NONLINEAR BEHAVIOR OF CANTILEVER-BASED PIEZOELECTRIC ENERGY SCAVENGERS UNDER LARGE EXCITATIONS

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Abstract: A common way to model piezoelectric cantilever based energy scavengers is to model the system as a simple single degree-of-freedom (1DOF) damped harmonic resonator, reducing the coupled multiple degree-of-freedom system to one-dimensional motion of an equivalent mass. In this work, we investigated nonlinear behavior of cantilever-based piezoelectric energy scavengers in cases when the proof mass center is located off axis and some distance from the cantilever tip. This was done by developing a three degree-of-freedom (3DOF) circuit-based model of the scavenger. As hypothesized, the model exhibited a softening type nonlinear behavior, resulting in a 1-3 % frequency shift. Our results are comparable, albeit smaller in magnitude, than the experimental data obtained from our Piezoelectromagnetic (PEM) AC energy scavenger. These results indicate that the geometric nonlinearities account for some of the nonlinear behavior observed in the PEM AC energy scavenger.

Keywords: Energy Scavenging, Piezoelectromagnetic, Nonlinear, Piezoelectric, Smart Grid

INTRODUCTION

The most common geometry for piezoelectric energy scavengers is a rectangular cantilever covered with piezoelectric material, with a proof-mass attached to its tip [1]. During alternating excitations of the proof mass, the strain in the piezoelectric layer converts the mechanical energy stored in the resonating spring-mass system to electric power. The common way to model piezoelectric cantilever based energy scavengers is to model the system as a simple single degree-of-freedom (1DOF) damped harmonic resonator, reducing the coupled multiple degree-of-freedom system to one-dimensional motion of an equivalent mass. However, in cases where the excitations of the proof-mass are large, and/or where the mass-center is offset from the neutral bending axis of the cantilever, a significant portion of the mechanical energy is used for rotating the mass, which may exuberate the effect of higher order interactions between the individual degrees of freedom. Such effects are often truncated in a 1DOF model.

This work was motivated by experimental results obtained from our Piezoelectromagnetic (PEM) AC energy scavengers [2], where we observed a 4-6 % change in the resonant frequency with increasing excitations. PEM AC scavengers use piezoelectric conversion from a mechanically resonating system excited by a magnet interacting with the current in a nearby conductor as the means for providing power for wireless Smart Grid sensing applications. We postulated that in our PEM AC scavenger, purely mechanical nonlinear effects cause the change in the resonant frequency of the scavenger as a function of the varying load. Such a shift is especially problematic for narrow-band scavengers, such as the PEM AC

scavengers [2], and a model that fully describes this phenomenon is highly desirable in order to remedy this effect.

Apart from intrinsic mechanical effects, nonlinear behavior of the energy scavengers can be among other caused by nonlinearity in the piezoelectric coupling [3], the design of nonlinear springs, or switchable power conditioning circuits. Purposefully nonlinear energy scavengers have been shown to generate higher power output under large bandwidth excitations. In narrow-band applications, such as AC energy scavenging, an accurate model of the nonlinear behavior may ultimately enable us to use the PEM AC energy scavenger as a current sensor, enabling compact and truly self-powered current sensing applications for the Smart Grid.

PEM AC ENERGY SCAVENGING

Piezoelectromagnetic (PEM) AC energy scavengers use a strong permanent magnet to couple a resonating mechanical mass-spring system (e.g., cantilever) to a current-carrying conductor, and then use piezoelectric transduction to convert the mechanical energy from the resonating circuit to electric power [2]. Fig. 1 shows a mesoscale PEM AC scavenger, consisting of a piezoelectric bi-morph cantilever (a) with magnets attached at its tip (b), placed in the vicinity of a single current-carrying conductor (c). Physical stoppers in the form of nonlinear springs (d) prevent excessive excitation in the case of overcurrent conditions. The magnetic circuit of this scavenger has been optimized to maximize the coupling to a single current-carrying conductor.
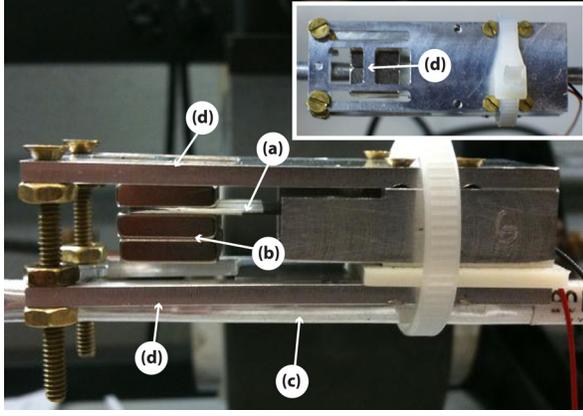
Fig. 1: A Mesoscale PEM AC energy scavenger.

