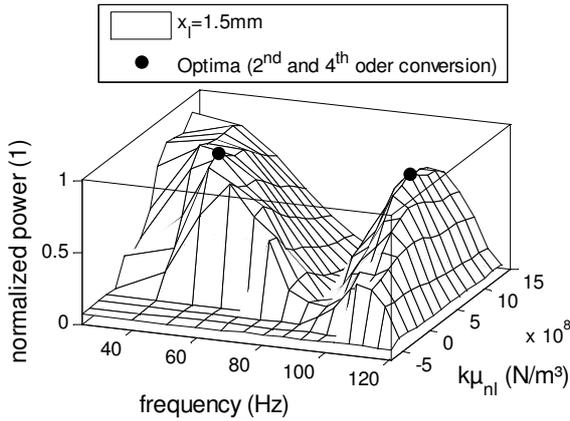


Fig. 4: Normalized output power for an example (country driving route) excitation and different spring characteristics.

where  $k$  is the spring constant,  $x$  the displacement and  $\mu_{nl}$  the nonlinearity parameter. A typical result of the mean output power for different linear ( $k\mu_{nl} = 0$ ) and nonlinear ( $k\mu_{nl} \neq 0$ ) springs and excitation with an acceleration profile of a country driving route (Fig. 2) is shown in Fig. 3. The corresponding simulation parameters (typical for meso-scale transducer) and the resulting optimal values are summarized in Table 1. The two efficient regions around 60 Hz and 110 Hz correspond to the 2<sup>nd</sup> and 4<sup>th</sup> engine order, respectively [5]. In this calculation example the output power definitely increases for hardening springs ( $k\mu_{nl} > 0$ ) and decreases for softening springs ( $k\mu_{nl} < 0$ ). In the simulation the output power of softening springs that are not monotonically increasing are set to zero. With respect to the best linear spring the output power can be

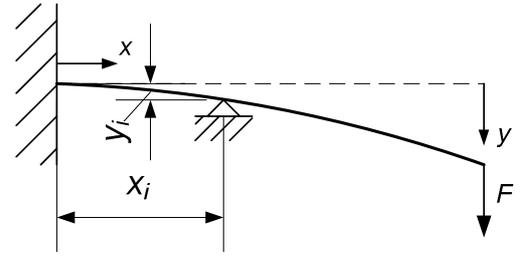


Fig. 5: The calculation approach is based on a cantilever beam with a single suspension point (offset in the y-direction).

Table 1: Simulation parameters and simulated optimal values in the calculation example of Fig. 4.

Sym- bol	Description	Value	Unit
Simulation parameters			
$m$	Oscillating mass	10	g
$d_m$	Mechanical damping	0.05	N/m/s
$d_e$	Electromagnetic damping	0.02	N/m/s
Resulting optimal spring characteristic			
$k\mu_{nl}$	Nonlinearity at 51 Hz (2 <sup>nd</sup> order conversion)	$2.4 \cdot 10^8$	N/m <sup>2</sup>
$k\mu_{nl}$	Nonlinearity at 102 Hz (4 <sup>th</sup> order conversion)	$7.1 \cdot 10^8$	N/m <sup>2</sup>

increased by about 20% using optimized hardening springs. Note that the total benefit as well as the optimal characteristic of the spring depends also on the inner displacement limit  $x_i$ . However more detailed analysis is beyond the scope of this paper and the basic question to be answered now is: How can the shape of the blocks be adapted to a predefined spring characteristic?

## CALCULATION OF THE BLOCK SHAPE

The calculation of the block shape is an approximation procedure which is based on the differential equation of a cantilever beam with a fixed end and a single suspension point with an offset in the y-direction (Fig. 5). Using the direct stiffness method (DSM) [6] the beam deflection curve for  $x < x_i$  can be found to be:

$$y_1(x) = -\frac{F(l-x_i)x^2}{4EI} + \frac{1}{4} \frac{F(l-x_i)}{EIx_i} x^3 + \frac{1}{2x_i^3}(3lx^2 - x^3) \quad (2)$$

and for  $x > x_i$ :

$$y_2(x) = y_i + \left( \frac{F(l-x_i)x_i}{4EI} + \frac{3y_i}{2x_i} \right) (x-x_i) + \frac{F}{6EI} (x-x_i)^2 (3l-2x_i-x) \quad (3)$$

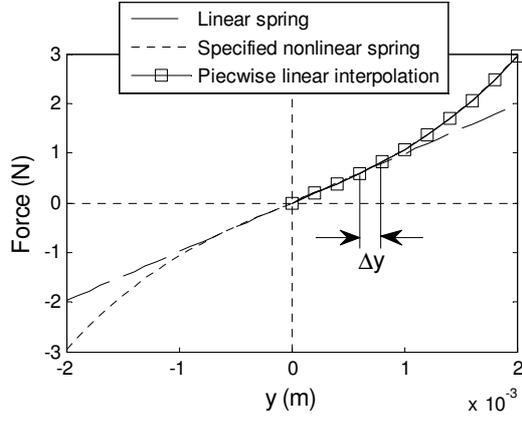


Fig. 6: The specified nonlinearity is divided into piecewise linear functions.

