THEORETICAL AND PRACTICAL LIMITS OF POWER DENSITY FOR PIEZOELECTRIC VIBRATION ENERGY HARVESTERS

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Abstract: This paper aims to clarify the achievable power density limits for resonant piezoelectric vibration energy harvesters. Four possible bottlenecks are discussed: the amount of energy transferred to the harvesting structure by inertial coupling, the amount of strain energy that the structure can sustain, the fraction of energy that is electrically converted and the fraction that can be extracted. For optimally designed resonant devices, the characteristics of the vibration source (acceleration and frequency) and the quality factor of the device define the achievable power density, suggesting 10-100 μW/cm^3 for typical vibrations. The materials and configuration of the piezoelectric beam, stress and fatigue considerations, as well as the circuitry will then define the beam size required to provide this power level. Analyzing results from four recently demonstrated resonant devices, the inertial limit appears to be commonly achieved, but varying the beam design and materials can significantly impact the beam power density. Figures of merit have been proposed to evaluate these characteristics independently of the operating conditions.

Keywords: energy harvesting, power density, vibration, inertial coupling, piezoelectric resonator

INTRODUCTION

Over the past 15 years, many research groups have built and tested miniature piezoelectric vibration energy harvesting (PVEH) devices with different configurations, geometries, materials and operating conditions. Although power densities on the order of μW/cm^3 are commonly achieved, the potential for further improvement is not clear. In practice, these results are far from the theoretical power density limits that were first considered by Roundy [1]. The goal of this paper is to discuss and identify the different limits on power density, quantify their impact, and decide whether the various devices reported in the literature have reached these limits. As a starting point, we can identify the following aspects that limit the power output of a piezoelectric harvesting device:

- How much energy is transmitted to the structure? This is defined by the vibration source itself, the quality of the inertial coupling of the device and the transmission principle (resonant or parametric);
- How much energy can the device sustain? Fatigue and stress concerns will limit the mechanical energy that can be stored in the material;
- How much mechanical energy can be converted into electrical energy? This depends on the achievable piezoelectric coupling factor, which itself depends on material properties and device geometry;
- Finally, how much electrical power can be extracted and stored? Only a fraction of the available electrical energy might be extractable, depending on the harvesting circuit used.

While all these aspects are interrelated for an inertial vibration energy harvester, each one can limit the achievable performance. The following sections will explore these limits theoretically and compare them to levels achieved in the literature.

LIMIT BASED ON INERTIAL COUPLING

This paper focuses primarily on resonant devices for vibration energy harvesting. This type of device must operate at a specific frequency, which implies a small operation bandwidth. Frequency tuning and non-linear compliant structures can help to increase the bandwidth by matching the resonator to the frequencies of vibrations, but these ideas are beyond the scope of this work. The focus here will be on resonators excited at their natural frequency. As a first step, the energy involved in the optimal operation of a resonator will be estimated. To do so, it will be assumed that the piezoelectric device has a large tip mass that occupies most of its volume. In this case, the effective mass M_{eq} nearly equals the tip mass. Table 1 presents the parameters that will be used to obtain this estimate as well as the other power density limit estimations to follow. These parameters include operating conditions, piezoelectric properties of common hard PZT [2], and density of silicon, which is used as the tip mass because it can be readily micromachined.

Table 1: Baseline parameters used for estimates

Young modulus Y (GPa)	Yield Strength σ_y (MPa)	Coupling factor k_{31}	Quality factor Q_m	Frequency f (Hz)	Acceleration amplitude A (m/s^2)	Density ρ (kg/m^3)
81.3	24	0.33	250	150	1	2330

For steady-steady oscillations at the resonant frequency, ω_n, the total mechanical energy stored in a resonating structure remains constant and can be evaluated as $E_M=½K_MX_{max}^2$, where X_{max} is the maximum tip amplitude and K_M is the mechanical stiffness. The rate at which this energy is converted from potential to kinetic energy during every cycle of oscillation is given by

$$\frac{P_M}{Vol} = \frac{1}{2}\frac{K_M X_{\max}^{\;\;2}}{Vol} f = \frac{1}{4\pi}\frac{\omega_n^3 M_{eq} X_{\max}^{\;\;2}}{Vol} \qquad (1)$$

The maximum displacement at resonance under a base acceleration is

$$X_{\max} = \frac{A}{2\omega_n^2(\zeta_m + \zeta_{el})} \qquad (2)$$

Hence, the internal mechanical power density is

$$\frac{P_M}{Vol} = \frac{1}{16\pi}\frac{\rho A^2}{\omega_n(\zeta_m + \zeta_{el})^2} \qquad (3)$$

