

A PHASOR METHOD FOR MECHANICAL DAMPING MEASUREMENT OF A PIEZOELECTRIC BENDING BEAM ENERGY HARVESTER

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Abstract: The paper describes and demonstrates a phase vector method for measurement of mechanical damping coefficient and characterization of a piezoelectric bending beam generator. The method uses the measured frequency responses of the beam under short- and open-circuit conditions to compare with the frequency responses based on an analytical model of the beam. Then, by using a model matching algorithm which minimizes the error between analytical and measured data, an accurate estimation for the mechanical damping of the beam will be obtained. The method is successfully tested on a cm-scale piezoelectric beam to determine the damping ratio of the vibrating beam in the air. Test results show a close match between the measured frequency responses and the analytical model with the estimated parameters.

Keywords: Damping ratio, Characterization, Piezoelectric beam, Energy harvester, Frequency response.

INTRODUCTION

Vibration energy harvesters including piezoelectric, electromagnetic, and capacitive micro-generators have been considered as promising sources of energy for low power portable devices. A piezoelectric energy harvester has been shown to be a suitable option for capturing energy from low frequency vibration. The reason is that the output voltage of a piezoelectric beam is independent of vibration frequency and it does not need any external sources to create electric or magnetic fields for energy conversion [1-3].

It has been well understood from the theory of vibration-to-electricity energy harvesters that the mechanical damping of a vibrating beam limits the amount of electrical energy that can be captured from a vibration source [4]. Thus, in design of a bending beam energy harvester, estimation of mechanical damping is of significant importance. Moreover, when characterizing a bending beam energy harvester, accurate estimation of mechanical damping is essential to evaluate the performance of a designed micro-power energy harvester.

The existing methods for measurement or estimation of the damping often consider a piezoelectric beam with pure mechanical properties. Analytical methods roughly estimate the dominant damping components (drag and squeezed film) using simplified Navier-Stokes equations [5] and these estimations are often used in a design procedure. For a vacuum packaged device, however, the internal material damping will become significant (instead of the drag) but is not well known, especially for multi-layer structures.

Experimental characterization methods for a fabricated piezoelectric device provide more accurate estimations for the device parameters. These methods often determine the damping ratio (ζ_m) of a beam based on measurement of the quality factor of the

beam (Q) about its resonant frequency (ω_n). It has been shown [6] that the piezoelectric coupling coefficient (θ) and the electrical load connected to a beam affect the values of Q and ω_n . Furthermore, θ depends on the beam fabrication process and it can be drastically different from θ of a bulk piezoelectric material, especially in piezoelectric MEMS devices. Thus, the accuracy of damping derived from Q and ω_n is hindered by the effects of an electrical load and by the quality of the piezoelectric film as fabricated.

Herein, we present a phase vector (*phasor*) method for the measurement of mechanical damping coefficient and characterization of a piezoelectric bending beam generator. The method analytically models a piezoelectric beam in the frequency domain based on phasor transfer functions under short- and open-circuit conditions. Then, the frequency responses corresponding to these transfer functions will be calculated based on measurement of the voltage, current, and the base acceleration of a vibrating beam. Finally, an accurate estimation for the model parameters including mechanical damping will be calculated by minimizing the error between frequency responses of analytical transfer functions and the measured frequency responses.

The proposed phasor method can be more accurate than other existing methods since this method uses data at several frequencies rather than only the resonant frequency of the beam. The validity of the method has been verified using a cm-scale piezoelectric beam setup.

MODEL OF THE BEAM

Phasor Model

The time-domain model of a piezoelectric bending beam includes coupled mechanical and electrical differential equations [2]. In frequency domain, and

under a steady-state sinusoidal condition, the model can be expressed in terms of phasors as:

$$(\omega_n^2 - \omega^2 + j2\zeta_m\omega_n\omega)\bar{u} - \frac{\theta}{M}\bar{v} = -Y_0\omega^2 \quad (1)$$

$$\bar{i} = -j\omega(\theta\bar{u} + C_p\bar{v}) \quad (2)$$

where M , θ , C_p and ζ_m are the beam effective mass, coupling coefficient, electrical capacitance and mechanical damping, respectively. ω_n denotes the short circuit resonant frequency which is defined as $\omega_n = \sqrt{K/M}$ where K is the beam mechanical stiffness.

