PASSIVELY TUNING HARVESTING BEAM LENGTH TO ACHIEVE VERY HIGH HARVESTING BANDWIDTH IN ROTATING APPLICATIONS

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Abstract: This paper presents the design and modeling of a new type of passively self-tuned energy harvester to extract energy from rotational motion. It achieves improved passive tuning by using centrifugal force both to tension the harvesting beam and to shorten the beam's length as rotational frequency increases. Length tuning is accomplished by augmenting the harvesting beam's true fixed support with a second, quasi-fixed support mounted on a radially-oriented spring. The length tuning and axial tensioning increase the harvester's resonant frequency to better match the driving frequency. An analytical model predicts a 3dB full bandwidth of 43Hz about a 28 Hz center frequency, which is much larger than in a comparable harvester without length tuning.

Keywords: passive tuning, energy harvester, length tuning, centrifugal force

INTRODUCTION

Vibration-based resonant energy harvesters extract electrical energy from mechanical motions through transduction mechanisms such as piezoelectric, electrostatic and electromagnetic, and they generate maximum power at their resonant frequencies [1]. Since most practical vibration sources are wide-band or have time-varying characteristic frequencies, much research (for example [2-9]) has focused on developing means to tune a harvester's resonance to match the ambient frequency with minimal required energy input. When successful, such tuning techniques effectively increase the harvester's operational bandwidth.

Many mechanical tuning methods have been reported such as varying the center of gravity of the proof mass [2], changing the cantilever's effective length by attaching a movable rigid support [3] or using a slider connected with a linear actuator [4], and using an axial preload to tune cantilever's stiffness [5, 6]. Passive tuning methods have an advantage over active tuning methods in that their operation does not require energy input and can therefore maximize output power. However, few examples of passive tuning have been proposed because it is challenging to create tuning methods that do not require any active power input.

Energy harvesters have been reported previously for rotational applications, and a few use centrifugal forces to achieve passive tuning. In [7], centrifugal force directly drove generation from piezoelectric actuators. In [8] and [9], centrifugal forces on radially-oriented cantilever beams mounted on a rotating platform passively tuned the beams' resonant frequencies. In [8], axial tensioning by centrifugal force increased the harvester's resonance frequency with increasing driving frequency to achieve an 11 Hz bandwidth about a center frequency of 15 Hz. The question that remains is whether centrifugal forces can be leveraged to provide still greater power bandwidths.

This paper presents a design and models of a new type of passively self-tuned energy harvester that is designed to extract vibrational energy from rotational motion with a greatly increased harvesting bandwidth. As shown in figure 1, the harvester comprises a low-modulus (e.g. plastic), non-piezoelectric cantilever mounted radially at a distance from the axis of the rotation. In addition to the cantilever's fixed support, it is also constrained by a moveable "quasi-fixed support". The quasi-fixed support is provided by a "slider", an element mounted on a radially-oriented spring that can translate radially outward under the influence of centrifugal force. The slider constrains the beam's oscillation, effectively shortening the harvesting beam. The harvesting beam has a tip-mounted proof mass and a PZT element mounted near the slider support.

As the platform rotates in the vertical plane, gravity drives vertical oscillation of the beam. As rotational frequency increases, increasing axial tension in the

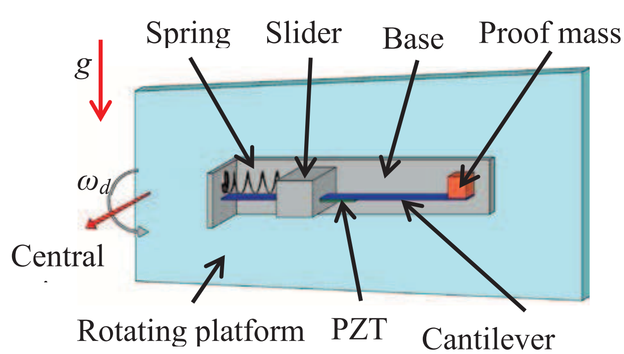

Fig. 1: Schematic diagram of the self-tuned energy harvester with length tuning.

harvester and decreasing harvester length increase the harvester's resonant frequency, ensuring that driving frequency approximately matches resonant frequency over a wide frequency range.

MODELING

Energy harvesters that are tuned solely by centrifugally-driven axial tensioning have been analyzed by applying the equations of motion of a cantilever beam for small deflections [10] and relevant boundary conditions. With appropriate choice of system parameters, this yields a matching relationship between driving frequency and resonant frequency over a relatively wide range of operating frequencies [8].

