THE INFLUENCE OF FERROELASTIC HYSTERESIS ON MECHANICALLY EXCITED PZT CANTILEVER BEAMS

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Abstract: A theoretical model for mechanically excited PZT cantilevers including the effect of ferroelastic hysteresis has been developed and experimentally verified. The nonlinear effect of the ferroeleastic hysteresis is of second order in the excitation amplitude and therefore typically much stronger than the commonly considered third-order nonlinearities that arise from a Taylor expansion of the Gibbs free energy. It is shown that the second order nonlinearities lead to a resonance frequency that decreases linearly and a mechanical damping that increases linearly with the excitation amplitude. Both features are clearly seen in the experiments.

Keywords: energy harvester, cantilever, PZT, nonlinearities, ferroelastic hysteresis

INTRODUCTION

Bimorph and unimorph PZT cantilevers are used in literature as generators in energy harvesters or as actuating elements. Typically the excitation is sinusoidal and the cantilevers are driven in resonance to increase their displacement. Linear models for these cantilevers are widely known and demonstrated in many publications e.g [1], but with increasing excitation amplitude nonlinear effects occur that are not fully understood yet.

Typically these nonlinear effects are considered by taking higher orders in the Gibbs free energy of the piezoelectric material into account, e.g in [2] for piezoelectric harvesters and in [3] for resonantly driven actuators. The nonlinear corrections found by this method are generally of odd order, because corrections of even order cancel out due to symmetry (bimorph cantilevers) or are reduced to higher odd order corrections (unimorph cantilevers). These models do not explain the experimentally observed nonlinearities in satisfactory way, as for example shown in [4].

In [4] it was concluded that the main nonlinearity is of purley elasto-mechanic origin. Therefore we examined the ferroelastic hysteresis of the piezoelectric material as a possible source of nonlinearities.

THE PZT CANTILEVER

In general, the developed theory can be adapted to any device geometry. It has been test with a harvester tzat is described in the following. The cantilever beam has the same dimensions as that examined in [4] (Figure 1). It consist of two PZT disks (PPT 11, Stettner) with a thickness h of 260 µm with a h_S = 20µm thick gluing layer of epoxy (Stycast 2057, Emmerson & Cummings). The length of the cantilever is L=20 mm and the width is b=5 mm. Both PZT disks are connected in parallel and drive an ohmic load R. The relevant material parameters of the piezoelectric material are d_{31}= -290 10^{-12} m/V s^E_{11}=1.5 10^{-11} m²/N, ε_r = 4800. The epoxy resin has a Young's modulus of about 6 MPa which is neglected compared to the much stiffer ceramics throughout the paper.

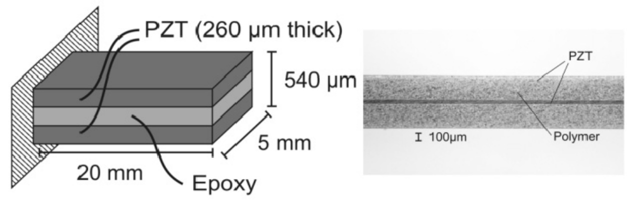

Fig. 1: Sketch and microscopic cross cut of the examined PZT cantilever.

THEORY

Hysteresis modeling

In general hysteresis modeling is a quite complicated task, but it is significantly simplified if only hysteresis loops have to be considered. The congruency theorem [5] allows to model a loop just by its amplitude, while the history of the material just leads to an offset S_{off} that does not enter our model (figure 2). Our modeling approach for a loop of stress amplitude T_0 is given in equation (1), where S denotes the strain, s^E symbolizes the linear compliance and α and γ are two dimensionless constants describing the change in compliance with the stress amplitude and the width of the hysteresis loop. Our model equals the simplest hysteresis model, the Rayleigh model, in case of $\alpha = \gamma$.

$$S = s^E(1 + \alpha \cdot s^E \cdot T_0) \cdot T \pm \frac{\gamma \cdot {s^E}^2}{2} \cdot (T^2 - T_0^2) \qquad (1)$$

For the purpose of this paper we need to know the inverted slope of $T(S)$ with S_0 as the strain amplitude (dashed line in figure 2) and the area of the loop w_{dis} that equals the dissipated energy density per loop.