Y_0 is the base vibration amplitude at the frequency ω . $\bar{u} = U_0\angle u$, $\bar{v} = V_0\angle v$, and $\bar{i} = I_0\angle i$ respectively represent the phasors corresponding to the beam deflection, voltage, and current with reference to the base vibration.

Short Circuit and Open Circuit Transfer Functions

Under a short circuit condition, the terminal voltage across the piezoelectric beam is zero. Under this condition, by substituting for u from (1) in (2), the short circuit transfer function can be defined as:

$$H_{sc} = \frac{I_0\angle i}{-Y_0\omega^2} = \frac{j\omega\theta}{\omega_n^2 - \omega^2 + j2\zeta_m\omega_n\omega}. \quad (3)$$

Similarly, under an open circuit condition the current phasor in (2) is zero and by substituting for u from (2) in (1), we obtain the open-circuit transfer function as:

$$H_{oc} = \frac{V_0\angle v}{-Y_0\omega^2} = \frac{\theta/C_p}{\omega_n^2 - \omega^2 + \frac{K_e}{M} + j2\zeta_m\omega_n\omega} \quad (4)$$

where $K_e = \theta^2/C_p$ is defined as the electrical stiffness [6]. These transfer function can be readily measured from open- and short-circuit tests which will be used for identification of the model parameters as follows.

MODEL PARAMETERS IDENTIFICATION

The parameters of the beam including damping can be estimated if the frequency responses of the short- and open-circuit transfer functions are known. To calculate these transfer functions, we define the following tests.

Short Circuit and Open Circuit Tests

Figure 1(a) and (b) depicts the two experiments which are used to obtain the frequency response of the short- and open-circuit transfer functions. In Fig. 1(a), the terminals of the beam are shortened via an ammeter

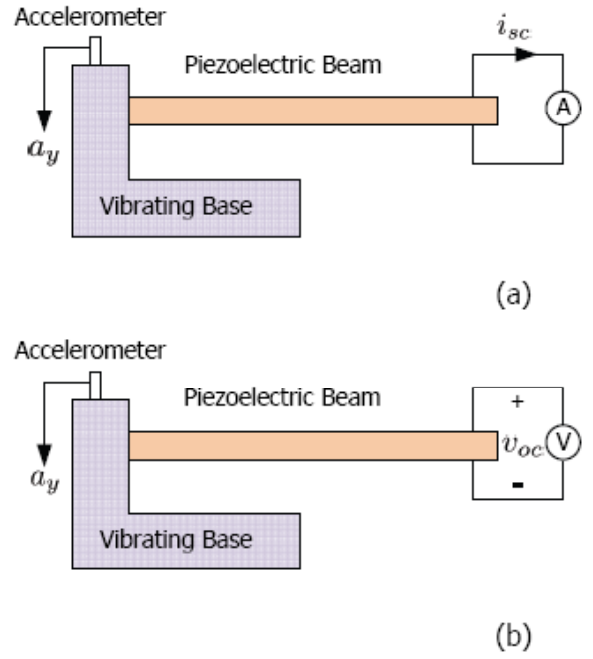


Fig. 1: (a) Short-circuit (b) Open-circuit test setups.

and the peak value of the current I_0 is calculated based on measured *rms* value by an ammeter. The base acceleration ($a_y = -Y_0\omega^2$) is measured using an accelerometer at the vibrating base. The output of the sensor is a voltage proportional to the acceleration which can be monitored and measured by an oscilloscope. Using a small series resistance with the ammeter, the current phase angle with respect to acceleration signal can also be measured through the voltage waveform across the resistor. Thus, for any arbitrary frequency, the amplitude and phase of the current and the base acceleration can be measured yielding the frequency response of the short circuit transfer function in (3).

