

# COUPLED-MOTION HARVESTING VS. PLUCKED HARVESTING: WHICH PERFORMS BETTER?

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**Abstract:** This paper uses a numerical implementation of analytical models to directly compare two forms of impact-based frequency up conversion technologies: coupled-motion harvesting and plucked harvesting. Plucked harvesting is observed at lower driving frequencies and a wide range of accelerations, and coupled-motion harvesting is observed at higher driving frequencies and low accelerations. The models show that plucked harvesting offers higher output power, but a system that undergoes plucked harvesting in some regimes and coupled harvesting in others offers the widest range of operating accelerations and frequencies.

**Keywords:** frequency up conversion, energy harvesting

## INTRODUCTION

Converting low frequency ambient motion to a high generating frequency can increase power output when harvesting energy from low frequency sources [1-7]. Up conversion can be accomplished by contact [1,2,4-6] or non-contact interactions [3,7] between a lower frequency driving element and a higher frequency generating element. This paper focuses on contact-mediated up conversion.

Coupled-motion frequency up converting harvesters were analyzed by Gu *et al* [1]. In the coupled-motion harvester, a lower-frequency driving beam bounces off of a higher-frequency generating beam, and power is generated primarily during the elements' coupled motion. Analytical models showed that coupled-motion harvesting offered a higher electrical damping ratio as compared with conventional harvesting at the low driving frequency. Plucked harvesters are a second type of contact-based, frequency up converting harvester. In a plucked architecture, a higher-frequency generating element is plucked by interaction with a slower-moving element and generates power while it rings down at its own resonant frequency. Examples of this type of harvesting include [6], in which the generating beam is plucked when a plectrum moves past its tip, and [5], in which generating beams are plucked when they first stick to and then are released from a passing magnet. Despite research on the two types of harvesters, the question of which type of harvesting (coupled motion or plucked) offers better performance has not previously been answered.

This paper directly compares coupled motion harvesting to plucked harvesting by modeling a single system that transitions from coupled-motion to plucked harvesting under small variations in the experimental set up or driving conditions. Figure 1 shows a schematic of the system, which comprises a plastic driving beam with a tip mass placed opposite and above a piezoelectric generating beam. The beams are separated vertically by the gap spacing, and they overlap horizontally by the beam overlap. For large values of beam overlap and/or small deflections, the tip of the driving beam cannot pass the tip of the generating beam, and the system operates in a coupled-motion mode. For small values of the beam overlap and/or large deflections, the tip of the driving beam can pass the tip of the generating beam, and the system operates in plucked mode.

Figure 2 illustrates the beam motions in both coupled-motion and plucked mode. Depending on acceleration, frequency, and overlap, when the beams come into contact they will either pass one another and pluck, or remain in coupled motion. In coupled-motion mode, the driving beam remains on one side of the generating beam. In plucked mode, the driving

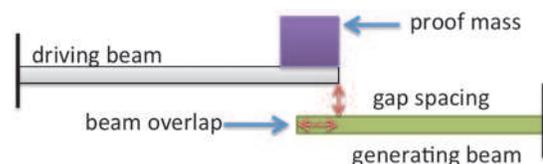


Figure 1. Schematic diagram showing the flexible driving beam with a proof mass and the high frequency piezoelectric generating beam.

beam first deflects the generating beam until the tips of the two beams just touch. Then the driving beam passes the generating beam's tip, leaving it to oscillate freely and generate power. In plucked mode, the driving beam crosses from above to below the generating beam and back.

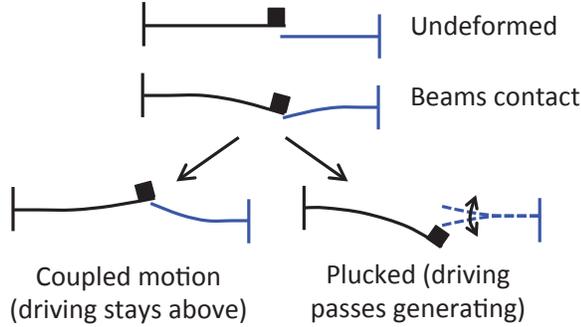


Figure 2 Schematic of impact based frequency up conversion techniques.

## MODELING

The harvester models are implemented with a lumped parameter approximation, and the harvesting dynamics are captured using direct integration over time. Impacts between the two beams are tracked to determine when the beams bounce off each other in coupled motion mode or pass each other in plucked mode. The model is allowed to run until it reaches steady state.