$$T = \frac{1}{s^E}\cdot S - \alpha\cdot\frac{1}{s^E}\cdot S_0\cdot S + O(\alpha^2) \tag{2}$$

$$w_{dis} = \frac{4}{3}\gamma\cdot\frac{1}{s^E}\cdot {S_0}^3 \tag{3}$$

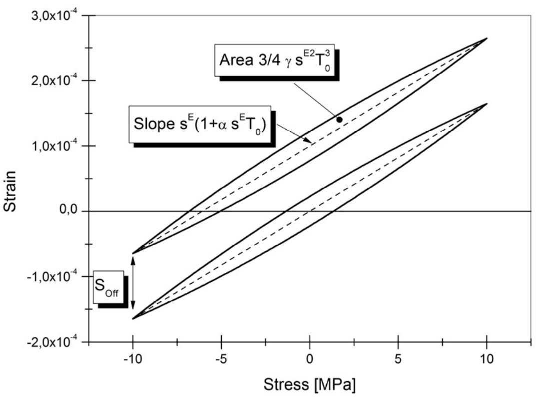

Fig. 2: Schematic explanation of the modeling approach with two loops of equal stress amplitude but different stress histories.

Cantilever model

The common theory of piezoelectric cantilevers is used with two extensions. The first extension accounts for the damping due to the dissipated energy of a strain loop. Secondly, equation (2) is used instead of the linear relation between stress and strain. It is important to note that the nonlinear term in (2) is quadratic in the strain amplitude but has the correct symmetry to enter the stiffness of the cantilever.

To model the extensions the mode shape of the cantilever has to be considered. We assume that the displacement curve *w(x,t)* is known. It consists of the dimensionless mode shape *f(x)* and the displacement z_{tip} of the tip of the cantilever (4). The tip displacement is a sinusoidal function of the excitation frequency ω and amplitude z_0.

$$w(x,t) = f(x)\cdot z_{tip} \tag{4}$$

Since the neutral plane of the beam is at $z = 0$ the resulting local strain amplitude can be expressed by equation (5).

$$S_0(x,y,z) = f''(x)\cdot z_0\cdot z \tag{5}$$

With (3) and (5) the total energy per period dissipated by the hysteresis can be calculated as volume integral of the dissipated energy density.

$$W_{hys} = \int_0^L\int_0^b 2\int_0^h w_{hys}\cdot dx\cdot dy\cdot dz$$

$$= \frac{2}{3}\alpha\cdot\frac{b\cdot h^4}{s^E}\cdot\left(\int_0^L f''^3 dx\right)\cdot {z_0}^3 \tag{6}$$

This energy loss can be approximately modeled by a hysteretic damping force proportional to the tip velocity and effective beam mass M_2 (7).

$$F_{hys} = -\delta_{hys}\cdot M_2\cdot \dot{z}_0 \tag{7}$$

With a hysteretic damping constant d_{hys} defined in (8) both equations (6) and (7) yield the same total energy dissipation per period.

$$\delta_{hys} = \frac{2}{3\pi}\cdot\frac{b\cdot h^4}{s^E\cdot M_2}\cdot\left(\int_0^L f''^3 dx\right)\cdot\gamma\cdot\frac{1}{\omega}\cdot z_0 \tag{8}$$

The consideration of (2) for the calculation of the elastic restoring force is straight forward and yields (9) to (11).

$$F_{elastic} = -(K_0 - \alpha K_1\cdot z_0)\cdot z \tag{9}$$

$$K_0 = \frac{2b\left(\left(h+\frac{h_S}{2}\right)^3 - \left(\frac{h_S}{2}\right)^3\right)}{3s^E}\cdot\left(\int_0^L f''^2\, dx\right) \tag{10}$$

$$K_1 = \frac{b\left(\left(h+\frac{h_S}{2}\right)^4 - \left(\frac{h_S}{2}\right)^4\right)}{2s^E}\cdot\left(\int_0^L f''^3 dx\right) \tag{11}$$

The total resulting system of differential equations is given in (12) and (13) with the abbreviations defined in (14).

$$q = \Phi\cdot z - \tau\cdot\dot{q} \tag{12}$$

$$M_1\cdot a(t) = M_2\cdot\ddot{z} + (K_0 - K_1\cdot z_0)\cdot z$$
$$+ M_2\cdot(\delta + \delta_{hys})\cdot\dot{z} + \Phi\cdot R\cdot\dot{q} \tag{13}$$

$$M_1 = \rho_{beam}\cdot\left(\int_0^L f dx\right)$$
$$M_2 = \rho_{beam}\cdot\left(\int_0^L f^2 dx\right) \tag{14}$$
$$\Phi = 2b\frac{d_{31}}{s^E}\cdot\frac{\left(h+\frac{h_S}{2}\right)^2 - \left(\frac{h_S}{2}\right)^2}{h}\cdot\left(\int_0^L f'' dx\right)$$

Here *a(t)* describes the sinusoidal excitation of amplitude *a*, ρ_{beam} is the mass per length of the cantilever beam, Φ is a piezoelectric coupling coefficient of dimensions force per voltage and τ is the *RC* timescale of the total generator capacitance *C* and the load resistance *R*. The mechanical damping constant δ has to be determined experimentally.

