

# PIEZOCAP: A HIGH POWER DENSITY VIBRATION ENERGY HARVESTER

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**Abstract:** In this study, modeling and experimental verification of a novel concept for vibration energy harvesting is presented. In this design, piezoelectric microfiber composites (MFCs) with magnets attached on top and bottom form a chamber where another magnet is levitated. We refer to this configuration as PiezoCap which allows harvesting energy at frequencies much lower than the natural frequencies of the MFCs. The natural frequency of the PiezoCap device was found to be 245Hz. It was also found that significant strains can be induced in the MFCs leading to high power output. Scaling analysis showed that reduction in the distance between the MFCs increases the power output. The numerical model was shown to agree with experimental results.

**Keywords:** vibration energy harvesting, MEMS, piezoelectric, magnetic levitation.

## INTRODUCTION

Energy harvesting via micro-electro-mechanical systems (MEMS) is a developing technology which currently depends on energy sources such as vibrations, heat and light. For energy harvesting from vibrations, piezoelectric transduction is currently the preferred energy harvesting technique at the MEMS scale [1]. Contrary to other forms of vibration energy harvesting at the MEMS scale, power generation via piezoelectric transduction is highly dependent on surface area [2]. Therefore, an improvement in power performance can be obtained by development of device configuration which optimizes the piezo surface area available for energy harvesting.

(attached to MFCs) were aligned to create repulsive force on the levitating magnet. As the device is excited through external vibrations (vertically), the levitating magnet vibrates and repels the attached magnets on the MFCs, thus inducing strain in the MFCs. When clamped on all four sides, the MFCs exhibit natural frequencies in the kHz range. However, the PiezoCap device enables the development of significant power output at low frequencies, thus addressing the problem of off-resonance energy harvesting.

## PIEZOCAP MODEL

The PiezoCap model incorporates a nonlinear magnetic system and the resultant harmonic displacements of the MFCs. The vibration of the levitating magnet is based on the mass-spring-damper system [3], and is governed by a summation of forces as:

$$m\ddot{z}(t) + c\dot{z}(t) + mg + F_{MAG} + F_D = 0 \quad (1)$$

where  $m$  is the mass of the levitating magnet,  $c$  is an empirically derived mechanical damping constant,  $z$  is the vertical displacement of the levitating magnet,  $t$  is the time, and  $g$  is the acceleration due to gravity.  $F_{MAG}$  is the nonlinear magnetic repulsion force and is defined as:

$$F_{MAG} = kz(t) + k_3z(t)^3 \quad (2)$$

where  $k$  and  $k_3$  are spring constants with units of N/m and N/m<sup>3</sup> respectively.  $F_D$  is the driving force from base excitation of the harvester and is defined by a cosine waveform as:

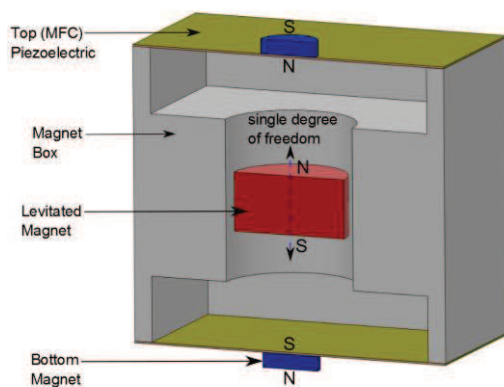


Figure 1: Section view of the PiezoCap model

In this paper, we present a novel concept (PiezoCap) for piezoelectric energy harvesting that relies on magnetic field induced strain in piezoelectric microfiber composites (MFCs). The PiezoCap device consists of one magnetic layer on the top and bottom of the MFCs with a levitating magnet in the middle of the device (Figure 1). The top and bottom magnets

$$F_D = F_0 \cos \omega t \quad (3)$$

where  $F_0$  is the amplitude of the driving force and  $\omega$  is the driving frequency. Overall, the governing equation for the nonlinear magnetic system is a modified form of the Duffing equation [4].

$F_{MAG}$  was obtained via finite element modeling (FEM) using ANSYS solid 236 element [5]. The set of  $F_{MAG}$  versus  $z$  values obtained from FEM were curve-fitted to obtain the spring constants  $k$  and  $k_3$ . Equations 1-3 were solved numerically and the damped natural frequency ( $\omega_d$ ) of the magnetic vibrating system was obtained as the value of the driving frequency ( $\omega$ ) which corresponds to the peak value of velocity ( $\dot{z}$ ).