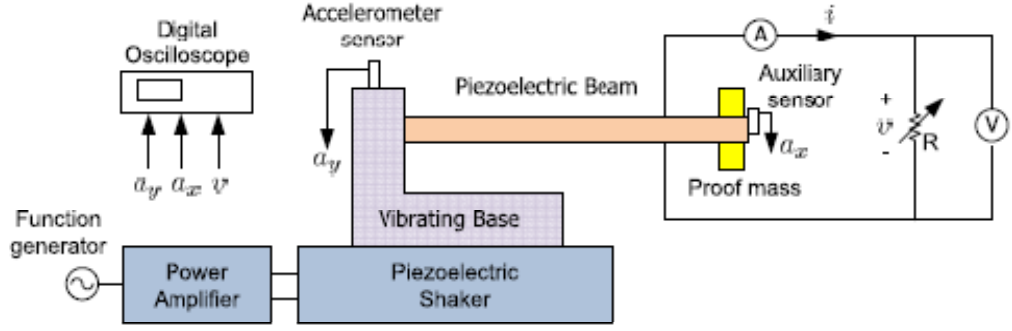
Similarly, the amplitude and phase of the open circuit voltage can be monitored and measured with respect to the base acceleration, using a test circuit shown in Fig. 2(b). Once the voltage and acceleration are measured, the frequency response of the open circuit transfer function can be obtained by calculating the ratio of voltage and acceleration in (4).

Model Matching Problems

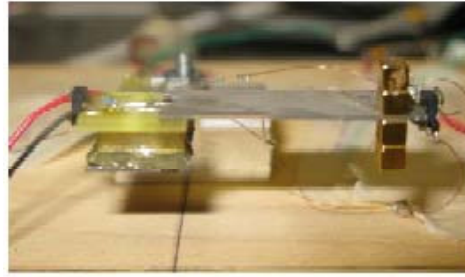
Assume that the measured resonant frequencies and frequency response of the short- and open-circuit transfer functions are denoted by $\omega_{sc_r}^{ex}$, $H_{sc}^{ex}(j\omega_i)$, $\omega_{oc_r}^{ex}$, and $H_{oc}^{ex}(j\omega_i)$, respectively, where ω_i is an arbitrary test frequency. Then, the following cost functions can be defined:

$$J_{sc} = \sum_i (\|H_{sc}(j\omega_i)\| - \|H_{sc}^{ex}(j\omega_i)\|)^2, \quad (5)$$

$$J_{oc} = \sum_i (\|H_{oc}(j\omega_i)\| - \|H_{oc}^{ex}(j\omega_i)\|)^2. \quad (6)$$



(a)



(b)

Fig. 2: (a) Schematic diagram of the experimental setup (b) Photo of the test setup.

An accurate estimation for the model parameters can be obtained by minimizing these cost functions. We perform the model parameter estimations in two steps. In the first step, we measure the short circuit frequency response to obtain θ and ζ_m such that J_{sc} in (5) is minimized. Then, using estimated θ and ζ_m in (4), and based on the measured frequency response in open circuit test, M and C_p are estimated such that J_{oc} in (6) is minimized.

EXPERIMENTAL VERIFICATION

Test Setup

Figure 2(a) shows the schematic diagram of the experimental test setup which was used to verify the method. The test setup includes a vibrating piezoelectric beam (Q220-A4-303YB, Piezo System Inc.) installed on a similar beam which served as a shaker. Fig. 2(b) shows the photo of the experimental setup. An accelerometer sensor is connected to the base of the vibrating beam and the beam terminals are connected to a variable resistance as a load. The voltage and current of the beam as well as the output of the accelerometer sensor were monitored on an oscilloscope. The variable resistor can emulate the short- and open-circuit condition in its extreme quantities. The vibration frequency of the base was adjusted by a function generator connected to the shaker via a power amplifier.

Experiment Results

The measured frequency responses of the short- and open-circuit conditions are depicted on Fig. 3. The measurements show that the maximum short circuit current is about $37\mu\text{A}$ at the resonant frequency of 118.19 Hz. The maximum open circuit voltage is about 0.8V which occurs at 119.10 Hz.

Using any optimization program, (such as MATLAB *fmincon.m* function), we can obtain the piezoelectric beam parameters to minimize the cost functions J_{sc} and J_{oc} in (5) and (6). An estimation of the beam parameter based on measured data is expressed in Table I.