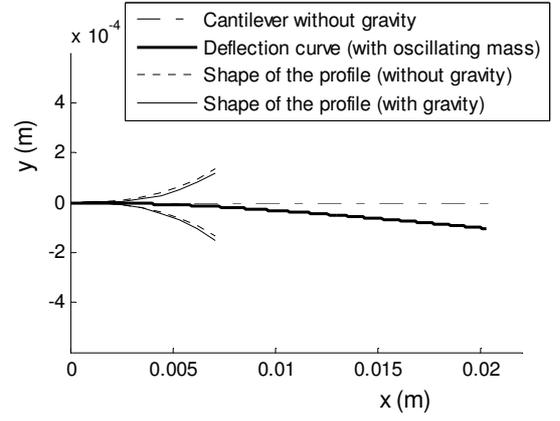


Fig. 9: Shape of the block for the specified nonlinear spring characteristic.

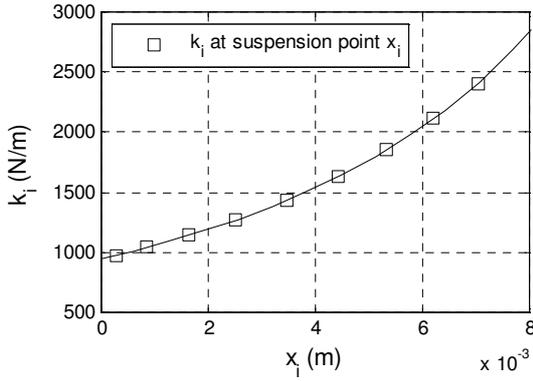


Fig. 7: The piecewise linear spring constants are assigned to a distance  $x_i$ .

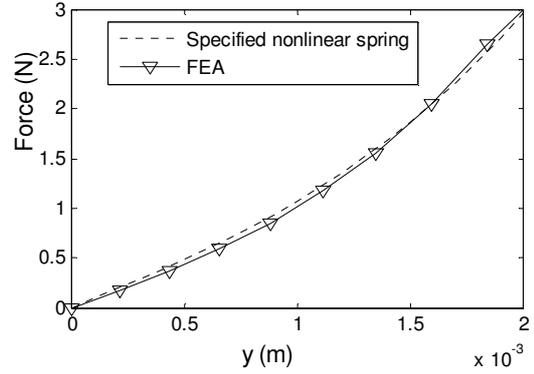


Fig. 10: Specified nonlinearity in comparison to FEA simulation of the spring characteristic based on the calculated block shape.

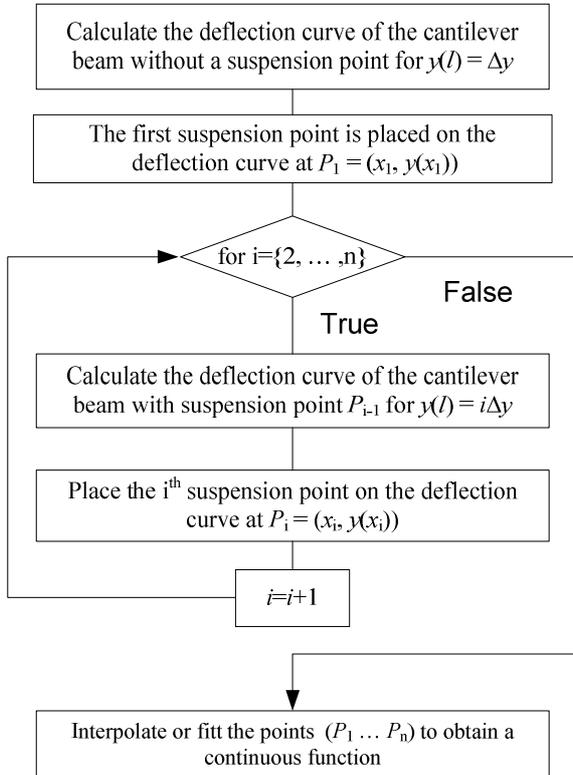


Fig. 8: Flow chart of the block shape calculation.

Table 2: Underlying parameters for the calculation of the shape profile.