Fig. 2 shows the drift in the resonant frequency of the PEM AC energy scavenger with increasing current flowing though a nearby conductor, recorded at near short-circuit conditions (1 kohm) and at near open-circuit conditions (1 Mohm). In both cases, the resonant frequency of the PEM scavenger was reduced by more than 4 Hz as the current I was increased from 1 A_{RMS} to 15 A_{RMS}. Similar trends recorded under dramatically different voltages across the piezoelectric layers suggest that the nonlinearity is confined to the mechanical domain.
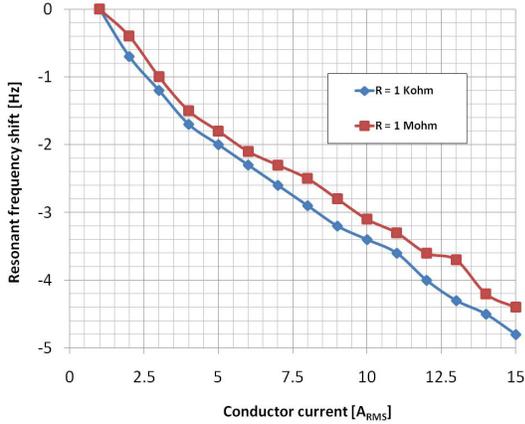

Fig. 2: Shift in the resonant frequency of the PEM AC energy scavenger with the increase in current flowing though the nearby conductor.

The goal of this work is to examine whether nonlinear terms of geometric origin in the equations of motion may contribute to this frequency shift. In order to maximize the coupling to the current in the conductor, the permanent magnets must be placed in a close proximity to the current-carrying conductor. This may cause the center of mass (COM) for the magnets (which also serve as the proof-mass of the resonating circuit), to be asymmetrically distributed around the beam, causing COM to be displaced away from the neutral axis of the cantilever (see Fig. 1). It was hypothesized that such a displacement may drastically increase the effect of the nonlinearities.

THE 3DOF MODEL

System Representation

A conceptual system representation of a piezoelectric energy scavenger is shown on Fig. 3. We assume a bi-morph piezoelectric cantilever beam made out of PZT, with the proof-mass suspended beneath the cantilever.
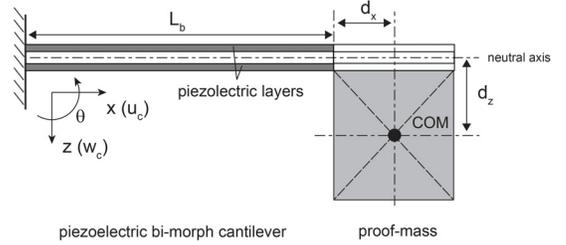

Fig. 3: A conceptual system representation of a general cantilever-based energy scavenger.

Constituent Equations

Using Euler-Bernoulli theory for piezoelectric beam of length L_B, the linear constitutive equations can be put on to the form

$$\begin{pmatrix} F_b \\ M_b \\ P_b \end{pmatrix} = \begin{pmatrix} k_t & k_c & 0 \\ k_c & k_r & 0 \\ 0 & 0 & k_a \end{pmatrix} \begin{pmatrix} w_b \\ \theta_b \\ u_b \end{pmatrix} + \begin{pmatrix} \Gamma_z \\ \Gamma_\theta \\ \Gamma_x \end{pmatrix} V \quad (1)$$

$$q = -\Gamma_z w_b - \Gamma_\theta \theta - \Gamma_x u_b + C_0 V \quad (2)$$

where F_b is the vertical force at the tip of the beam, M_b the moment, P_b the axial force and V the voltage across the electrodes, w_b is the vertical tip displacement, θ is the tip rotation angle, u_b is the longitudinal tip displacement and q is the charge on the electrodes.