DISCUSSION

The experiment shows that the short circuit resonant frequency is about 1Hz less than that of open-circuit. The discrepancy between the resonant frequencies can be justified based on the electrical stiffness terms, K_e in (4), which increases the open circuit resonant frequency compared to ω_n .

Figure 4 shows the frequency responses of short- and open-circuit transfer functions based on estimated parameters against the experimentally measured data. A close match among the analytical and experimental results show the validity and accuracy of the method for estimation of mechanical damping as well as other parameters.

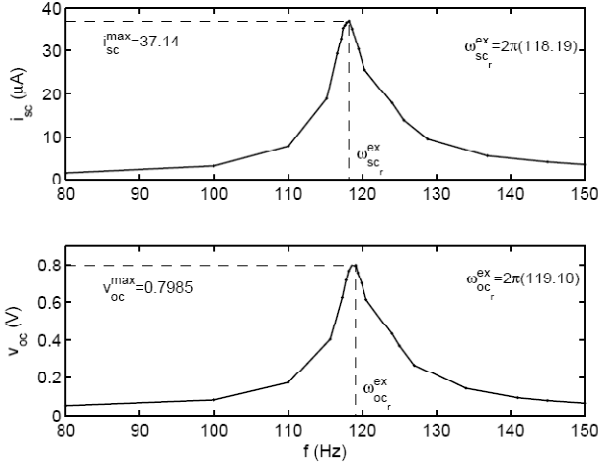


Fig. 3: Measured frequency response under short- and open-circuit conditions.

Table I. Estimation of the Model parameters

Test	Estimated Parameter
min. J_{sc}	$\zeta_m = 0.0191$ $\theta = -0.631$ mN/V
min. J_{oc}	$C_p = 61.3$ nF $M = 0.847$ mgr $K = 467.5$ N/m

As an alternative to this method, instead of assuming $\omega_n = \omega_{sc_r}^{ex}$ and minimizing J_{sc} , one can also consider ω_n as a parameter to be estimated. Then, the minimization of (5) will change to a constrained minimization problem with $|\omega_n - \omega_{sc_r}^{ex}| < \varepsilon$ as a constraint where $\varepsilon > 0$ is an arbitrary small quantity. Adding this constraint to the problem will provide more flexibility in estimation of parameters which can lead to more accurate results.

CONCLUSION

A method for estimation of mechanical damping of a piezoelectric beam has been presented which identifies the model parameters of the beam based on two experiments. The experiments include short- and open-circuit tests which can be readily performed for a piezoelectric energy harvester. The method uses the experimentally measured data of these tests to obtain the frequency response of the beam and thereby accurately estimates the model parameters.

The method has been successfully tested on a cm-scale piezoelectric energy harvester to determine mechanical damping and other parameters of an electromechanical model of the beam. The method is useful for performance evaluation of various bending beam piezoelectric energy harvesters. Also it is useful to determine both mechanical damping and material properties such as coupling coefficient of a fabricated piezoelectric beam.

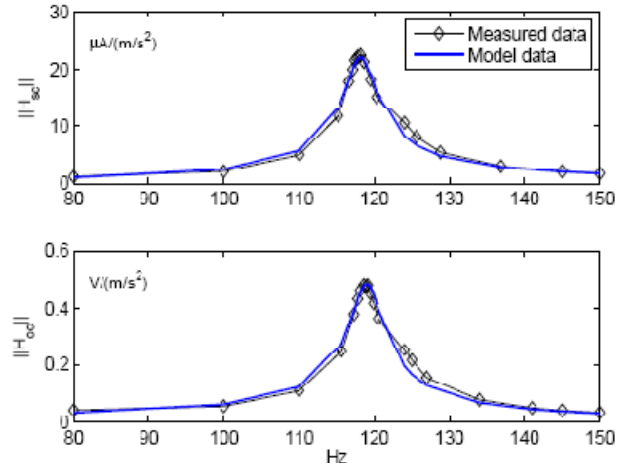


Fig. 4: Comparing the measured frequency response against analytical model with estimated parameters.

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