IMPROVED PARAMETRIC STUDY OF A PIEZOELECTRIC ENERGY HARVESTER WITH EXPERIMENTAL VALIDATION

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Abstract: This paper presents the findings from an improved parameter study in energy harvesting and includes results from its experimental validation. The previously developed model [5] has been extended to incorporate a non-constant damping ratio to take account of geometric changes and a form of energy storage. The results show that an optimum beam length exists due to increased damping being present in longer beams. In a parameter study the natural frequency of harvester designs should ideally remain constant. A model has been created which generates such configurations by using the substrate thickness as a frequency control variable. Results indicate a beam partially covered by piezoelectric material can yield a 16% increase in the power dissipated by a resistor. A similar advantage is observed when using such a configuration to charge a capacitor with the maximum stored voltage increasing by 18%. In the case of an electrical scenario comprising a resistor, samples were manufactured and used for model validation – the difference between theoretical and experimental results was less than 2.5%.

Keywords: Energy harvesting, Piezoelectric beam, Damping, Harvester storage circuit

INTRODUCTION

Currently a large proportion of research in energy harvesting focuses on design improvements to maximise efficiency and power output. Numerous authors have published work on frequency tuning mechanisms, e.g. [1, 2], which are essential to maintain a structure's resonant frequency at the excitation frequency. Others are working on devices which make use of frequency-up conversion [3, 4]; such devices are useful when designing at the MEMS scale where the natural frequency is inherently high.

Although these areas are important a more basic analysis has been somewhat overlooked. The conventional energy harvester is a rectangular cantilevered beam with piezoelectric material spanning the entire length of the substrate layer. A number of researchers have shown that this design can be improved by altering geometric parameters [5-7]. Patel et al. have used a finite element method to model the beam, enabling the structure's natural frequencies and mode shapes to be calculated accurately during their parameter studies [5]. Their results show that an increase in power is achievable by solely altering the piezoelectric layer's geometrical parameter. In this paper the model developed by Patel et al. [5] is improved and extended in terms of both the mechanical and electrical aspects. In the mechanical domain, structural damping is allowed to change with geometrical parameters. In the electrical domain the authors have extended the model to incorporate a form of energy storage. These models enable trends in voltage across a capacitor to be investigated.

THEORY

Damping

The previously developed analytical model, for a cantilever beam energy harvester [5], has been extended to better represent the mechanical affects of parameter changes, namely on the damping ratio, γ, of the structure. This property is especially important as it has a significant influence on the magnitude of the power generated by the harvester. The damping matrix is assumed to be proportional to the mass and stiffness matrices.

$$[C] = \alpha [M] + \beta [K] \tag{1}$$

Modal decoupling and the modal orthogonality conditions are used to obtain an expression for the damping ratio in terms of the structure's fundamental natural frequency, ω_{nat},

$$\gamma = \frac{\alpha}{2\omega_{nat}} + \frac{\beta\omega_{nat}}{2} \tag{2}$$

where the constants α and β are obtained from experimental frequency response data. As geometrical parameters are altered, the natural frequency of the structure will change causing changes to the damping ratio. The theoretical model generated results indicting an increased damping is present in longer beams. To validate this theoretical trend, six samples of varying length were manufactured and their fundamental damping ratios obtained using the half-power bandwidth method. Fig. 1 shows good agreement between theoretical and experimental results.

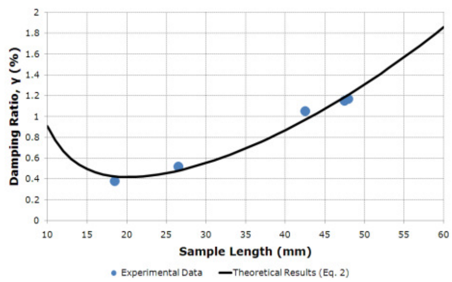

Fig. 1: The effect of sample length on the mechanical damping ratio.

Constant Natural Frequency

The design of an energy harvester is often centred on a specified application and so the excitation characteristics are already known. To perform a fair comparison between different harvester configurations, the fundamental natural frequency must be kept constant while varying system parameters. The dimensions of the conventional energy harvester used in this study are shown in Table 1.