C_0 is the clamped capacitance. The short circuit stiffness matrix elements have the standard forms $k_t = 12 K_B / L_B^3$, $k_c = -6 K_B / L_B^2$, $k_r = 4 K_B / L_B$ and $k_a = K_A / L_B$, where K_B and K_A are respectively the flexural and axial rigidities of the beam.

For a straight beam, the electromechanical coupling is parameterized by $\Gamma_z=0$, $\Gamma_\theta=-N$ and $\Gamma_x=J$ where N is the bending moment per unit voltage and J is the axial force per unit voltage in the beam.

Equations of Motion

We derive the equations of motion for the 3DOF model using Eq. (1)-(2). The forces and moments on the proof mass are those from the beam, the force at the center of mass, and the damping forces.

It is most convenient to write the equation of motion for the system in terms of COM displacements, but we know the forces from the beam only in terms of beam tip displacements. The relations between the displacements are

$$w_b = w_c - d_x \sin\theta - d_z(1 - \cos\theta) \quad (3)$$
$$u_b = u_c + d_x(1 - \cos\theta) - d_z \sin\theta \quad (4)$$

which can be reduced to

$$w_b \sim w_c - d_x\theta \quad (5)$$
$$u_b \sim u_c - d_z\theta \quad (6)$$

in the linear case.

We use energy methods to derive the equations of motion. The internal energy in terms of the COM displacements $U_c(w_c,\theta,u_c,q)$ is expressed in terms of the internal energy $U^0_b(w_b,\theta,u_b,q)$ corresponding to the linear constitutive equations for the beam. We then consider $w_b,\theta,u_b,q,V\sim O(\varepsilon)$ where ε is a formal smallness parameter, and treat the distances as $d_x,d_y\sim O(\varepsilon^{-1})$ in order to render them large. Finally U_c is expanded to a second order form in ε as

$$
\begin{aligned}
U_c(w_c,\theta,u_c,q) &= U^0_b(w_b,\theta,u_b,q) \\
&= U^0_b(w_c - d_x\theta, \theta, u_c - d_z\theta, q) \\
&\quad -\frac{1}{2}k_{zz}d_x\theta^2(w_c - d_x\theta) + \frac{1}{2}k_{xx}d_x\theta^2(u_c - d_z\theta) \\
&\quad -\frac{1}{2}(k_{z\theta}d_z - k_{\theta x}d_x)\theta^3 + \frac{1}{8}(k_{zz}d_z^2 + k_{xx}d_x^2)\theta^4 \\
&\quad +\frac{1}{2C_0}\Gamma_x d_x q\theta^2 + \mathcal{O}(\epsilon^3).
\end{aligned}
\quad (7)
$$

In this expression we have used the open circuit stiffness k_{zz}, $k_{z\theta}$, k_{zx}, $k_{\theta\theta}$, $k_{\theta x}$, and k_{xx} which are related to the short circuit stiffness as $k_{zz}=k_t$, $k_{z\theta}=k_c$, $k_{zx}=0$, $k_{\theta\theta}=k_r+\Gamma_\theta^2/C_0$, $k_{\theta x}=\Gamma_\theta\Gamma_x/C_0$, and $k_{xx}=k_t+\Gamma_x^2/C_0$.

We can now write down the equations of motion for the model as

$$m\ddot{w}_c = -\frac{\partial U_c}{\partial w_c} - f_z^d + F_z^{\text{ext}} \quad (8)$$

$$I_c\ddot{\theta} = -\frac{\partial U_c}{\partial\theta} - f_\theta^d \quad (9)$$

$$m\ddot{u}_c = -\frac{\partial U_c}{\partial u_c} - f_x^d \quad (10)$$

where m is the proof mass, I_C is the moment of inertia of the mass, f^d_z, f^d_θ, and f^d_x are damping forces and F_Z^{ext} is the external vertical excitation force.