Table 1: Properties of the harvester

Cantilever Beam	
Young's modulus	2.3 Gpa
Density	1048 kg/m^3
Width	9 mm
Thickness	0.5 mm
Initial effective length	80 mm
Quasi-fixed Support (Slider)	
Weight	5 g
Axial length	20 mm
Initial position of the fix-beam face	65 mm
PZT (X-poled)	
Thickness	0.51 mm
Width	3.2 mm
Length	31.8 mm
Proof Mass at the Tip	2 g

The dependence of the resonant frequency of a cantilever on its length can be used to provide additional frequency tuning beyond what is available with pure centrifugally-driven axial tensioning. The resonant frequency f_r of a cantilever with a tip mass is given as [11]

$$f_r=\frac{1}{2\pi}\sqrt{\frac{Y\omega h^3}{4l^3(m+0.24m_c)}} \qquad (1)$$

where Y is the beam's Young's modulus, w, h and l are its width, thickness and length respectively, m is the tip mass, and m_c is the mass of the beam. The resonant frequency increases monotonically with decreasing beam length; this fact is used here to provide additional passive frequency tuning by varying the effective beam length l. The effective beam length l is related to the movement of the slider and the centrifugal force as

$$f_c=m_s(r_0+x)\omega_d^2 \qquad (2)$$

where r_0 is the initial radial distance of the slider from the center of the rotating platform, x is the displacement of the slider caused by centrifugal force, ω_d is the driving frequency of the platform, and m_s is the mass of the slider. The effective length of the beam is thus

$$l=L \qquad (3)$$

where L is the zero-speed operating length of the beam.

The slider's displacement from its origin is designed to be constrained by a linear spring with a spring characteristic of

$$f_c=kx \qquad (4)$$

or by a nonlinear spring with a spring characteristic of

$$f_c=k_1x+k_3x^3 \qquad (5)$$

where k_1 , k_3 and k are the corresponding stiffness coefficients. For practical implementation, the behavior of a nonlinear spring can be approximated by a linear spring with a motion stop or by a combination of two linear springs that act over different but overlapping ranges of slider displacement.

Note that the resonant frequency and stiffness were calculated approximately using the whole length of the beam (starting from the end of the slider) without considering the added stiffness from the attached PZT near the base. The effect of PZT on the stiffness can be found by modeling a composite structure through moment equilibrium analysis.

These equations were used along with the equations for tuning by axial tensioning to design wide-band, passively tuned rotational energy harvesters. Table 1 describes the geometry and material properties of an optimized macroscale harvester design. The analytical model was implemented numerically to create the optimized design and to predict its performance. The slider displacement was allowed to vary from 0 mm to 9 mm, corresponding to effective beam lengths from the full 80 mm to 71 mm. The optimal linear and nonlinear spring constants were determined by fitting the calculated resonant frequency to the drive

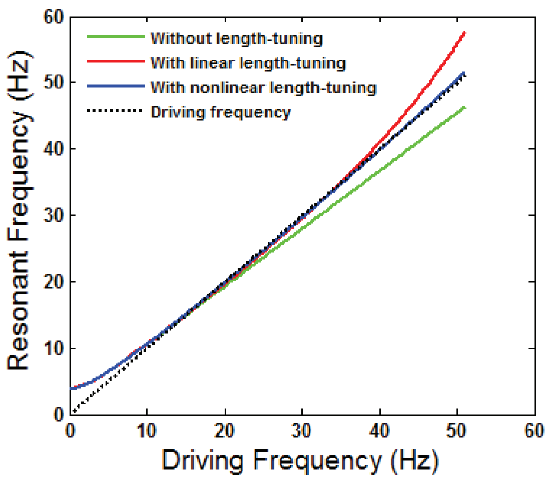

Fig. 2: Plot of predicted resonance frequencies vs. drive frequency for no length tuning, length tuning with a linear spring, and length tuning with a nonlinear spring. All cases include tension tuning.

frequency over a frequency range from 14.5 Hz to 52.5 Hz. Fig. 2 plots the calculated resonant frequencies vs. driving frequency for three cases: with length tuning using an optimized linear spring, with length tuning using an optimized nonlinear spring, and without length tuning. In each case, tuning by axial tensioning was also taken into account. Fig. 3 plots the mismatch frequency (the difference between driving and resonant frequency) for the same three cases.