Table 1: Structural dimensions and mechanical properties of the conventional harvester used throughout this analysis

Substrate length (mm)	50
Substrate width (mm)	5
Substrate thickness (mm)	0.5
PZT length (mm)	50
PZT width (mm)	5
PZT thickness (mm)	0.5
Young's modulus of the substrate (GPa)	69
Density of the substrate (kg/m^3)	2700
Young's modulus of the PZT (GPa)	62.1
Density of the PZT (kg/m^3)	7800

A MatLab model has been created which generates dimensions for various harvester configurations having identical natural frequencies. Firstly, the parameter under investigation is selected, for example the distance x1 in Fig. 2. Secondly a control parameter is selected – a parameter used to change the structures natural frequency, for example substrate thickness, t_s. If a tip mass is added to the system its inertia can easily be utilised as a control parameter.

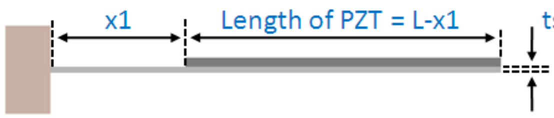

Fig. 2: Adjusting the piezoelectric layer length by changing its start position from clamped end.

The targeted fundamental frequency for all configurations is the natural frequency of a conventional energy harvester.

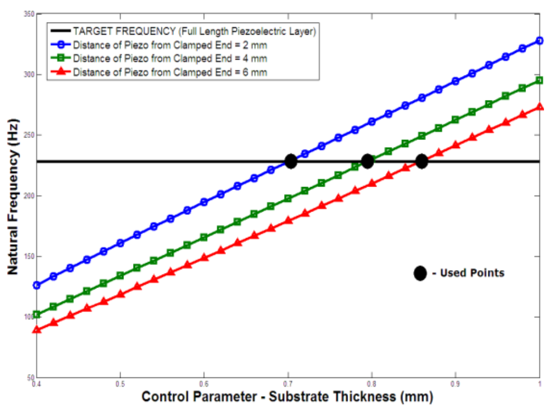

Fig. 3: Visual representation of constant natural frequency model.

In the analysis undertaken here, for each length of piezoelectric material a new magnitude for t_s is obtained. This produces a range of energy harvesters with the same natural frequency, see Fig. 3.

Charging Circuit

The analytical model [5] has been extended in the electrical domain. A capacitor along with a diode bridge for rectification is now used to store the voltage generated by an energy harvester. The energy harvester is represented by a current source in parallel with a capacitor, see Fig. 4. The following equations can be used to model the behaviour of such a circuit:

$$\left.\begin{array}{l} I_p(t) = I(t) \\ I_s(t) = 0 \end{array}\right\} \quad V_p(t) < V_s(t) + 2V_{th}$$
$$\left.\begin{array}{l} I_p(t) = I(t)\dfrac{C_p}{C_s + C_p} \\ I_s(t) = I(t)\dfrac{C_s}{C_s + C_p} \end{array}\right\} \quad V_p(t) \geq V_s(t) + 2V_{th} \qquad (3)$$

where V_{th} is the diode threshold voltage, assumed to be 0.6 V. C_p is the piezoelectric internal capacitance, Eq. 4, and C_s is the storage capacitance, 500 nF. I is the current generated by the piezoelectric material, I_p and I_s represent the current flowing through C_p and C_s respectively. V_p is the voltage across the piezoelectric material, V_s is the voltage across the storage capacitor,

$$C_p = \frac{\varepsilon_{33}^{S} b_p L_p}{t_p} \qquad (4)$$

where ε^{S}_{33} is the permittivity, L_p, b_p and t_p are the piezoelectric length, width and thickness, respectively.

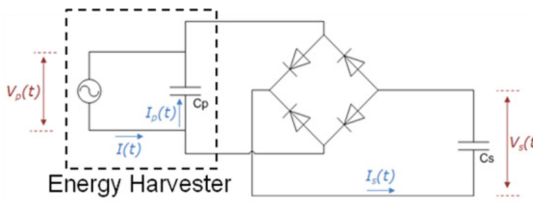

Fig. 4: Charging circuit used in analysis. C_s = 500 nF.

The mechanical and electrical behaviour of the system are governed by the following coupled equations:

$$I(t) = \sum_{r=1}^{\infty} -\varphi_r \dot{\eta}_r(t) \qquad (5)$$

$$\ddot{\eta}_r(t) + 2\gamma_r \omega_r \dot{\eta}_r(t) + \omega_r^2 \eta_r(t) = N_r - R_r V_p(t) \qquad (6)$$

where η_r is the modal coordinate, γ_r is the damping ratio, ω_r is the natural frequency, N_r is the modal forcing term, φ_r is the forward modal coupling term and R_r is the backward modal coupling term. In all of the proceeding analysis the effects at the fundamental frequency are examined and so only one mode (r=1) is sufficient for obtaining practical results. For the equations defining N_r, φ_r, and R_r, refer to [5].

