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PERFORMANCE LIMITS OF PIEZOELECTRIC VIBRATION TO ELECTRICAL ENERGY CONVERTERS

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Abstract: A novel figure of merit for benchmarking resonant piezoelectric vibration to electrical energy converters is proposed. The dimensionless product *G* of maximum power and relative bandwidth has two advantages compared to other benchmarking schemes. First the product is finite for coupling constants smaller than 1, even under idealized conditions. Second it is more meaningful compared to the maximum output power because it describes the achievable power under realistic conditions, i.e. frequency mismatch and varying frequency excitation. The theoretical background of the figure of merit is described and the optimization regarding *G* instead of maximum power is discussed.

Key words: energy harvesting, piezoelectric converter, bandwidth power product

1. INTRODUCTION

Many schemes for normalizing the output power of piezoelectric energy harvesters have been proposed [1,2,3,4,5], but no theoretical limitation of output power has been established. This is due to the fact that the excitation is typically modeled as a sinusoidal vibration with fixed amplitude. An idealized system without any parasitic damping can draw an infinite amount of power from this excitation. Therefore no efficiency can be deduced as a figure of merit. The problem of infinite power can be circumvented by introducing a displacement limit [3], but nevertheless no efficiency can be defined. Another approach is to calculate the efficiency of the energy conversion from the mechanical to the electrical domain [4]. This approach does not take into account that the output power depends on both the amount of power absorbed from the vibration and the conversion efficiency.

As a second drawback benchmarking is typically done for a sinusoidal excitation in resonance. This idealized condition drastically overestimates the achievable power in comparison to real applications. Therefore the bandwidth of the resonance curve should also be included in a useful benchmarking scheme.

Following these considerations we propose the “gain factor” *G* as product of the extracted peak power and power bandwidth. *G* is made dimensionless by excitation amplitude, frequency and total mass of the harvester to obtain a comparable figure of merit. The usefulness of *G* will be demonstrated in next section. Then *G* is calculated for idealized generators. In the following section approximate solutions are compared to exact numeric results and to the results of a model with parasitic damping. Differences concerning the optimization for *G* and for maximum peak power are discussed. Some conclusions are drawn in the last section.

2. THEORY

2.1 *G* as a meaningful figure of merit

The number *G* is defined as the product of the peak power P_{max} extracted in resonance and the power bandwidth $\Delta\omega_R$ measured between the frequencies where a 50% decrease from P_{max} occurs. This product is made dimensionless by the total mass of the harvester *M*, the resonance frequency ω_R and the amplitude of the excitation *a*.

$$G = \frac{P_{\max} \cdot \Delta\omega_R}{M \cdot \omega_R^4 \cdot a^2} \tag{1}$$

This figure is meaningful for harvesters under realistic conditions. Typically excitation frequency and resonance frequency are not perfectly matched, e.g. due to fabrication tolerances. Furthermore the resonance frequency varies in time due to temperature changes and/or aging. Therefore an ideal harvester with a narrow bandwidth is not the optimum choice, as it will only deliver energy for perfect matching of excitation and resonance frequency. It has been shown in [6] that the optimum load for harvesters driven with broadband excitation is not equal to that for a perfectly matched sinusoidal excitation. The load is reduced compared to the value for sinusoidal excitation, which reduces the maximum achievable power, but increases the bandwidth. The best compromise of maximum power and bandwidth depends on the nature of the exciting signal.

2.2 *G* for idealized converters

A resonant piezoelectric harvester driven at frequencies around the first resonance frequency and loaded by a resistive load *R* can be modeled by a system of differential equations for the vibration amplitude *z(t)* and the charge *Q(t)* on the piezo element. Many slightly different non-dimensional forms of these equations have been published [1,2,3,4,5,6]. Our choice is described in equations (2) to (4).

The two governing differential equations are given in (2). The first equation describes the force balance with the piezoelectric coupling to the electric domain, while the second describes the electrical circuit with piezoelectric coupling to the mechanical domain.

$$\begin{aligned} &\Delta z' + \frac{k^2}{\omega_{el}} \cdot \dot{Q}' + \Delta \ddot{z}' = \omega'^2 \cdot \sin(\omega' \cdot t) \\ &Q' + \frac{1}{\omega_{el}} \cdot \dot{Q} + \Delta \ddot{z}' = \omega'^2 \cdot \sin(\omega' \cdot t) \end{aligned} \tag{2}$$

The non-dimensional quantities are defined in equation (3), where a and ω are the amplitude and angular frequency of the excitation, K_{eff} the effective stiffness, M_{eff} the effective mass and C^T the capacitance of the piezo element at constant mechanical stress.

$$\begin{aligned} &t' = t \cdot \omega_m; \quad z' = \frac{z}{a}; \quad Q' = \frac{Q}{k \cdot \sqrt{K_{eff}^E C^T} \cdot a} \\ &\omega_m = \sqrt{\frac{K_{eff}^E}{M_{eff}}}; \quad \omega_{el} = \frac{1}{\omega_m \cdot RC^T}; \quad \omega' = \frac{\omega}{\omega_m} \end{aligned} \tag{3}$$

A simple representation of this system is depicted in figure 1, with modeling quantities according to equation (4). For more complex devices the modeling quantities can be calculated as shown in [7].