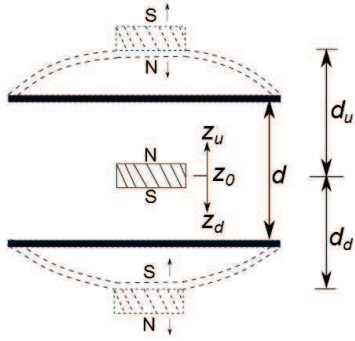


Figure 2: Schematic description of PiezoCap.

A schematic diagram of the PiezoCap device is shown in Figure 2. In the absence of the attached magnets, the distance between the MFCs is  $d$ . The distance between the levitated magnet and the top magnet is  $d_u$  and the distance between the levitated magnet and the bottom magnet is  $d_d$ . As the levitated magnet vibrates, it is displaced by a distance  $z_u$  upwards and downwards  $z_d$  from its rest position  $z_0$ . The top and bottom MFCs are displaced harmonically during vibration due to the repulsion of the attached magnets towards the levitating magnet. Therefore the PiezoCap device is modeled iteratively since the distances between the magnets ( $d_u$  and  $d_d$ ) are initially unknown.

To find the output power of a PiezoCap device, (1)  $F_{MAG}$  is obtained based on  $d$  and equations 1-3 are solved. (2) Next, the  $F_{MAG}$  values which correspond to the maximum displacements (up and down) of the levitating magnet are used to perform FEM harmonic analyses on the MFCs using ANSYS solid 186 element. (3) The center displacements of the MFCs are obtained and steps 1 and 2 are repeated until the solution converges.

The values which define the PiezoCap device in this preliminary study are given in Table 1. The levitated and attached magnets are of the same dimensions and

are made of NdFeB material. The MFCs (small and big) have the same MFC and piezo thicknesses thickness ( $H$  and  $H_p$ ). The small MFC is in  $d_{31}$  mode while the big MFC is in  $d_{33}$  mode.

Table 1: Important values for the PiezoCap device

Magnet diameter	3/8 inch
Magnet thickness	1/32 inch
Magnet material	NdFeB
Magnet mass ( $m$ )	0.424g
Small MFC ( $L_b \times B_b$ )	27mm x 13mm
Big MFC ( $L_s \times B_s$ )	28mm x 16mm
$d_{33}$ constant (big MFC)	$4 \times 10^{-10}$ C/N
$d_{31}$ constant (small MFC)	$1.7 \times 10^{-10}$ C/N
$g_{33}$ constant (big MFC)	$22.2 \times 10^{-3}$ Vm/N
$g_{31}$ constant (small MFC)	$10.9 \times 10^{-3}$ Vm/N
Total MFC thickness ( $H$ )	305 $\mu$ m
Piezo thickness ( $H_p$ )	185 $\mu$ m
MFC piezo material	PZT 5A1
MFC elastic modulus	30.336GPa
MFC density	5.55g/cm <sup>3</sup>
MFC Poisson's ratio	0.31
Distance between MFCs in the absence of magnets ( $d$ )	3.2mm

## RESULTS AND DISCUSSION

The magnetic force ( $F_{MAG}$ ) profile obtained from FEM (Figure 3) is nonlinear as expected and the spring constants were obtained as  $k = 1007$  N/m and  $k_3 = 4.374 \times 10^8$  N/m<sup>3</sup>. The  $F_{MAG}$  profile is an important determinant of the natural frequency: As the device becomes less nonlinear (i.e as  $k/k_3$  increases), the frequency increases.

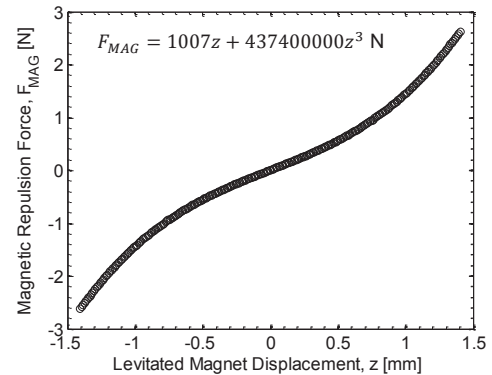


Figure 3: The magnetic repulsion force on the levitating magnet as a function of its displacement.

The velocity and maximum displacements of the levitating magnet as a function of frequency are shown in Figure 4. The dynamics of the levitating magnet was modeled for two cases:

- A. With small MFC on top and big MFC at the bottom; and
- B. With big MFC on top and small MFC at the bottom.