Each beam is modeled as spring-mass-damper system, and the displacement  $x$  of each beam is assumed to be governed by

$$m\ddot{x} + c\dot{x} + kx = F(t), \quad (1)$$

where  $m$ ,  $c$ , and  $k$  are the lumped parameter mass, damping coefficient, and spring constant. The external driving force  $F(t)$  is taken to be sinusoidal for the driving beam and zero for the generating beam. The spring-mass-damper approximation is appropriate for the driving beam, which has no electrical damping. It is not a strictly accurate approximation for a piezoelectric generating beam, as piezoelectric power extraction goes as displacement rather than as velocity. The approximate model is used here in the interests of simplicity.

The equations are solved simultaneously using numerically implemented differential equations. The motions of the two masses are tracked independently, and impact is identified whenever the positions of the two masses pass each other. When impact is detected, the approach velocities are calculated from the positions before and after impact and the time step. Assuming a coefficient of restitution (COR) of 0.5

and conservation of momentum, the departure velocities are then calculated. (The chosen value of COR affects the dynamics, but the effects are relatively mild except for extreme values such as COR=0.) The model then solves the differential equations using the impact position and departure velocities as the new initial conditions.

Along with tracking impact, the model tracks the positions of the beam tips to determine whether the driving beam has passed from above to below the generating beam or vice versa. The horizontal and vertical beam deflections are calculated as in [8], and the positions of the beam tips in the plane are compared in order to determine whether passing has occurred. Since there is a one-to-one correspondence between the horizontal and vertical deflections of a given beam, the beam-passing criteria are readily implemented in the lumped parameter model. Note that for the 10 mm driving beam and 31.8 mm generating beam used here, the horizontal displacements are very small, on the order of 10  $\mu\text{m}$ .

Electrical power extraction and its corresponding contribution to system dynamics are approximated as an additional damping term, as described above. The electrical damping ratio was calculated as in [1] and added to the mechanical damping. The electrical damping ratio depends on the electrical frequency at which power is being extracted, which in turn depends on whether the system is undergoing coupled motion (with harvesting at an intermediate, coupled-motion frequency) or plucked motion (with harvesting at the generating beam's higher, free-vibration frequency). Whether the system undergoes coupled-motion or plucking dynamics in turn depends on the amount of electrical damping. To resolve this self-consistently, the dynamics are calculated for each operating condition assuming first a coupled-motion electrical extraction frequency and then a plucked, higher electrical extraction frequency. The result that provides a self-consistent operating condition is used.

The driving beam is taken to be a 100mm x 10mm x 1.5875mm beam of ABS plastic with a 10g proof mass, and the generating beam is taken to be a 31.8mm x 12.7mm x 0.51mm PZT bimorph beam with an effective mass of 0.194 g. The mechanical damping coefficients of the driving and generating beams are taken to be 0.03 Ns/m and 0.005 Ns/m respectively, equal to those measured for coupled-motion harvesters of the same geometry [1]. The stiffnesses of the driving and generating beams are taken to be 22.4 N/m and 416 N/m, respectively. The permittivity,  $d_{31}$ , and capacitance of the PZT bimorph are taken respectively to be  $1.59 \times 10^{-8}$  F/m, -190 pC/N, and 7.4 nF.

The displacements, voltage, instantaneous power, and total extracted energy were calculated while varying acceleration, driving frequency, and amount of overlap between the two beams. Both plucked and coupled-motion harvesting are typically observed. To ensure direct comparability, each map of performance vs. parameters was repeated twice, once with a load resistance that was well-matched to plucked harvesting and one with a load resistance that was well-matched to coupled-motion harvesting.

### MODELING RESULTS

Figures 3 and 4 plot the beam’s displacements, open circuit voltage, instantaneous power, and energy converted vs. time for coupled-motion and plucked harvesting, respectively. During coupled-motion harvesting, the majority of the energy is extracted during the coupled motion phase, even if the load resistance is chosen to match the free oscillation frequency. During plucked harvesting, the coupled-motion phase is brief, and the majority of the energy is extracted during free oscillation.

Figure 5 maps the harvesting regime vs. acceleration and beam overlap. Driving frequency is fixed at 8.66 Hz, and the vertical gap spacing is 0.5 mm. Plucked harvesting occurs at low beam overlaps, typically 0.01%, and a wide range of accelerations, from 0.2 g to 1.5 g. Coupled-motion harvesting occurs at lower accelerations and larger overlaps. Mixed harvesting, which exhibits both plucking and coupled motions, is also observed.