For optimal power generation, the electrical damping, ζ_{el}, should equal the mechanical damping, ζ_m [3-4]. Using this condition, and recognizing that $Q_m = (2\zeta_m)^{-1}$, Eq (3) can be written as

$$\frac{P_M}{Vol} = \frac{1}{16\pi}\frac{\rho A^2 Q_m^2}{\omega_n} \qquad (4)$$

Using the parameters found in Table 1, we obtain a power density of 3 mW/cm^3, which is about two orders of magnitude greater than that typically achieved by inertial PVEH devices. During operation, energy is constantly removed from the device due to mechanical losses (damping) and energy harvesting. Therefore, mechanical energy must be continuously supplied to maintain steady-state oscillations. This is achieved by the work done by the inertial force acting on the mass, $F_{in}=M_{eq}A$. Noting that the input power will be the average product of force and velocity over a cycle, we obtain

$$P_{in} = P_E + P_{m,loss} = F_{in,RMS}(\omega X_{RMS}) \qquad (5)$$

which, for the specific case where mechanical and electrical damping are equal ($\bar{\zeta} = \zeta_{el}/\zeta_m = 1$), is

$$\frac{P_{in}}{Vol} = \frac{\rho A^2}{4\omega_n \zeta_m (1+\bar{\zeta})} = \frac{\rho A^2 Q_m}{4\omega_n} \qquad (6)$$

For the baseline parameters, this input mechanical power density is 154 µW/cm^3, approximately 20 times less than the internal mechanical power density. This ratio is directly a function of the quality factor: $P_M/P_{in} = Q_m/4\pi$, stating simply that a larger fraction of the mechanical energy must remain in the structure so as to sustain the high amplitude oscillations at high Q_m.

The electrical power density can be estimated as [3-4]

$$\frac{P_E}{Vol} = \frac{\rho A^2}{4\omega_n}\frac{\zeta_{el}}{(\zeta_m + \zeta_{el})^2} = \frac{\rho A^2 Q_m}{2\omega_n}\frac{\bar{\zeta}}{(1+\bar{\zeta})^2} \qquad (7)$$

which, for the optimal case of mechanical and electrical damping being equal, is the theoretical electrical power density limit due to inertial coupling

$$\frac{P_E}{Vol} = \frac{\rho A^2 Q_m}{8\omega_n} \qquad (8)$$

Here, the resulting electrical power density is estimated to be 77 µW/cm^3, exactly half the input mechanical power density. This ratio of input mechanical to output electrical powers can be expressed as: $P_E/P_{in} = \bar{\zeta}/(1+\bar{\zeta})$. A value of 1/2 means that half of the energy pumped into the resonator at each cycle is lost through mechanical damping and the other half is channeled into useful electrical energy, which is expected since $\zeta_m = \zeta_{el}$. If ζ_{el} is adjusted to be higher than ζ_m, a larger fraction of the energy pumped into the system is converted into electricity ($1/2 < P_E/P_{in} < 1$). However, the electrical power output will not increase since the excess damping reduces the amplitude, X_{max}, and consequently reduces the input mechanical power.

Equations (1) to (8) show that the power density is a function of the characteristics of the vibration source (amplitude and frequency) as well as the properties of the device (quality factor and density). Therefore, it can be misleading to use the power density as a metric to compare devices operating under different conditions. A better choice for a figure of merit (FOM) is $FOM1=P_E\omega/A^2Vol$. The power density can be increased by using metals such as nickel, tungsten or gold instead of silicon. The density of these metals is 4 to 8 times that of silicon, but their use can have adverse implications for manufacturability and cost. Finally, high values of Q_m should lead to devices with high power densities, but also to high deflections and stresses. The next section addresses this additional limit on performance.

LIMIT BASED ON STRESS

In practice, the maximum stress must be limited to ensure reliable operation for the lifetime of the device. The maximum mechanical power density would then be given by the rate at which this maximum strain energy can be induced to the structure without failure. If the material is uniformly stressed to its yield limit (σ_y) in a harmonic fashion and all the electrical energy is extracted, the average electrical power density limit is given by [1]:

$$\frac{P_E}{Vol_b} = \frac{1}{4}\frac{(\sigma_{RMS})^2}{Y}k_{31}^2 f = \frac{1}{4}\frac{\sigma_y^{\;2}}{Y}k_{31}^2 f \qquad (9)$$