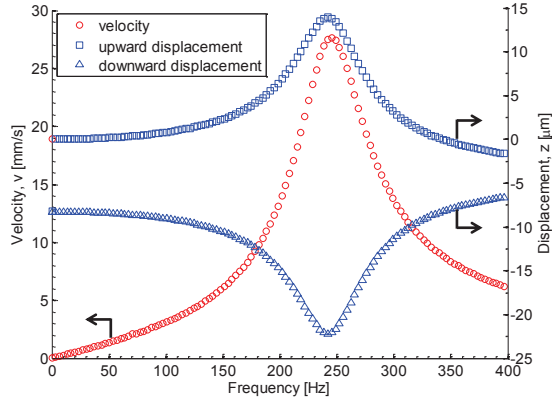


Figure 4: Dynamics of the levitating magnet as a function of frequency (values of velocity ( $v$ ) and maximum values of the upward ( $z_u$ ) and downward ( $z_d$ ) displacements).

In both cases, the results are similar except with respect to the displacement of the MFCs. This is mainly because the natural frequencies of the MFCs are in the kHz range while the magnetic system has a natural frequency less than 300Hz. Since the base excitation frequency is varied below 1000Hz, the governing frequency of the PiezoCap device is that of the magnetic system.

The maxima (or minima) values of velocity and displacement correspond to the resonance of the system. Therefore, the natural frequency of the PiezoCap system was 245 Hz. The maximum velocity was 27.62mm/s, while the upward ( $z_u$ ) and downward ( $z_d$ ) displacements (relative to the center of the PiezoCap device) were 13.92 $\mu$ m and -22.17 $\mu$ m respectively. This shows the actual rest position of the levitating magnet ( $z_0 = -4.13\mu$ m) was influenced by gravity.

Table 2: Maximum displacements of the MFCs at resonance

	Max. upward displacement ( $\mu$ m)	Max. downward displacement ( $\mu$ m)	Total piezo displacement at center ( $\mu$ m)
Small piezo on top	68.96	397.70	466.66
Big piezo on top	297.95	54.07	352.02

The natural frequency and displacement values indicate that the device in this study is quite stiff (large values of  $k$  and  $k_3$ ). However, despite the low displacement values of the levitating magnet, the displacements induced in the MFCs (at resonance) were quite significant as shown in Table 2.

Due to the weight of the levitating magnet, the PiezoCap device favors the small-MFC-on-top configuration as indicated by the 32.6% increase in total piezo displacement above the case with big MFC on top. The piezo displacement results are particularly crucial for power output calculations since power is dependent on the strain rate.

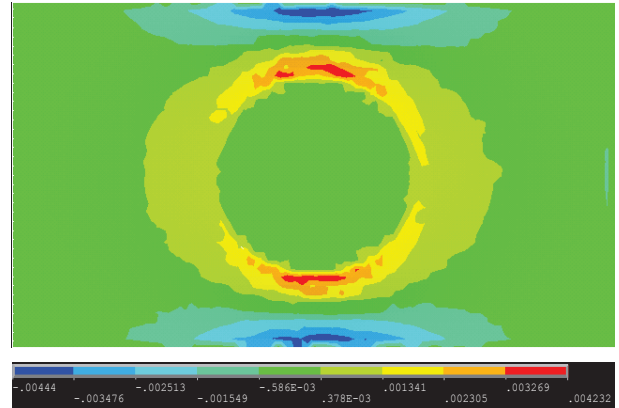


Figure 5: Resonance strain profile of the big MFC in the small-MFC-on-top configuration.

The maximum power output will be obtained by placing the big MFC at the bottom of the PiezoCap device. As seen in Figure 5, the strain profile of the MFC indicates that maximum strain occurs around the edge of the attached magnet placed under the MFC. Otherwise, the strain is uniform for most of the MFC.