It is remarkable that the differential equations are still linear in the displacement function *z*, only the amplitude z_0 enters the equation nonlinearly. Therefore the system can be solved for the displacement amplitude z_0 and the charge amplitude q_0 by the linear relation between z_0 and q_0 (15) and the nonlinear oscillator equation (16) for the displacement amplitude.

$$z_0 = \frac{\sqrt{1+(RC)^2\cdot\omega^2}}{\Phi} q_0 \quad (15)$$

$$\left(\omega_0^2 + 2\Delta\omega_{el}\cdot\omega_0 - 2\Delta\omega_{hys}\cdot\omega_0 - \omega^2\right)^2\cdot z_0^2 + \left(\delta_0+\delta_{el}+\delta_{hys}\right)\cdot\omega^2\cdot z_0^2 = \left(\frac{M_1}{M_2}\right)^2\cdot a^2 \quad (16)$$

$$\Delta\omega_{el} = \omega_0\cdot\frac{k^2}{2}\cdot\frac{\tau^2\cdot\omega^2}{1+\tau^2\cdot\omega^2} \quad (17)$$

$$\Delta\omega_{hys} = \omega_0\cdot\alpha\cdot\frac{K_1}{2K_0}\cdot z_0 \quad (18)$$

$$\delta_{el} = k^2\cdot\omega_0^2\cdot\frac{\tau}{1+\tau^2\cdot\omega^2} \quad (19)$$

In summary, equation (16) describes an oscillator with frequency shifts due to the electrical loading (17) and due to the ferroelastic hysteresis (18) as well as electrical and hysteretic contributions to the damping (8) and (19).

Assuming that the modeshape of a non-piezoelectric cantilever is approximately valid all parameter, except α, γ and δ can be calculated. These parameters have to be fitted to the experimental data

EXPERIMENT

The harvester of figure 1 was characterized with the measurement set-up shown in figure 3. The harvester is excited by a computer-controlled shaker (Tira TV 51110). The output voltage is connected to an electronically controlled variable resistor box and is recorded by a data acquisition board (Meilhaus Electronic ME AB-D78M). Furthermore the tip displacement of the cantilever beam is measured by laser triangulation (Welotec AWL 7/0.5). More details concerning the measurement set-up can be found in [6].

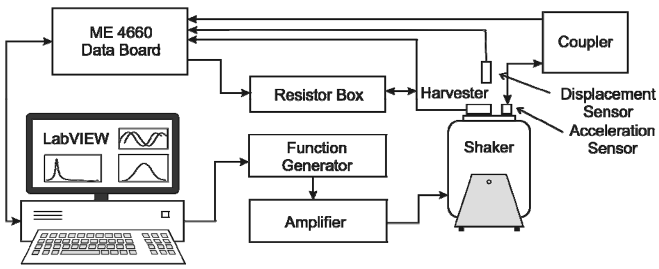

Fig. 3: Schematic of the measurement set-up [6]

RESULTS

The linear behavior of the harvester was characterized by measuring frequency response curves for different load resistances between 50 Ω and 4.4 MΩ. Good fits to the developed theory neglecting the nonlinear terms were obtained with the parameter values shown in table 1. The experimental values are very near to the simulated ones and are used as the basis of the nonlinear experiments.

Table 1: Fitted and simulated parameters of tests harvester.

Parameter	Experiment	Simulation
K_0	$1.664\ 10^3$ N/m	$1.645\ 10^3$ N/m
Φ	$2.30\ 10^{-3}$ N/V	$2.04\ 10^{-3}$ N/V
C	$3.27\ 10^{-8}$ F	$3.27\ 10^{-8}$ F
M_2	$1.022\ 10^{-3}$ kg	$1.001\ 10^{-3}$ kg
M_1/M_2	1.59	1.59

To check the nonlinear behavior frequency response curves were measured for six different excitations levels from 0.1 g to 1.1 g in steps of 0.2 g at quasi open circuit (R=4.4 MΩ) and quasi short circuit (R=50 Ω) conditions. The results for displacement and voltage are shown in figures 4 to 7.

The simulated values are generated with α = *650,* γ = 455 and δ = 26.4 s^{-1}. A fairly good agreement is observed between experiment and simulation. Especially the linear frequency shift and linearly growing mechanical damping is clearly seen.