Here, Vol_b is the volume of the beam. This limit suggests an ultimate power density of 29 mW/cm^3 (in terms of beam volume) using our baseline parameters. For equivalent beam and mass volumes, this value is three orders of magnitude higher than the inertial limit established previously. In practice, most resonators are not subject to a uniform stress field because they are cantilevers. For this configuration, the power density can be expressed in terms of stiffness and tip displacement of the cantilever beam as

$$\frac{P_E}{Vol_b} = \frac{1}{2}\frac{K_M X_{RMS}^{\;\;2}}{Vol_b}k_{31}^2 f = \frac{1}{4}\frac{K_M X_{\max}^{\;\;2}}{Vol_b}k_{31}^2 f \qquad (10)$$

For simplicity, it is assumed that the stress distribution is that of a statically deformed rectangular cantilever beam with a force applied at the free end. Using well known beam equations for deformation, stress, and stiffness, the power density can be written as

$$\frac{P_E}{Vol_b} = \frac{1}{36}\frac{\sigma_{max}^2}{Y}k_e^2 f \qquad (11)$$

with $k_e^2=k_{31}^2$. Equation (11) reveals that the power density is reduced by a factor of 9 (to 3.2 mW/cm^3) due to distribution of strain along the length, and across the thickness of the beam (Fig. 1). A triangular tapered beam would maintain the stress over the entire length and only have a reduction factor of 3.

Fatigue failure could impose a more severe limit, and reduce the allowable stresses under cycling to values 10 times less than σ_y, thus reducing the power density by two orders of magnitude. Fatigue limits of piezoelectric materials are not well known, so this is a field that requires more investigation. When exposed to large vibrations, or if a device has a high Q_m, stress and fatigue concerns could take priority on the inertial coupling aspect, limiting the power density. The design process should therefore evaluate the maximum stresses and apply a design constraint to prevent mechanical failure. Piezoelectric fatigue due to high operating voltages and environmental conditions (high T and %RH) should also be considered while defining the maximum allowable stress.

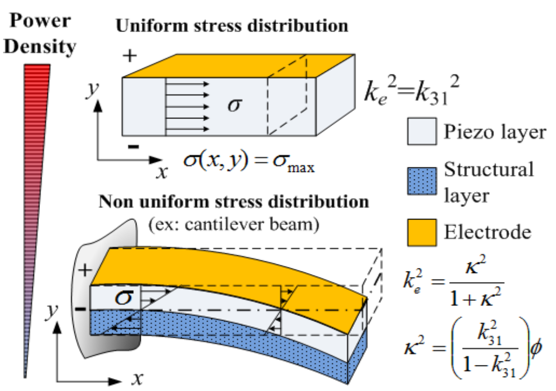

Figure 1: Effect of stress distribution on power density

Thus far, we have not considered the effect of the non uniform stress distribution on the electromechanical conversion factor. As shown in the next section, this factor can also reduce the power density of the device.

ELECTROMECHANICAL CONVERSION

The previous section assumed that the effective piezoelectric conversion factor, k_e^2, was only dependant of the piezoelectric material ($k_e^2=k_{31}^2$). This is true only for structures that are uniformly stressed. The coupling factor is related to the electrical-to-mechanical energy ratio, κ, by [6]

$$k_e^2 = \frac{\kappa^2}{1+\kappa^2}\ ;\ \kappa^2 = \frac{\theta^2}{K_m C_p} = \left(\frac{k_{31}^2}{1-k_{31}^2}\right)\phi \qquad (12)$$

for a d_{31} mode harvester. In this expression, θ is the piezoelectric coupling coefficient of the harvester, C_p is the capacitance and ϕ is a geometric correction factor that is a function of the elasticity and thickness ratios between the piezoelectric and structural layers [4]. For an elasticity ratio of one, the optimal thickness fraction of a unimorph beam is h_{piezo}/h_{total} = 0.33, with ϕ = 0.33; these values are doubled for a bimorph configuration. Other configurations, such as air spaced cantilevers, have been studied and have shown improved conversion factors [7]. Finally, for an optimized unimorph cantilever with the baseline parameters, the electromechanical conversion decreases from k_{31}^2=10.9% (uniform stress) to k_e^2=3.9%, which gives a power density of 1.1 mW/cm^3.

ELECTRICAL ENERGY EXTRACTION

As previously stated, the optimal load should be chosen such that $\zeta_{el}=\zeta_m$. Since $\zeta_{el}\approx\kappa^2/4$ [4] and $\zeta_m = 1/2Q_m$, we can rewrite this condition as $\kappa^2Q\approx2$; therefore, devices with low piezoelectric coupling or high mechanical losses will not be able to reach this optimal condition. Once this condition is reached, $\kappa^2Q>2$, further increasing the coupling will not increase the output power, although increasing Q is always beneficial for power density (see Eqn. 8). Non-linear harvesting circuits have been shown to increase the electrical damping and therefore the power output [5], but only for highly damped or lightly coupled devices ($\kappa^2Q < 2$). In either case, the PVEH cannot overcome the inertial limits identified above.