Figure 6 maps harvesting regime vs. driving frequency and acceleration for a gap spacing of 0.5 mm and 0.02% beam overlap, with load resistance

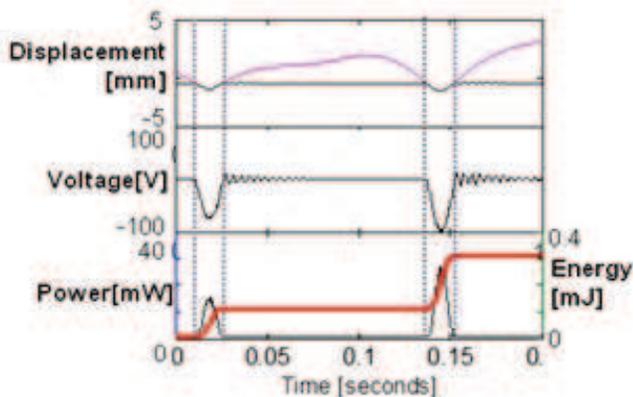


Figure 3. Plots of driving (purple) and generating (black) beam displacements (top), open circuit voltage (middle), and both instantaneous power (black) and converted energy (thick red) (bottom) vs. time for a coupled-motion harvester with matched load resistance. Dotted lines indicate the times at which the beams come in contact and separate.

chosen to match coupled-motion harvesting. Figure 7 plots power vs. driving frequency in the plucked and coupled-motion regimes for a fixed acceleration of 0.5 g and for load resistances that match the plucked or coupled-motion frequencies. Plucked harvesting offers higher power outputs than coupled-motion harvesting, and matched load resistance maximizes power output in both regimes. For coupled motion, the power output approaches the mW scale for matched load resistances but is much lower for unmatched load resistances. Matching load resistance to plucked harvesting offers the highest peak powers, but matching load resistance to coupled-motion harvesting offers a wider range of operating conditions in which power output is at the mW scale.

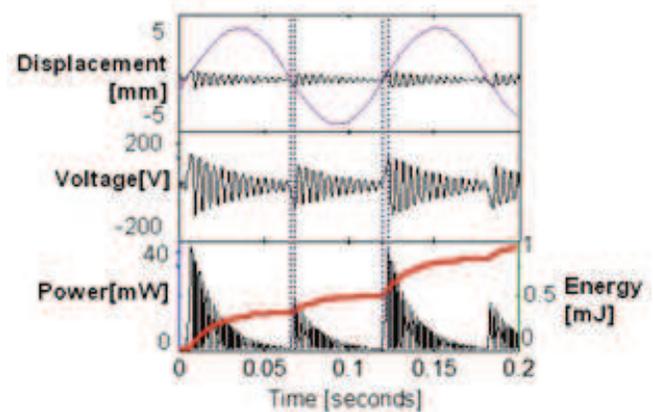


Figure 4. Plots of driving (purple) and generating (black) beam displacements (top), open circuit voltage (middle), and both instantaneous power (black) and converted energy (thick red) (bottom) vs. time for a plucked harvester with matched load resistance. Dotted lines indicate the times at which the beams come in contact and separate.

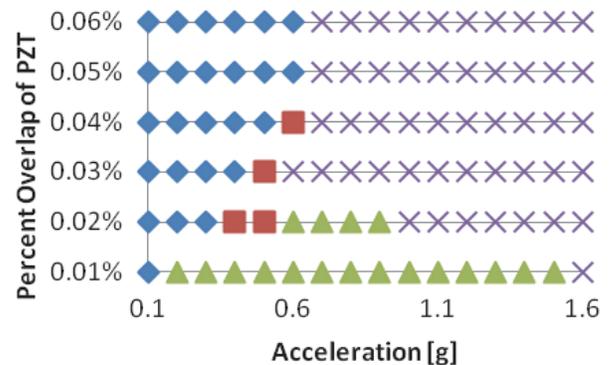


Figure 5. Map of harvesting regime vs. acceleration and ratio of lateral beam overlap to generating beam length. Coupled-motion = blue diamonds, plucking = green triangles, mixed = red squares, and breakage = purple Xs.