All three approaches result in matching of the harvester's resonant frequency to the driving frequency over some frequency range. The smallest matched frequency range is obtained by the harvester that tunes only by axial tensioning and not by changes

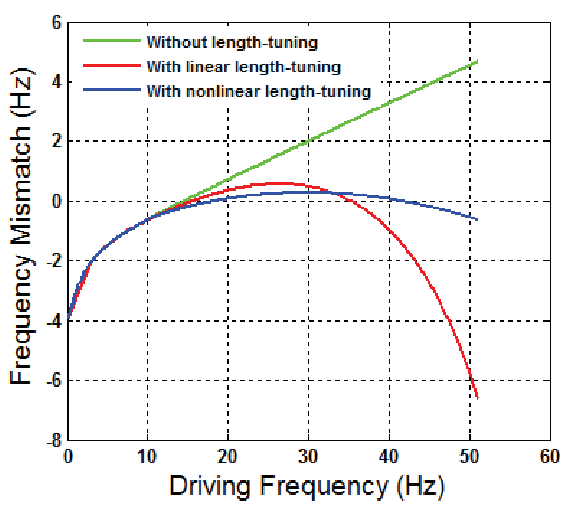

Fig. 3: Plot of mismatch vs. driving frequency for no length tuning, length tuning with a nonlinear spring, and length tuning with a linear spring. All cases include tension tuning.

in beam length (green). Tuning with a linear spring (red) adds about 20 Hz to the driving frequency range in which the driving and resonant frequencies differ by less than 0.5 Hz, except for a window around 26 Hz where it increases slightly to 0.56 Hz of mismatch. Tuning with a nonlinear spring (blue) extends the matched frequency range out to 49 Hz, more than 30 Hz further than is obtained without length tuning.

To estimate the predicted power output of length-tuned energy harvesters, a simple 1-D electromechanical analytical model was applied [12]. The use of a single degree of freedom model is a substantial oversimplification of the bending case, as it assumes that strain is uniform within the harvester rather than having the characteristic spatial variations of a bending beam. It will therefore overestimate the power output. The benefits of using this model are its computational simplicity. Although the results are not exact, they are at the correct order of magnitude and will capture the impact of frequency matching on the power bandwidth. The power per unit of acceleration squared is given as follows [12]:

$$\left|\frac{P_{out}}{(\ddot{\omega}_B)^2}\right| = \frac{m_{eff}(1/\omega_N) r k_e^2 (R_{eq}/R_l)\Omega^2}{[1-(1+2\zeta_m r)\Omega^2]^2 + [(1+k_e^2) r\Omega + 2\zeta_m \Omega - r\Omega^3]^2} \qquad (6)$$

The parameter ζ_m is the damping ratio, which can be derived from measured results [13]. The effective mass m_{eff} approximately equals the proof mass. The parameters $\Omega = \omega/\omega_N$, $r = \omega_N R_{eq} C_p$ and $k_e^2 = \frac{k_{31}^2}{(1-k_{31}^2)}$ are dimensionless parameters defined to simplify the analysis. The equivalent resistance R_{eq} is assumed to be approximately equal to the load resistance R_l as the

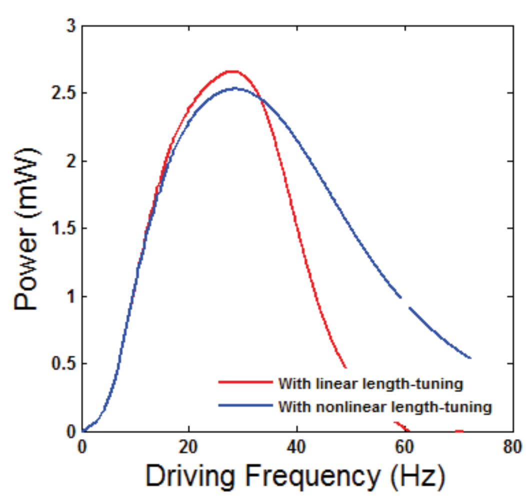

Fig. 4: Plot of the predicted power vs. driving frequency for length tuning with both linear and non-linear springs.

leakage resistance is small compared with load resistance. The power per unit of acceleration squared is converted to power by multiplying it by the square of gravitational acceleration (g^2), because the platform on which the harvester is mounted rotates in a gravitational field.