RESULTS

Damping

The analytical model incorporating a changing damping ratio was used to determine the effect of sample length on the power generated by an energy harvester. The width, thickness and material properties of the substrate and piezoelectric layer are shown in Table 1. Table 2 shows the electrical properties of the piezoelectric material. The base acceleration is maintained at 1g throughout the simulation. All power measurements are taken at the fundamental frequency. Fig. 5 shows that if the mechanical damping is assumed to have a constant value of 0.5%, the power increases exponentially with beam length over all magnitudes of load resistance.

Table 2: Piezoelectric material electrical properties

Piezoelectric constant, d_{31} (m/V)	-190×10^{-12}
Permittivity, ε^{S}_{33} (F/m)	15.83×10^{-9}

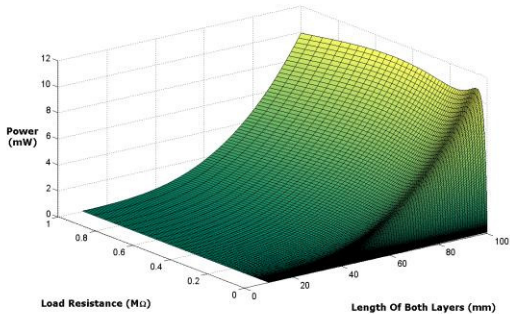

Fig. 5: Effect of beam length and load resistance on the *harvester's power output while damping ratio remains* constant.

This observation is incorrect because longer beams are seen to experience a greater degree of damping. Fig. 6 shows the results again, only this time the damping ratio is changed in accordance with Eq. 2. The new trend indicates there is an optimal beam length and the advantage of using an excessively long beam is minimal. Over a 177 kΩ load resistor the power generated by a 100 mm long sample is 0.49 mW. A 5.3% increase can be obtained if a 75 mm long sample is used instead.

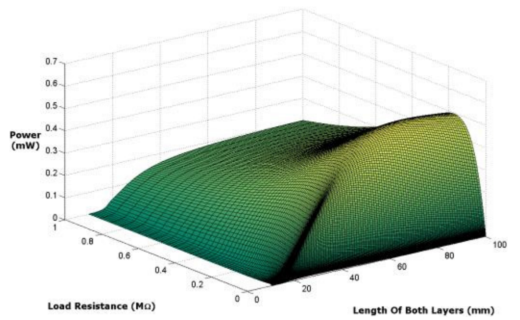

Fig. 6: Effect of beam length and load resistance on the *harvester's power output while damping ratio is* changing.

Constant Natural Frequency

Using the constant natural frequency model, different configurations of energy harvester were generated possessing identical natural frequencies of 228.1 Hz. Configuration 1 is a conventional rectangular harvester in which both the substrate and piezoelectric layer have the same length. The length of the piezoelectric layer is reduced by 0.25 mm (and the thickness of the substrate layer is increased) for each subsequent configuration, while the ends of both layers remain coincident. The results in Fig. 7 show that a 16% increase in power, from 0.0149 mW/m^2/s^4 to 0.0173 mW/m^2/s^4, is achievable if configuration 13 is used, see Table 3. The optimum load resistance depends on the harvester's configuration – 149 kΩ for the conventional design and 497 kΩ for configuration 13. It is seen to increase with configuration number since piezoelectric capacitance is decreasing.

Table 3: Dimensions of optimum configuration 13

Substrate length (mm)	50
Substrate and PZT width (mm)	5
Substrate thickness (mm)	0.5
PZT length (mm)	47
PZT position from clamped end (mm)	3
PZT thickness (mm)	0.753

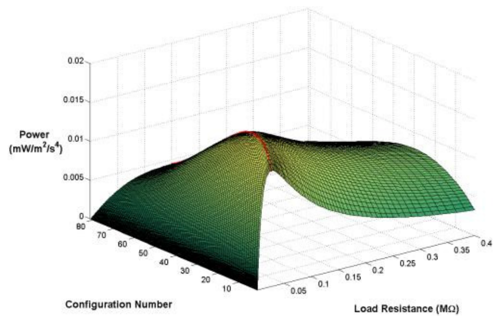

Fig. 7: Power dissipated across various load resistances for a range of harvester configurations with the same natural frequency.

Charging Circuit

Lastly, the charging circuit model was combined with the constant natural frequency model and compiled in Simulink. The results, in agreement with those obtained when a resistor is modelled, indicate it may be better to utilise a shorter piezoelectric layer.