The average stresses on the MFCs in the small-MFC-on-top configuration were obtained from ANSYS FEM harmonic analysis as  $\sigma_{big} = 1.7 \times 10^5$  N/m<sup>2</sup> and  $\sigma_{small} = 1.5 \times 10^5$  N/m<sup>2</sup>. Therefore maximum power estimates for the device can be calculated for the MFCs at resonance as [2]

$$P_{big} = \sigma_{big}^2 g_{33} d_{33} L_b B_b H_p f_r = 5.39 \mu W, \text{ and} \quad (4)$$

$$P_{small} = \sigma_{small}^2 g_{31} d_{31} L_s B_s H_p f_r = 0.71 \mu W \quad (5)$$

where  $f_r$  is the resonance frequency. Overall, the maximum power density of the PiezoCap device (MFC volume only) is estimated at resonance as

$$\text{Power density} = \frac{P_{big} + P_{small}}{(L_s B_s + L_b B_b) H} = 25 \mu W / \text{cm}^3 \quad (6)$$

## MEMS SCALING ANALYSIS

Scaling analysis was carried out to show the effects of reducing the spacing between MFCs ( $d$ ) as depicted in Figure 6. Reduction in  $d$  reduces nonlinearity ( $k/k_3$  increases) and leads to an increase in resonance frequency. The pressure on the bottom MFC also increases with reduced  $d$ , leading to more strain and energy harvesting potential at the MEMS scale. However, the decrease in nonlinearity implies that peak performance may occur close to resonance only as the device bandwidth is reduced.

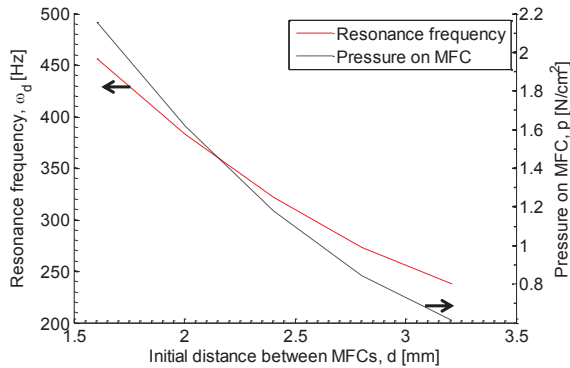


Figure 6: Scaling analysis of the PiezoCap device: The effect of initial distance between the levitating magnet and the MFC ( $d$ ) on the resonance frequency, and the maximum force impacting the bottom MFC.

## EXPERIMENTAL VERIFICATION

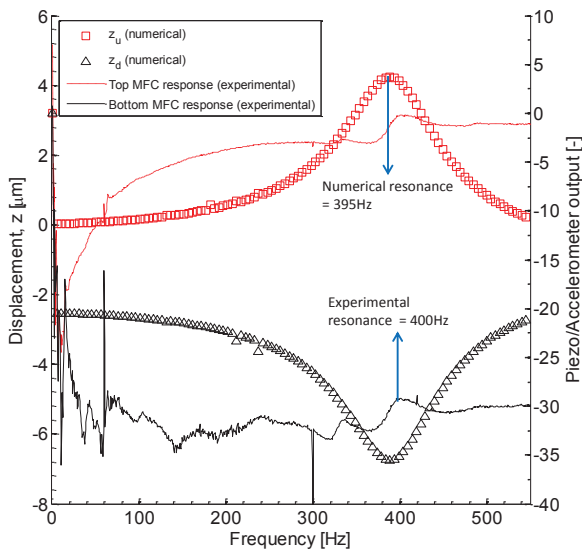


Figure 7: Results for PiezoCap setup with  $d = 2$ mm. Magnet and MFC dimensions remain the same as in Table 1.

The numerical model for the PiezoCap device was experimentally verified by vibrating the device with an accelerometer attached at its base and recording the response of the top and bottom MFCs. The

experimental setup had an initial MFC separation distance  $d = 2$ mm. The results were compared to the numerical displacement calculations with the same value of  $d$  as shown in Figure 7.

From these results, the error in predicting the natural frequency was 1.3%. The under-prediction of the frequency may be due to the measured value of  $d$ , since any slight variation in the value of  $d$  (especially at the MEMS scale) will have a significant effect on the natural frequency (see Figure 6).

## CONCLUSIONS

Modeling of the novel PiezoCap device was conducted. The model involves iterative analysis of the magnetic levitation system and the resulting piezo response. The natural frequency of the device was 245Hz. Despite low displacement values of the levitating magnet, the MFCs exhibited significant strain, and thus significant power output potential. The small-MFC-on-top configuration produced the best displacement results due to the effect of gravity on the levitated magnet's rest position. MEMS scaling analysis showed that a reduction in the distance between the MFCs increases the power output, but increases the frequency and reduces nonlinearity. The numerical modeling error was shown to be 1.3% when compared with experimental results.

## ACKNOWLEDGMENTS

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