The voltage is obtained from

$$V = -\frac{\partial U_c}{\partial q}. \quad (11)$$

Taking the partial derivatives of Eq. (7) according to Eqs. (8)-(11) results in the following expanded equations of motion:

$$m\ddot{w}_c = -k_t w_c - \alpha m\dot{w}_c - \mathbf{A}\theta - \mathbf{B}\theta^2 - \frac{\Gamma_z}{C_0}q + F_z^{\text{ext}} \quad (12)$$

$$
\begin{aligned}
I_c\ddot{\theta} &= -\mathbf{C_K}\theta - \alpha I_C\dot{\theta} - \mathbf{A}w_c - \mathbf{D}u_c - \mathbf{E}w_c\theta \\
&\quad -\mathbf{G}\theta u_c - \mathbf{H}\theta^2 - \mathbf{I}\theta^3 - \frac{\hat{\Gamma}_\theta}{C_0}\theta q - \frac{\tilde{\Gamma}_\theta}{C_0}q
\end{aligned}
\quad (13)
$$

$$m\ddot{u}_c = -k_a u_c - \alpha m\dot{u}_c - \mathbf{D}\theta - \mathbf{J}\theta^2 - \frac{\Gamma_x}{C_0}q, \quad (14)$$

as well as the electrical side

$$V = \frac{\Gamma_z}{C_0}w_c + \frac{\tilde{\Gamma}_\theta}{C_0}\theta + \frac{\Gamma_x}{C_0}u_c + \frac{\breve{\Gamma}_\theta}{C_0}\theta^2 + \frac{1}{C_0}, \quad (15)$$

where

$$\mathbf{A} = k_c - d_x k_t \quad (16)$$
$$\mathbf{B} = \frac{1}{2}k_t d_z \quad (17)$$
$$\mathbf{C_K} = k_r - d_x^2 k_t - 2d_x k_c + d_z^2 k_a \quad (18)$$
$$\mathbf{D} = -d_z k_a \quad (19)$$
$$\mathbf{E} = -d_z k_t \quad (20)$$
$$\mathbf{G} = d_x k_a \quad (21)$$
$$\mathbf{H} = \frac{3}{2}(d_x d_z(k_t - k_a) - k_c d_x) \quad (22)$$
$$\mathbf{I} = \frac{1}{2}(k_t d_z^2 + k_a d_x^2) \quad (23)$$
$$\mathbf{J} = \frac{1}{2}k_a d_x \quad (24)$$
$$\tilde{\Gamma}_\theta = \Gamma_\theta - d_x\Gamma_z - d_z\Gamma_x \quad (25)$$
$$\hat{\Gamma}_\theta = \Gamma_x d_x - \Gamma_z d_z. \quad (26)$$
$$\breve{\Gamma}_\theta = -\frac{1}{2}\hat{\Gamma}_\theta \quad (27)$$
$$\Gamma_z = 0. \quad (28)$$

Eqs. (12) – (28) can now be used to construct a circuit that represents the lumped-parameter 3DOF model (shown on Fig. 4) of the energy scavenger, which now includes the effect of the higher order terms. The contribution of these nonlinear terms is represented through behavioral voltage and current sources defined through functions f_z, f_θ, f_x, f_q, and f_y.
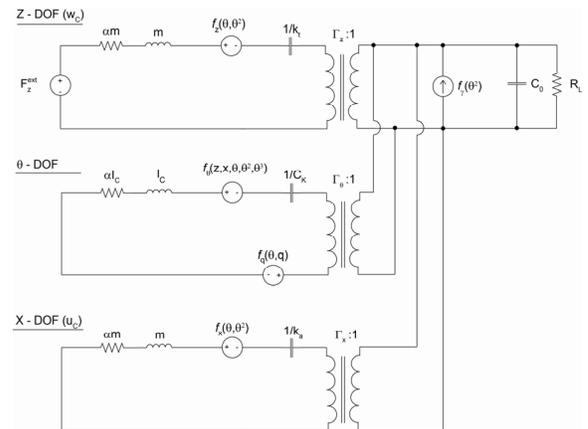

Fig. 4: Circuit representation of the lumped-parameter 3DOF model.

