COUPLED FDM AND FEM SIMULATION OF HIGH-SPEED FOIL BEARINGS FOR MICRO GAS TURBINES

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Abstract: Foil bearings are a very promising bearing technology for micro gas turbines. Although a lot of papers about foil bearing modelling exist, basic aspects are not yet fully clarified. A good example is the influence of the mechanical structure and more specifically of the top foil on the bearing characteristics. Moreover, not much attention is paid to the simulation of the no-contact zone in a foil bearing. In this paper we propose a more advanced method to handle the no-contact zone. Based on a small parameter study we show that the top-foil stiffness is not negligible. Furthermore we propose a new calculation scheme where FEM for the mechanical structure is combined with FDM for the air film calculations. Also the influence of a top-foil with finite dimensions on the steady-state properties is discussed.

Key Words: foil bearings, JFO boundary conditions, FEM-FDM coupling

1. INTRODUCTION
Mesoscopic gas turbines are subject to intensive research as they are a promising alternative for batteries as a portable power source. However, the combined high speed, on the order of 500,000 rpm and high temperature, on the order of 1000K, requirements lead to bearing designs operating in unexplored areas. Air bearings and more in particular deformable air bearings like foil bearings are seen as possible bearing candidates. Although many papers are published about foil bearing modelling, relatively basic mechanical models are used. Also the non-obvious behaviour of the top-foil loosening contact with its supporting flexible element is treated without careful attention. In this paper we focus on both of those shortcomings in the literature. First, we focus on an improved and more robust method to simulate the no-contact region between the top-foil and the supporting elements. In section 4 we discuss, based on a parametric study with a representative load, the need for an improved mechanical model. In order to implement this improved mechanical model we propose in section 5 a new solution strategy based on an air bearing model, solved with finite differences, and a separate mechanical model solved with finite elements. To conclude, the influence of the top foil thickness on the load capacity is discussed.

2. FOIL BEARING MODEL
Figure 1 shows a schematic of a journal foil bearing in which the flexible elements are modelled as simple springs. In practice the flexible elements can range from corrugated bump foil to silicone rubber. In order to provide a smooth continuous surface for the air bearing gap, a top foil is mounted between the air gap and the spring elements.

![Figure 1 Foil bearing schematic](image)

The modelling of the deformable bearing can be done by adding a pressure dependent gap height-term \( \alpha(P-1) \) in the isothermal compressible Reynolds equation as first proposed by Heshmat [1], resulting in the following equation:
\[
\frac{\partial}{\partial x} \left( p \left( H + \alpha(p-1) \right) \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left( p \left( H + \alpha(p-1) \right) \frac{\partial p}{\partial y} \right) = 0
\]

\[\lambda \frac{\partial}{\partial x} \left( p \left( H + \alpha(p-1) \right) \right) \]

with

\[\lambda = \frac{6\alpha}{p_s} \left( \frac{R}{C} \right) \quad \sigma = \frac{12\mu p_s}{p_s} \left( \frac{R}{C} \right) \quad \gamma = \frac{\rho \omega}{\nu} \]

\[X = \frac{x}{C} \quad Y = \frac{y}{C} \quad P = \frac{P}{p_s} \quad T = \omega t \quad \alpha = \frac{p_s}{k^2} \quad (2)\]

The left hand side of equation (1) is the diffusion or Poiseuille term and the right hand side the convection or Couette term of the Reynolds equation. Often this equation is solved by using the finite difference method (FDM). Due to the non-linear character, caused by the compressibility, it is solved iteratively. Important to mention in equation (1) is the dimensionless flexibility parameter \( \alpha \) which contains the surface stiffness \( k \).

The top-foil cannot withstand sub-ambient pressures as it is not fixed to the flexible element. As a result of this, the minimum pressure in a foil bearing is the atmospheric pressure. A comparable behaviour is also noticed in liquid film bearings where the pressure cannot be lower than the cavitation pressure. In liquid film bearings this leads to the Reynolds or more generally the JFO (Jakobson-Floberg-Olsson) boundary conditions. For foil bearings, the same boundary conditions can be used which are applicable at the angle \( \theta_0 \):

\[ P = 1 \quad \frac{\partial p}{\partial x} = 0 \text{ at } X = R \theta_0 \quad (3) \]

Comparable with liquid film cavitation, the angle \( \theta_0 \) is unknown in advance. It can be determined only while solving the Reynolds equation. Up to now, no adequate solution for a cavitation-like behaviour for foil bearings exists.

3. SOLUTION STRATEGIES FOR THE NO-CONTACT ZONE IN FOIL BEARINGS

As mentioned in the previous section, the loss of contact between top foil and supporting flexible elements leads to additional boundary conditions on an in advance unknown angle \( \theta_0 \). In this section we discuss three different ways of treating these boundary conditions.

In the first method, the region with sub-ambient pressure is neglected without taking the boundary conditions into account. Nevertheless most authors mention the correct boundary conditions [1]. This is clearly not a correct implementation as the pressure gradient at the border is not zero and mass conservation is not fulfilled. The advantage of this method is its simplicity.

In the second method, the pressure gradient becomes zero at the border by keeping the height constant in the no-contact zone and resetting the sub-ambient pressures.

At first sight, this seems a correct implementation. However, the constant height in the no-contact region does not prevent pressure generation in the no-contact zone. In order to prevent any pressure generation and to fulfill mass conservation it is needed to modify the differential equation in the no-contact region by eliminating the diffusion term. To conclude, although the second method gives good results, it has the problem that in the no-contact region the differential equation still allows pressure generation.