Symbol	Description	Value	Unit
$E$	Young's modulus	118	GPa
$h$	Cantilever beam height	0.3	mm
$b$	Cantilever beam width	10	mm

where  $F$  indicates the force at the end of the cantilever,  $E$  indicates the Young's modulus of the beam material and  $I$  indicate the moment of inertia. The force  $F$  which causes the deflection  $y$  at the end of the beam is given by:

$$F = -6EI \frac{(y_i x_i - 3y_i l + 2yx_i)/x_i}{(x_i - l)/(-5x_i l + x_i^2 + 4l^2)}. \quad (4)$$

Fig. 6 shows a nonlinear characteristic that is used to explain the calculation of the block shape. In the first step the spring characteristic in the first quadrant is piecewise linear interpolated (at equally spaced  $\Delta y=0.2$  mm). This yields a vector containing linear spring constants  $K = [k_0, k_1, \dots, k_n]$ . The first spring constant  $k_0$  (942 N/m) must be provided by the cantilever beam itself without a suspension point. With the parameters of Table 2 the beam length becomes:

$$l = \sqrt[3]{\frac{3EI}{k_0}} = 20.4 \text{ mm}. \quad (5)$$

In the next step the remaining spring constants ( $k_1, \dots$

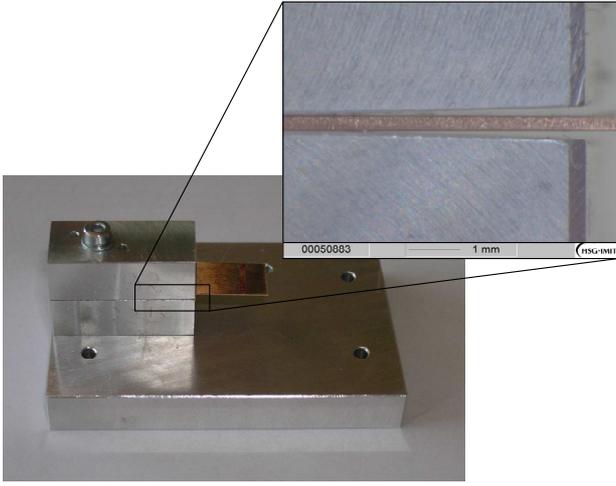


Fig. 11: Fabricated nonlinear hardening spring using standard milling technology.

$k_n$ ) are assigned to particular distances ( $x_1, \dots, x_n$ ) of a single suspension point using:

$$k_i = \frac{1}{\left(x_i(l-x_i)^2/4EI\right) + \left((l-x_i)^3/3EI\right)}. \quad (6)$$

In this step  $y_i$  is set to zero which results in an appropriate approximation because the deflections are very small. The result of this step is shown in Fig. 7. The shape of the profile is found by following the flow chart in Fig. 8. This has been done with and without the influence of gravity. The resulting block shapes are shown in Fig. 9. Note that the axes of the diagram have a different scaling. For the predefined characteristic the shape is rather flat. The last point of the shape has the coordinates  $P_9 = (7.049, 0.151)$  mm. In general the higher the nonlinearity the flatter the block shapes. To verify the approach 2-dimensional static mechanic Finite element analyses (FEA) calculations have been performed where the spring characteristic have been calculated based on the calculated block shape. The results are in good agreement (Fig. 10).

## CONCLUSION AND OUTLOOK

The approach presented in this paper can be used to design a nonlinear hardening spring with predefined characteristic. The benefit of nonlinear hardening springs have been verified using transient simulations of a vibration transducer with measured stochastic acceleration profile as the excitation. The optimal spring characteristic has been identified and the result shows that the output power can be increased by 20% with the optimized characteristic. Consequently a realization approach for nonlinear hardening springs with predefined characteristic has been introduced. This approach is based on a cantilever beam clamped between two shaped blocks. The calculation of the block shape is an approximation method based on the differential equation of a cantilever beam with a single suspension point (offset in  $y$ -direction). Based on a given arbitrary spring characteristic the block shape calculation has been performed. Static mechanic FEA has

been used to verify the result which is finally in very good agreement. Because the approach is based on a simple cantilever beam it can be applied to piezoelectric- and electromagnetic vibration transducers.

Issues like the fatigue strength have not been considered. However the stress in the cantilever will somehow limit the nonlinearity in application. Nevertheless for the optimized characteristic in the calculation example (Table 1) this is irrelevant. A prototype of a nonlinear hardening spring designed with the presented approach has already been manufactured using standard milling technology (Fig. 11). However the tolerances are rather small and milling is possibly not the best manufacturing method. The next step is to verify the realization approach experimentally.

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