For model implementation, the piezoelectric coupling coefficient k_{31} is taken to be equal to 0.35 [14]. The capacitance C_p of the PZT $C_p$is determined by the active dimensions of the capacitor, which changes as the slider varies the effective length of the active region of the PZT. The load resistance was chosen as 1838 kΩ to be well matched for a specific capacitance at a frequency of 30 Hz.

The predicted power output of the self-tuning energy harvester with both linear length tuning and nonlinear length tuning were calculated and plotted in Fig. 4. For length tuning with a linear spring, the power peaks at 28 Hz, giving a 2.6 mW output and a 30 Hz bandwidth. For length tuning with a nonlinear spring, the power peaks at 28 Hz with a similar maximum power output but has a 43 Hz bandwidth. The calculated power output values are overestimates as described above and must be treated accordingly. Nonetheless, this new type of passive self-tuning architecture offers effective power conversion over nearly 80% of the frequency range from 0 to 55 Hz. For this harvester with a quality factor of about 10, the frequencies at which power drops to half of its maximum value correlate well with the points at which the difference between driving and resonant frequency exceeds about 0.5Hz, as presented in Fig. 3.

CONCLUSION

This paper presents a design and models of a new type of passively self-tuned energy harvester to extract vibrational energy from rotational motion. Analytical models predict that passive length tuning in rotational systems will offer dramatically improved passive frequency tuning as compared with harvesters that are only tuned by centrifugal axial tensioning. As compared with an 11 Hz bandwidth about a 15 Hz center frequency obtained experimentally for the purely axially tuned case, the present harvester design offers a 3dB full bandwidth of up to 43 Hz about a 28 Hz center frequency, corresponding to a full normalized 3dB power bandwidth of greater than 1.5. In the future, experiments will be necessary to confirm the extremely large normalized frequency bandwidth enabled by this passive tuning technique.

REFERENCES

[1] Williams C. B. and Yates R. B. 1996 Analysis of a micro-electric generator for microsystems *Sensors and Actuators* **A 52** 8-11

[2] Wu X., Lin J., Kato S., Zhang K., Ren T., Liu L. 2008 A frequency adjustable vibration energy harvester. *Proceedings of PowerMEMS 2008 (Sendai, Japan)* 245–248

[3] Huang S-C, Lin K-A 2012 A novel design of a map-tuning piezoelectric vibration energy harvester *Smart Mater. Struct.* **21** 085014

[4] Gieras J. F., Oh J-H, Huzmezan M., Sane H. S. 2007 Electromechanical energy harvesting system WO2007070022 (A2), WO2007070022 (A3)

[5] Leland E. S. and Wright P. K. 2006 Resonance tuning of piezoelectric vibration energy scavenging generator using compressive axial preload *Smart Mater. Struct.* **15** 1413-1420

[6] Eichhorn C., Goldschmidtboeing F., Woias P. 2009 Bidirectional frequency tuning of a piezoelectric energy converter based on a cantilever beam *J. Micromech. Microeng.* **19** 094006

[7] Manla G., White N. M., Tudor M. J. 2012 Numerical model of a non-contact piezoelectric energy harvester for rotating objects *Sensors Journal* **12**(6) 1785-1793

[8] Gu L., Livermore C. 2010 Passive self-tuning energy harvester for extracting energy from rotational motion *Appl. Phys. Lett.* **97** 081904-1

[9] Gu L., Livermore C. 2012 Compact passively self-tuning energy harvesting for rotating applications *Smart Materials and Structures* **21**, 015002 (9pp)

[10] Armand S. C., Lin P. P. 1993 Influence of mass moment of inertia on the normal modes of preloaded solar array mast *Finite Elements in Analysis and Design* **14** 313-324.

[11] Blevins R. D. 2001 *Formulas for natural frequency and mode shape* (New York, Krieger Publishing)

[12] duToit N., Wardle B., Kim S-G 2005 Design considerations for MEMS-scale piezoelectric mechanical vibration energy harvesters *Integr. Ferroelectr.* **71** 121–60

[13] Gu L., Livermore C. 2011 Impact-driven, frequency up-converting coupled vibration energy harvesting device for low frequency operation *Smart Mater. Struct.* **20** 045004 (10pp)

[14] Low T. S., Guo W. 1995 Modeling of a three-layer piezoelectric bimorph beam with hysteresis *J. MEMS* **4**(4) 230-237