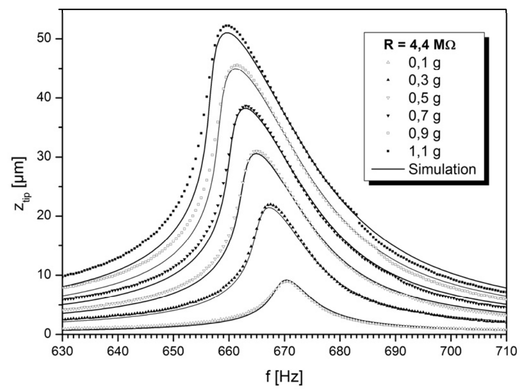

Fig. 4: Quasi open circuit: tip displacement vs. frequency

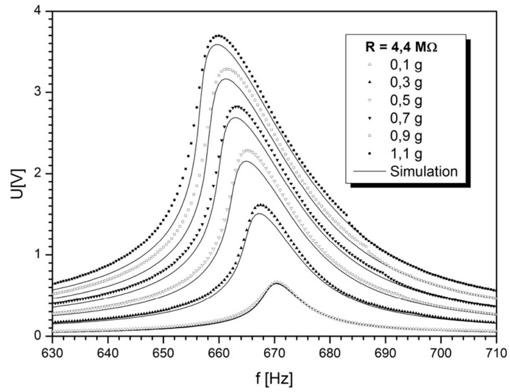

Fig. 5: Quasi open circuit: voltage vs. frequency

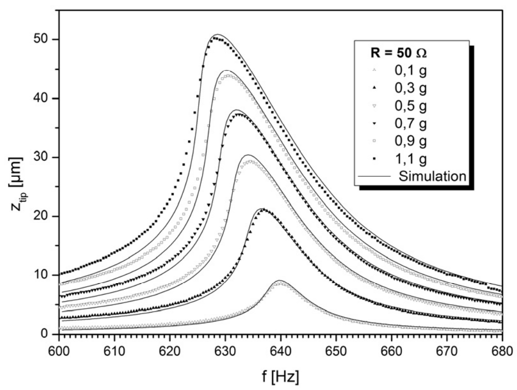

Fig. 6: Quasi short circuit: tip displacement vs. frequency

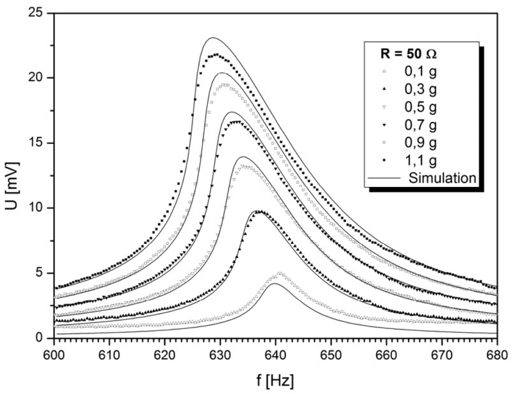

Fig. 7: Quasi short circuit: voltage vs. frequency

DISCUSSION

A nonlinear model for PZT cantilevers at low electrical fields was developed and verified by experiments. The experimental results show the predicted behavior. To our knowledge the linear frequency shift of PZT cantilevers with excitation amplitude was theoretically explained for the first time.

The two nonlinear parameters γ and α have a ratio of 0.7 which differs from the result of 1 that would be expected from the Rayleigh hysteresis model. Nevertheless both parameters are of the same order of magnitude supporting the assumption that both effects are based on the same origin. As the Rayleigh model is a special case of the Preisach model of hysteresis that occurs for a constant density of switching states it is likely that an improved model based on the Preisach approach could solve this discrepancy easily.

Still, small differences in curve shapes of experiments and simulations are observed. These are supposed to be caused by systematic measurement errors, especially for the very low voltages in figure 7, a non-optimal parameter fitting and by the disregard of higher order nonlinearities. These higher order nonlinearities are described in detail in [2] and could be added to our model without difficulty.

Nevertheless the proposed model clearly describes the most prominent nonlinearity for PZT cantilevers.

Further nonlinear corrections might be necessary for high electrical fields that typical do not occur in energy harvesters but might be applied to actuators.

The described nonlinearities are a material property of the used PZT material, other materials might have higher or lower nonlinear parameters. Cantilevers with a significant amount of non-piezoelectric material, e.g. unimorph beams show a less pronounced nonlinear behavior because the fraction of nonlinear material is reduced compared to our bimorph design.

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