OPTIMIZATION OF RESONANT MECHANICAL HARVESTERS IN PIEZO-POLYMER-COMPOSITE TECHNOLOGY

F. Goldschmidtböing, B. Müller, P. Woias
Laboratory for Design of Microsystems, IMTEK, University Freiburg, Germany

Abstract: We present our recent developments in the field of piezoelectric resonant beam energy harvesters. The piezo-polymer-composite technology is used to generate piezoelectric bimorph structures with various shapes. A dedicated theory for the modeling of these structures is reviewed and different approaches for the shape optimization of the generators are evaluated with this theory. Experimental data support the theoretical findings.

Key Words: energy harvesting, piezo-polymer-technology, shape optimization

1. INTRODUCTION

The recently growing application of distributed sensor systems and the need of battery-less power supplies calls for the development of micro energy harvesting devices [1]. Generally these devices should be low-prized and highly efficient to fit into a broad area of applications. The piezo-polymer-composite technology (PPC) developed in our laboratory allows the development of piezoelectric beam harvesters with a great variety of shapes enabling low-prized structures with optimized and therefore efficient beam shapes [2]. As the PPC is generally transferable to injection molding processes future mass products will profit from this development. After a short introduction to the elements of the PPC, the theoretical modelling of the devices will be reviewed based on the theoretical frame work by Hagood [3]. Different types of energy harvesters will be presented and discussed in terms of efficiency based on the theoretical modelling and experimental data.

2. FABRICATION

The PPC is basically an insert molding process as shown in figure 1. First silicone mold segments are replicated from metal masters of a desired micro device. PZT disks of 100µm to 200µm thickness are placed into the silicon mold which is subsequently filled with resin. After a thermal curing step the final device can easily be released from the soft mold. Besides the piezo disk additional elements may be inserted, i.e. wires for electrical contacts, fibers or steel wires as hinges or even electronic circuits. The main advantage of this technology is a reduction in packaging effort as both device fabrication and packaging are combined in one molding step. Many different devices, for example scanning mirrors [4] and free jet dispensers [5] have been fabricated with this technology.

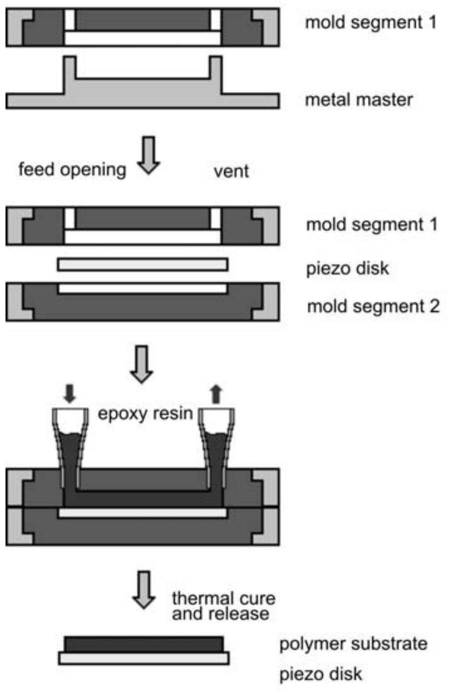

Fig. 1: Process chart of the PPC

3. THEORY

The theory is briefly reviewed in this chapter. The reader is referred to the original work [3] for a more detailed derivation. The type of device considered here is sketched in figure 2. An appropriate theory of piezoelectric vibration-to-electricity converters must include a two-way coupling of the elastic and electric domains, provided by the piezoelectric equations (1) and(2).

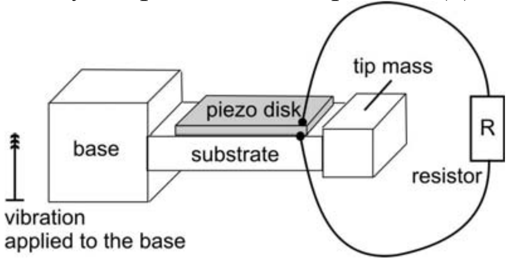

Fig. 2: Generic piezoelectric vibration to electricity converter

$$\overline{D} = \overline{\overline{\varepsilon^{S}}} \cdot \overline{E} + \overline{\overline{e}} \cdot \overline{S} \quad (1)$$

$$\overline{T} = -\overline{\overline{e}}_{tr} \cdot \overline{E} + \overline{\overline{C^{E}}} \cdot \overline{S} \quad (2)$$

Here the one upper bar indicates a three or six component vector and two upper bars denote a matrix of appropriate dimensions. The electric displacement is written as D and the electric field as E. S symbolizes the six components of the elastic strain and T is the stress vector. The material parameter are given in a non-standard way. ε^{S} is absolute dielectric matrix at constant strain and e is the tensor of electric displacement per strain. C^{E} symbolizes the elastic compliance at constant electric field. These matrices can be calculated from the standard d- matrix and the free dielectric matrix by simple algebra [3].