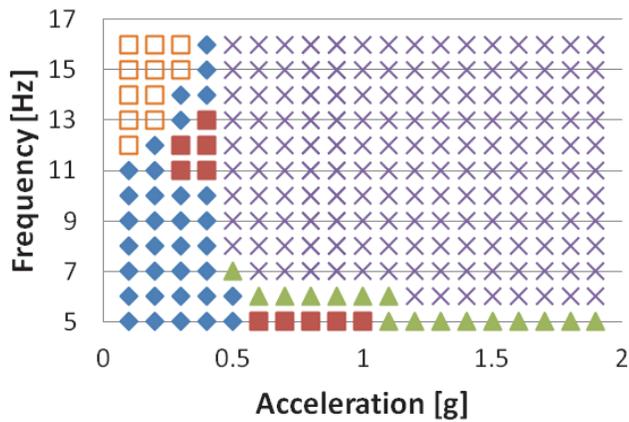


Figure 6. Map of harvesting regime vs. driving frequency and acceleration for a 0.02% beam overlap. Coupled-motion = blue diamonds, plucking = green triangles, mixed = red squares, no impact = orange open squares and breakage = purple Xs.

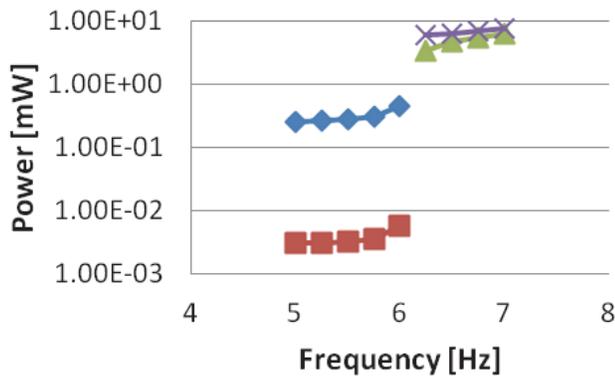


Figure 7. Plot of power and frequency for 0.5 g. Coupled motion with matched resistance = blue diamonds, coupled motion with a plucked resistance = red squares, plucked harvesting with matched resistance = purple Xs, and plucked harvesting with coupled motion resistance = green triangles.

## CONCLUSION

We have presented the first direct modeling comparison between coupled motion and plucked energy harvesting. Coupled-motion harvesting offers a relatively wide operating frequency range but is prone to breakage at larger accelerations. Plucked harvesting is effective over a large range of accelerations, but takes place primarily at lower driving frequencies. Plucked harvesting produces the highest output powers whether it is matched to a plucked or coupled-motion load resistance. Coupled-motion harvesting still approaches the mW scale as long as it is matched to a coupled-motion load resistance. If the goal is to maximize the range of frequency and acceleration over which power is generated, it is advantageous to take advantage of both the plucking and coupled-motion harvesting

behaviors, using a load resistance that is matched to the coupled-motion regime.

The present models considered impact-driven harvesting in the worst case scenario, when the driving beam comes directly into contact with the generating beam. It is expected that the operational limits of this technique may be expanded by adding a low-stiffness tip to the driving beam to mediate contact between the beams.

## REFERENCES

- [1] L. Gu and C. Livermore, "Impact-driven, frequency up-converting coupled vibration energy harvesting device for low frequency operation," *Smart Materials and Structure*, vol. 20, pp. 1-10, 2011.
- [2] S. M. Jung and K. S. Yun, "Energy-harvesting device with mechanical frequency-up conversion mechanism for increase power efficiency and wideband operation," *Appl. Phys. Lett.*, vol. 96, p. 11906, 2010.
- [3] H. Klah and K. Najafi, "Energy Scavenging From Low-Frequency Vibrations by Using Frequency Up-Conversion for Wireless Sensor Applications", *IEEE Sensors*, pp. 261-268, 2008.
- [4] D.-G. Lee, G.-P. Carman, D. Murphy and C. Schulenburg, "Novel micro vibration energy harvesting device using frequency up conversion," *Proc. Int. Conf. on Solid-State Sensors, Actuators and Microsystems (IEEE Transducers 07)*, pp. 871-874, 2007.
- [5] P. Pillatsch, E. M. Yeatman and A. S. Holmes, "Piezoelectric Impulse-Excited Generator for Low Frequency Non-Harmonic Vibrations," *Proc. of PowerMEMS 2011*, 245-248, 2011.
- [6] M. Pozzi and M. Zhu, "Plucked piezoelectric bimorphs for knee-joint energy harvesting: modelling and experimental validation," *Smart Mater. Struct.*, vol. 20, no. 5, 2011.
- [7] Q. C. Tang, Y. L. Yang and L. Xinxin, "Bistable frequency up-conversion piezoelectric energy harvester driven by non-contact magnetic repulsion," *Smart Mater. Struct.*, vol. 20, p. 125011, 2011.
- [8] T. Belndez, C. Neipp and A. Belndez, "Large and small deflections of a cantilever beam," *European Journal of Physics*, vol. 23, no. 3, pp. 371-379, 2002.