In order to solve this fundamental problem with the previous methods it is needed to implement a slightly modified Elrod’s cavitation algorithm which is used to take cavitation into account in liquid film bearings [2]. Elrod proposed to add a switching function to the differential equation which eliminates the diffusion term in the equation if the point of interest lies in the cavitated zone or in our case the no-contact zone. Furthermore he changed the variable \( P \) to a new variable representing the fractional film content in the cavitated zone and the density ratio in the full-film zone. Afterwards, the density ratio is transformed to the pressure.

In order take use of Elrod’s algorithm in foil bearings some modifications have to be made. The basic principle of adding a switching function in order to eliminate the diffusion term from the Reynolds equation in the no-contact region remains. But instead of replacing the pressure variable with the fill-fraction variable, one has to keep the pressure as the variable (voids cannot exist in air). Furthermore the height is kept constant in the no-contact region, which represents its actual behaviour. Important to mention is the upwind difference scheme (UDS) for the convection term as proposed by Elrod. For the diffusion term a central difference scheme can be used. An advantage of this method is its...
robustness. After only a few iteration steps, the mass errors are extremely small, meaning that full convergence for the complete bearing surface is obtained. Inherent to numerical simulations, the no-contact border is only known up to one grid point. This results in a border switching between 2 grid points. Therefore the algorithm blocks the border if it is stabilized. One or two additional iteration steps then lead to full convergence of the solution.

Figure 2 shows the resulting pressure distribution for 2 cases: only neglecting the subambient pressure and the proposed modified Elrod’s algorithm. Although the different algorithms handle the no-contact region differently, only the zone close to the border between contact and the no-contact is influenced. The load capacity does not change significantly. The advantage of the proposed modified Elrod algorithm is its robustness and the more correct representation of the actual behaviour.

![Figure 2 Pressure profiles for two different no-contact algorithms for \( \Lambda=2, \alpha=2, \varepsilon=0.5 \) and \( \text{L/D-ratio}=1 \)](image)

4. PARAMETER STUDY ON THE INFLUENCE OF A NON-IDEAL TOP-FOIL

An important factor in the mechanical model of a foil bearing is the top-foil. In literature two assumptions can be found: an ideal zero-stiffness top foil and the infinitely stiff (in the meridional plane, RZ-plane with \( \theta \) fixed) top-foil. The former assumes that the deflection at each point only depends on the local pressure. The latter supposes that the deflection depends on the average pressure in the meridional plane. This assumption is based on the observations of Carpino [3]. However, his observation is based on the simulation of both air film and mechanical structure, which are both very strongly connected with each other. Therefore, it is interesting to do a small parameter study on only the mechanical structure with a representative load. This load is constant along the circumference and parabolic in the meridional plane. The mechanical model is a circular spring bed with a top-foil modelled as a shell with all 4 edges free.

![Figure 3 Foil deflection in the meridional plane for different top foil thicknesses with \( d=8\text{mm}, b=8\text{mm}, k=5 \times 10^5 \text{N/m}^2 \) and a parabolic pressure load with \( P_{\text{max}}=0.5\text{bar} \)](image)

![Figure 4 Top foil deflection as function of bearing radius and top foil thickness with \( d=8\text{mm}, b=8\text{mm} \) and \( k=5 \times 10^5 \text{N/m}^2 \)](image)

Figure 3 shows the deflection in the meridional plane. The applied parabolic pressure has an amplitude of 0.5bar. The total deflection can be split up into two components: the average or DC component and deflection difference \( \Delta \) which is the most interesting in this case.

A small deflection difference \( \Delta \) means that the assumption of an infinitely stiff top foil is more valid while a large deflection difference tends more to the ideal, zero-stiffness top foil. The maximal resulting deflection of an ideal zero-stiffness top foil in the symmetry plane is thus:

\[
\Delta_{\text{max}} = \frac{P}{k} = \frac{5 \times 10^5 \text{Pa}}{5 \times 10^5 \text{N/m}^2} = 10^{-6} \text{m} \hspace{1cm} (3)
\]
5. COUPLED FEM-FDM SIMULATION

The previous section clearly showed that the assumption of an ideal zero stiffness top foil does not fully hold true, nor the assumption of an infinitely stiff top foil. However, it is still not clear how a top foil with finite stiffness influences the bearing characteristics. A reliable mechanical modelling of a foil bearing requires the combination of several element types, ranging from shell elements for the top-foil to special damping sources for the flexible elements. Because of this, it is not easy to reduce the mechanical system to an equation as for the air gap. Therefore it is very beneficial to uncouple the air film and mechanical FEM model. In this way it is possible to use commercially available finite element (FEM) programs, e.g. Nastran™, for the mechanical model and the finite difference method implemented in e.g. Matlab™ for the air gap model. In the first step, the pressure distribution coming from the air gap model is calculated. This pressure is applied as load on the mechanical model. The resulting displacement is afterwards fed back to the air gap model which is used to calculate the new pressure distribution. This procedure is carried out till convergence is achieved.

Figure 5 shows the load capacity for several top foil thicknesses for two different working conditions. The load capacity of a very thin top foil corresponds with the load capacity with the ideal top foil assumption. An increasing top foil thickness reduces the load capacity slightly. At first glance this is strange as previous simulations[4] showed that a stiffer bearing, e.g. lower $\alpha$, gives a higher load capacity. However, the contrary behaviour here can be explained with increased leakage in the axial direction.

CONCLUSION

In this paper we proposed a new robust method to simulate the no-contact region in foil air bearings. Furthermore a small parameter study is carried out, clearly showing that the assumption of an ideal zero-stiffness top-foil does not hold fully true. Also the effect of a top-foil with finite stiffness is analysed with a coupled FEM-FDM algorithm. The load capacity tends to decrease (slightly) with a finite stiffness top foil.

REFERENCES


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