Table 2: Assessment of several PVEH devices reported in the literature.

Ref.	Piezo material (process)	Piezo thick-ness (μm)	Beam thick-ness (μm)	Thick-ness fraction	Mass* (mg)	f (Hz)	A (m/s^2)	A^2/ω (m^2/s^3)	Q_m	k_e^2	κ^2Q_m	σ^* (MPa)	P_E (μW)	*FOM1* *(g/cm^3)*	*FOM2*	*FOM3* (nJ/mm^3)
[8]	d_{33} PZT (sol-gel)	2	9	0.22	0.76	877	19.6	0.070	220*	0.05	11.6*	84	1.4	62	0.98	550
[9]	d_{31} PZT (epitaxial)	2.8	56	0.05	38	126	5	0.032	85	0.013	1.12	7	5.3	33	0.41	8.3
[10]	d_{31} AlN (sputtering)	0.8	45.8	0.017	28	572	19.6	0.11	170	0.0022	0.38	460	60	43	0.94	454
[11]	d_{31} AlN	1.2	51.2	0.023	14	1082	3.14	0.0015	1200	0.0025	3	170	3	333	1	46

*****Estimated values

DEVICE ASSESSMENTS

Table 3 summarizes data extracted from the literature for several PVEH resonators. All these devices produce power on the order of 1 to 100µW.

These devices have different dimensions and operate under different excitations, but their performance can be compared using the first figure of merit introduced earlier ($FOM1=P_E\omega/A^2Vol$). *FOM1* essentially scales with ρQ_m, motivating the choice of high density materials and devices with high Q_m. For example, device from Ref. [11], with its very high Q_m, has a *FOM1* larger than all the others. However, a low value of Q_m can be compensated by using materials with higher density, as can be seen from the devices reported in Ref. [9] and Ref. [10]. The former uses stainless steel as the structural layer instead of silicon.

A second figure of merit based on Eqn 8 is introduced to evaluate whether the device has approached the inertial limit: $FOM2 = P_{E,m}/(MA^2Q_m/8\omega_n)$. Only the device reported in Ref. [9] has a value of *FOM2* that is significantly less than unity, which is expected because $\kappa^2Q < 2$*. The device in Ref. [9] could be improved by increasing its thickness fraction, which would increase k_e^2; this modification could allow it to reach the inertial limit. As illustrated by the device in Ref. [11] however, an optimum thickness fraction is not required to achieve the inertial limit if the quality factor is sufficiently high.

A third figure of merit is introduced to evalue the power density of the beam, based on Eq. 11. To account for different operating frequencies, it is defined as $FOM3 = P_E/Vol_b f$, which scales as $\sigma_y k_e^2$. As expected, highly stressed [10] and highly coupled [8] beams have higher beam power density. Although increasing coupling will not increase the power density based on the total volume (*FOM1*), it will reduce the required beam volume for a desired electrical power.

CONCLUSION

This paper presented a discussion of the various aspects that can limit the power density of piezoelectric energy harvesters. The first section addressed the energy flow in resonant devices, which is limited by inertial coupling, parameterized by the mass, damping, and excitation (A^2/ω). With typical values for PZT, an acceleration source of 1 m/s^2 at 150 Hz, and a Q_m factor of 250, a power density limit of 77 µW/cm^3 was estimated. This power density is further limited by coupling or by the strength of the material, if very large accelerations or forces are available. An assessment of several resonant devices published in the recent literature leads to the conclusion that many of these devices are already at the inertial limit, or could easily be. A new figure of merit has been proposed to evaluate the beam design and guide the selection of materials and geometry to achieve a specified power level in a compact beam volume. These metric, along with cost and manufacturability considerations, provide a framework for comparing inertial piezoelectric energy harvesters. Further reflections are however required to define figures of merit that account for broadband operation and to identify the actual stress limits of beam materials under high cycling.

* However, the results for the device from Ref. [10] are inconsistent because a similarly low value of κ^2Q should lead to a low *FOM2*.

ACKNOWLEDGEMENT

This research was supported by General Motors of Canada and by the Natural Sciences and Engineering Research Council (NSERC) of Canada through the Canada Research Chairs, Post Graduate Scholarship and CREATE ISS programs.

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