The material equations for passive materials can be obtained by setting all components of e to zero.

With these equations the action integral S can be deduced. It consists of kinetic, elastic, electric and work (vibration) terms.

$$S = \int_{t_1}^{t_2} W_{kin} - W_{elas} + W_{elek} + W_{vib} \, dt \quad (3)$$

This integral is minimized by the Ritz-Rayleigh method. The vector of elastic displacement u and electric potential φ are expressed as superposition of n elastic and m electric ansatz functions Ψ and ϑ.

$$\overline{u}(\overline{x},t) = \sum_{i=1}^{n} r_i(t) \cdot \overline{\Psi}(\overline{x}) \quad (4)$$

$$\varphi(\overline{x},t) = \sum_{j=1}^{m} v_j(t) \cdot \vartheta(\overline{x}) \quad (5)$$

With these functions the minimization of (3) yields a set of linear equations for the coefficients r and v.

$$\overline{\overline{M}} \cdot \ddot{\overline{r}} + \overline{\overline{K}} \cdot \overline{r} - \overline{\overline{\Theta}} \cdot \overline{v} = \overline{\overline{B}}_f \cdot \overline{f} \quad (6)$$

$$\overline{\overline{\Theta}}_{tr} \cdot \overline{r} + \overline{\overline{C}}_p \cdot \overline{v} = \overline{\overline{B}}_q \cdot \overline{q} \quad (7)$$

The coupling matrices M, K, Θ, B_f, B_q and C_p depend on the material parameters, the geometry and the set of ansatz functions. The external forces (vibration) are combined in the vector f, while q denotes the charge on the electrodes of the piezo material. Equation (6) is completed by an empirical damping term. For the sake of simplicity a damping term proportional to the mass matrix M is chosen, more dedicated damping models may be applied with the drawback of a higher number of empirical coefficients.

$$\overline{\overline{M}} \cdot \ddot{\overline{r}} + \alpha \cdot \overline{\overline{M}} \cdot \dot{\overline{r}} + \overline{\overline{K}} \cdot \overline{r} - \overline{\overline{\Theta}} \cdot \overline{v} = \overline{\overline{B}}_f \cdot \overline{f} \quad (8)$$

The set of equations (7) and (8) has to be completed by equations that link the charges q to the voltages on the electrodes. In our case a resistor is switched between the electrodes to consume the harvested energy, i.e. the missing equation is a simple linear relation between charge and the time derivative of the voltage U, that can be determined from the electric ansatz functions.

$$q = -\frac{1}{R} \cdot \frac{\partial U}{\partial t} \quad (9)$$

With this procedure the problem may be reduced to a set of linear equations that can numerically be solved within seconds, provided that an appropriate set of ansatz functions is found. Two premises have to be fulfilled by these functions. First they have to fit into all boundary conditions and second a superposition of these functions must match the expected real solutions. While the first premise is easy to fulfil, the second premise asks for some experience of the designer.

This theory has already been successfully applied to rectangular piezoelectric beam harvesters by other groups e.g. [6].

4. DEVICES

When optimizing piezoelectric beam harvesters, the main task is to adapt the system's resonance frequency to the frequency of the excitation, because a beam harvester may be orders of magnitude more efficient if it is driven in resonance. This frequency adaption may be performed by adding a tip mass or by adjusting the beam length. Another optimization criterion is the curvature of the beam. As the locally generated charge density is directly proportional to the curvature κ of the beam, this quantity should be maximized. On the other hand a certain maximum curvature κ_{max} corresponding to a maximum stress safely below the fracture stress of the piezoelectric material must not be exceeded. In consequence a homogeneous curvature of κ_{max} is desired. We introduce the ratio of mean curvature to maximum curvature $\kappa_{mean} / \kappa_{max}$ ranging from zero to one as a figure of merit for piezoelectric beam harvesters. Typical rectangular cantilever beams exhibit a decreasing curvature from base to tip along the beam axis. Therefore an optimization of the beam shape or the suspension is necessary. Two approaches are presented in the following subchapters.