The model was simulated in LTSPICE (v. 4.12a). A typical transient simulation of 500 ms simulation time would complete in approximately 30 seconds real time on an Intel Core 2 Duo CPU at 2.53 GHz, with 3 GB RAM. A 1DOF model of the same scavenger system executes in approximately 1 second.

MODELING RESULTS

To allow a good comparison between the experimental data and the 3DOF model, we chose parameters such that the model corresponded to the PEM AC scavenger shown on Fig. 1, with a separation of 2.5 mm between the bottom magnet and the current carrying conductor. Current I from 5 A_{RMS} − 20 A_{RMS} was modeled. In addition, a 1DOF linear model of the system was constructed. Since we were interested in analyzing the mechanical nonlinearities, the models were primarily executed under near short-circuit conditions (R_L = 1 kohm).

Fig. 5 shows the voltage response from 3DOF model. The response is normalized both in frequency and the voltage domain with respect to the resonant peak at 5 A_{RMS}.
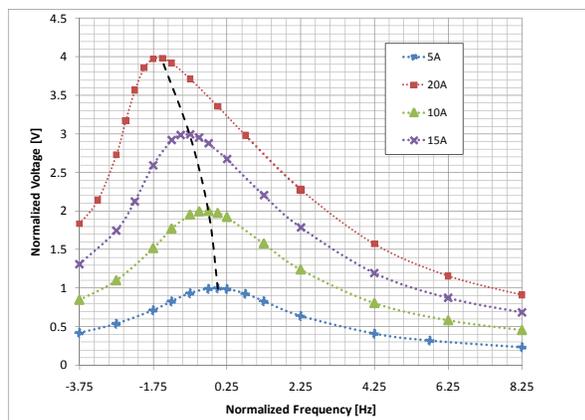

Fig. 5: Normalized voltage response of the 3DOF model as a function of increasing input excitation.

A significant nonlinear effect can be observed in the response of the 3DOF model. The resonant frequency was reduced by 1.5 Hz, indicating a *softening* type of behavior of the corresponding mass-spring system. This trend is consistent with our experiments. The normalized voltage response of the linear 1DOF model is for comparison shown in Fig. 6.

Table 1 shows the frequency shift (ΔHz/ΔA) for R_L = 1kohm (~short-circuit) and 1Mohm (~open circuit) for the 3DOF model compared to the experimental data. In both cases, the experimentally recorded frequency shift was significantly larger than of the 3DOF model. This suggests that is not describing all of the nonlinear effects of the PEM system. Nonlinearities in the beam itself is a possible additional mechanism that may be significant.
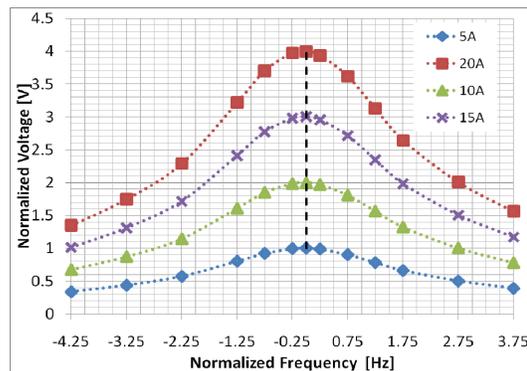

Fig. 6: Normalized voltage response of the linear 1DOF model as a function of increasing input excitations.

Table 1: Frequency shift (ΔHz/ΔA) of the 3DOF model compared with experimental results.

R_L	Experimental	3DOF Model
1 kohm	-0.28	-0.1
1 Mohm	-0.26	-0.05

CONCLUSION

In this paper, we developed a lumped parameter 3DOF model of a cantilever-based energy scavenger. It includes higher order nonlinear terms, with the hypothesis that under large excitations these terms may contribute to a nonlinear response. Indeed, the results from our model show a softening-type nonlinear behavior, which is consistent with our experimental results. However, the magnitude of the resonant frequency shift was 3-5 times smaller than what we observed experimentally, suggesting that not all nonlinear effects are described by our 3DOF model. Further work needs to be done to model all the remaining nonlinear contributions, such that an accurate model of the response of the scavenger could be created, ultimately perhaps enabling the fabrication of a resonant current sensor.

ACKNOWLEDGEMENTS

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