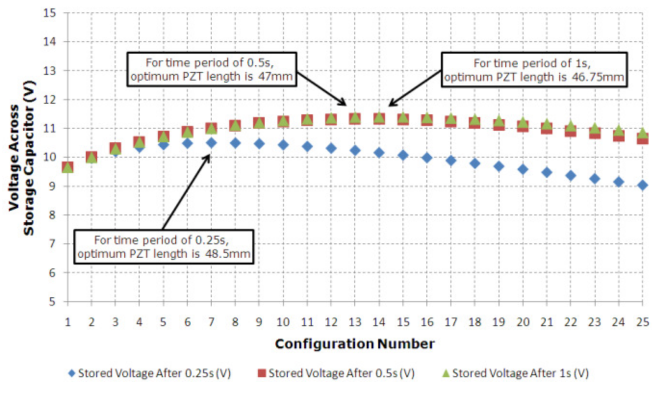

Fig. 8: Voltage across 500 nF capacitor against piezo length at three different time intervals. Substrate thickness used to maintain constant natural frequency.

From Fig. 8 the optimum configuration comprises of a $46.75 \times 5 \times 0.5$ mm piezoelectric layer attached 3.25 mm from the clamped end. A substrate thickness of 0.764 mm is required to maintain the 228.1 Hz natural frequency. A trade-off between maximum voltage and time taken to reach steady state is required with this time period increasing by 54% when using the optimum configuration.

EXPERIMENTAL VALIDATION

Partial validation of the analytical model has been completed through experimental work. 'Homemade' samples were used to achieve this. The samples were manufactured using the following procedure:

- Piezoelectric sheets purchased from Physik Instrumente [8] are cut, along with the aluminium substrate layer, using a carbide abrasive wheel. Grit paper is used to remove any jagged edges. The d_{31} and ε^S_{33} values are -180×10^{-12} m/V and 15.49×10^{-9} F/m, respectively.
- Isopropyl alcohol is then used to clean the bonding surfaces.
- Heavy duty Scotch-Brite pads are used to scratch and remove dirt from the substrate surface.
- Isopropyl alcohol is applied once more to the bonding surfaces and wiped using cotton buds.
- Epoxy adhesive DP460 is used to bond layers.

The samples are all unimorph with sizes ranging from $50 \times 10 \times 1$ mm to $18 \times 5 \times 1$ mm. Throughout the experiment the harmonic base excitation was provided by a GW-V4 shaker connected to the source output of a Stanford Research Systems SR785 dynamic signal analyser. A PolyTec OFV-055 laser vibrometer was used to measure the velocity of the sample while the base acceleration was measured using a PCB Piezotronics accelerometer, model number - 352C23. The frequency response of both sample velocity and base acceleration were recorded using the dynamic signal analyser.

Non-uniform samples were tested and the results are presented in Fig. 9.

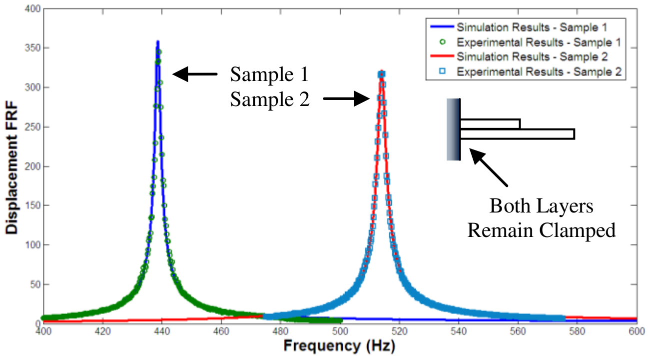

Fig. 9: Comparison between theoretical and experimental results for non-uniform samples.

A base acceleration in the region of 0.01g was applied to the system. Sample 1 comprised of a $31.95 \times 5 \times 0.67$ mm substrate layer and a $12 \times 5 \times 0.5$ mm piezoelectric layer. Whereas sample 2 was fabricated from a $24 \times 5 \times 0.67$ mm substrate layer and a $21.49 \times 5 \times 0.5$ mm piezoelectric layer. Experimental results from both samples show a good agreement with theoretical results in terms of the displacement FRF. The difference in peak magnitudes and natural frequencies were found to be below 2.5%.

CONCLUSION

Several model extensions have been presented along with experimental validation. By changing the damping ratio according to the mass and stiffness of the system more realistic trends can be obtained during parameter studies with results showing that an optimum sample length exists. On the electrical side, modelling a capacitor as a form of energy storage was performed. Results show an 18% increase in stored voltage when a shorter piezoelectric layer is used. The natural frequency was kept constant during the parameter studies by altering the substrate thickness.

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