4.1 Triangular beam harvester

Static considerations lead to triangle shape for curvature homogenisation. A triangle shaped cantilever that is loaded at the tip has linearly falling bending moment. As the bending stiffness also decreases linearly, the curvature (ratio of moment to stiffness) is constant. Since a beam with relatively high tip mass may be treated in a quasistatic fashion, i.e the inertia of the beam does not contribute to the movement, a homogeneous curvature ($\kappa_{mean}/\kappa_{max}=1$) can be expected for high tip masses. The same argument results in a curvature ratio of 0.5 for rectangular beams.

We have fabricated and characterized a triangular shaped beam with different additional tip masses. A photograph and a sketch with dimensions are shown in figures 3 and 4. The triangle had to be truncated to attach the additional tip mass.

Fig. 3: Photograph of the triangle beam harvester

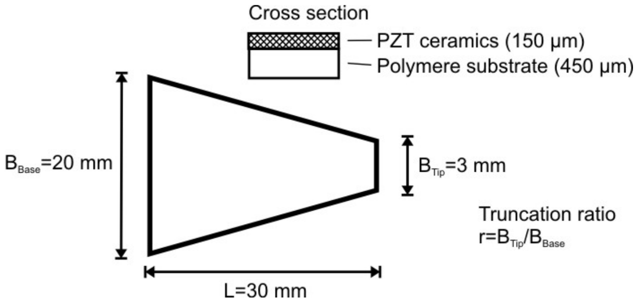

Fig. 4: Sketch and dimensions of the triangle beam harvester

We have measured the electrical power across a 18 kΩ resistor for the additional tip masses of 0, 20g and 52g at a 2.5 g sinusoidal excitation. Figure 5 shows the measurement results in combination with simulated results according to the theory of chapter 3. Only the damping parameter α was used as fit parameter with a value of $\alpha = 80\ s^{-1}$. Details on the simulations (ansatz functions and resulting mode shapes) will be explained in a following journal publication.

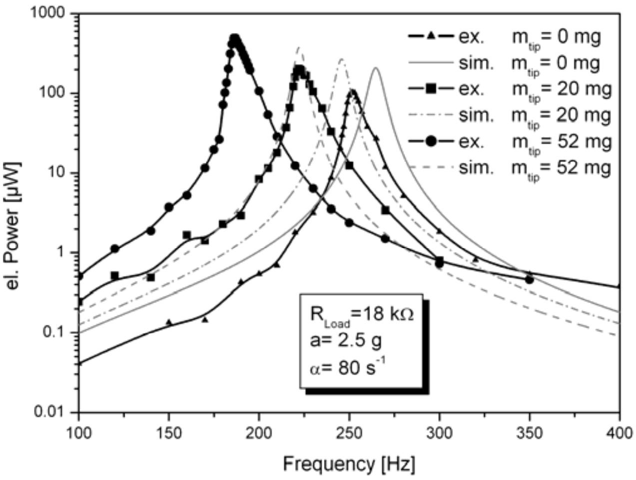

Fig. 5: First resonance of the triangle beam harvester (simulation and experiment)

As can be seen in figure 6 the resonance frequencies are slightly shifted and the peak height shows a significant deviation between theory and experiment. Nevertheless both theory and experiment show the same trends and orders of magnitude. The resonance frequency decreases and the power output increases with the additional tip mass. As the main discrepancy can be seen in the peak height it may be concluded that the empirical damping model is the major source of inaccuracy.

We have theoretically analyzed the effect of truncation and tip mass on the curvature ratio with our simulation model. The results are summarized in figure 6. The curves show that the curvature ratios of 1 and 0.5 for triangular respectively rectangular beams with high tip masses hold. The truncation of our triangle has only a small negative effect of approximately 10% compared to a fully triangular beam. The results show a potential for further optimization at small tip masses. At zero tip mass the triangle is about 30% less efficient compared to the (unknown) optimum shape.

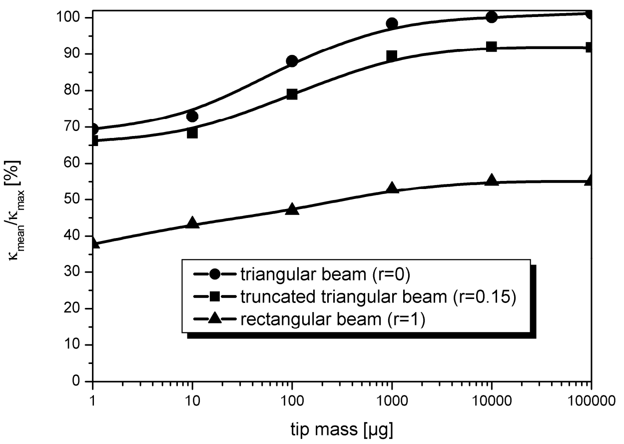

Fig. 6: Theoretical curvature ratio versus tip mass and truncation ratio

4.2 Swivel joint harvester

Another approach to homogenize the curvature is the use of swivel joints at base and tip of the cantilever, as shown in figure 7.

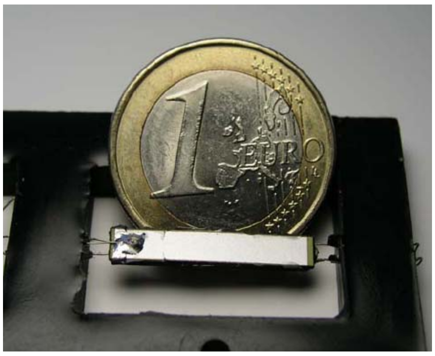

Fig. 7: Photograph of the swivel joint harvester (beam dimensions: 5 mm x 27 mm)

Here the swivel joints are approximated by steel wires with a lower bending stiffness compared to the beam. The nominal curvature ratio is $\kappa_{mean} / \kappa_{max} = 67\%$ which is significantly better compared to the value 37% of a rectangular cantilever beam (see figure 6). The simulated measured first resonance peak is shown in figure 8. The measured resonance frequency is higher compared to the simulated one. This effect is presumably caused by the finite hinge moment at the wire hinge that is neglected in the analytical theory.

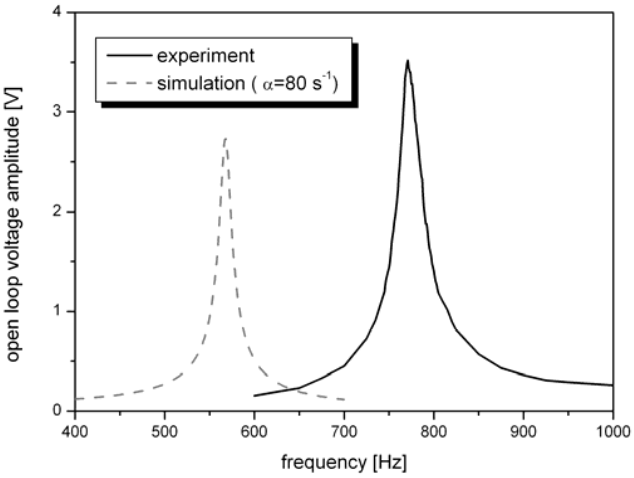

Fig. 8: Measured and simulated first resonance peak of the swivel joint harvester

5. CONCLUSION

A proven fabrication and simulation procedure for PPC based harvesters is demonstrated. We have shown two different approaches to optimize the performance of piezoelectric beam harvesters. Further work will focus on a more realistic modelling of damping effects and the transfer of the PPC to injection molding processes.

REFERENCES

[1] P. Woias, 3.GMM-Workshop, Kassel, Germany, 2004.

[2] C. Friese, et al., Transducers ´03, Digest of Technical Papers, vol. 2 (2003), 1007-1010

[3] N. W. Hagood, et al., J. Intell. Mater. Syst. Struct. 1, 327–354 (1990)

[4] E. Just et al, IEEE Transducers '05, Seoul, Korea, June 5-9, 2005, Digest of Technical Papers, Vol. 1, 2005, 753-756

[5] E. Just, et al., Proc. Actuator 2006, Bremen, Germany, June 14-16, 2006, 285-288

[6] N. G. Elvin et al., Smart Mater. Struct. 10 (2001) 293–299