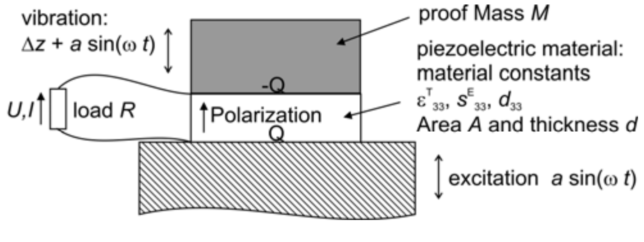

Fig. 1: Model representation of a piezoelectric energy harvester.

$$k_{33}^2 = \frac{d_{33}^2}{s_{33}^E \cdot \varepsilon_{33}^T}; \quad C^T = \frac{\varepsilon_{33}^T \cdot A}{d}; \quad K^E = \frac{A}{s_{33}^E \cdot d} \tag{4}$$

The quasi stationary solution of equation (2) for arbitrary excitation frequencies can be found by algebraic manipulation, but the results are not easy to handle due to their complexity. As our aim is to find reasonably handy equations for a fast benchmarking, we have chosen a much simpler approximate solution. Therefore we exploit the similarity of the force equation (first part of (2)) to a simple harmonic oscillator. The force equation differs from that of a harmonic oscillator only by the coupling term which is typically small. We therefore use the known phase relations between excitation and oscillator signal at resonance ω_R and the two frequencies of half maximum power ω_1 and ω_2 and feed them into equation (2). The phase shift between displacement *z(t)* and the charge *Q(t)* depends on the electric circuitry. It varies between zero for $\omega_{el} \to \infty$ ($R \to 0$) and $\pi/2$ for $\omega_{el} \to 0$ ($R \to \infty$). To get the exact values for the limiting cases a small deviation δ from these phase shifts must be introduced. In the following we demonstrate the calculation for the most interesting limit $\omega_{el} \to 0$.

$$\begin{aligned} &\Delta z' = \Delta z'_R \cdot \sin\left(\omega'_R \cdot t' - \frac{\pi}{2}\right), \quad Q' = Q'_R \cdot \sin(\omega'_R \cdot t' - \delta_R) \\ &\Delta z' = \Delta z'_1 \cdot \sin\left(\omega'_1 \cdot t' - \frac{\pi}{4}\right), \quad Q' = Q'_1 \cdot \sin\left(\omega'_1 \cdot t' + \frac{\pi}{4} - \delta_1\right) \\ &\Delta z' = \Delta z'_2 \cdot \sin\left(\omega'_2 \cdot t' - \frac{3\pi}{4}\right), \quad Q' = Q'_2 \cdot \sin\left(\omega'_2 \cdot t' - \frac{\pi}{4} - \delta_2\right) \end{aligned} \tag{5}$$

As these solutions are not the analytically correct ones, but good approximations the equation system (2) is only approximately fulfilled. The solution for the limit $\omega_{el} \to 0$ is found by using the small angle approximation for δ and neglecting all contributions of order ω_{el}^2. As an example substituting (5) in (2) for the resonance case leads to the following results (6).

$$\begin{aligned} &Q'_R = -\frac{1}{k^2} \\ &\Delta z' = \frac{1}{\omega_{el}} \cdot \frac{\sqrt{1-k^2}}{k^2} \\ &\omega'_R = \frac{1}{\sqrt{1-k^2}} \\ &\delta_R = \frac{\omega_{el}}{\sqrt{1-k^2}} \end{aligned} \tag{6}$$

The frequencies of half maximum power ω_1 and

ω_2 can be found in an analogous manner leading to two cubic equations (7) and (8).

$$-1-\frac{1}{\omega_{el}}\cdot\omega_1+\omega_1^2+\frac{1-k^2}{\omega_{el}}\cdot\omega_1^3=0 \tag{7}$$

$$1-\frac{1}{\omega_{el}}\cdot\omega_2-\omega_2^2+\frac{1-k^2}{\omega_{el}}\cdot\omega_2^3=0 \tag{8}$$

From these results the maximum power and the bandwidth can be calculated. The maximum power is given in equation (9).

$$\begin{aligned}P_{\max}&=\frac{1}{2}\cdot R\cdot\omega_R^2\cdot Q_R^2=\\&=\frac{1}{2}\cdot\frac{1}{\omega_{el}}\cdot\frac{\sqrt{1-k^2}}{k^2}\cdot\left(\omega_R^3\cdot M_{eff}\cdot a^2\right)\end{aligned} \tag{9}$$

As already mentioned in the introduction this idealized generator may draw infinite power from the vibration for $\omega_{el}\to 0$ (i.e. $R\to\infty$).

To find the bandwidth the cubic equations of (7) and (8) have to be solved. The results are quite complicated, but it turns out, that the limiting value for $\omega_{el}\to 0$ can be given in the compact form (10)

$$\Delta\omega_R=\omega_2-\omega_1=\omega_m\cdot\omega_{el}\cdot\frac{k^2}{1-k^2} \tag{10}$$

Therefore the power bandwidth product is finite in the limit of $\omega_{el}\to 0$.

$$G=\frac{P_{\max}\cdot\Delta\omega_R}{\omega_R^4\cdot M_{eff}\cdot a^2}=\frac{1}{2}\quad \omega_{el}\to 0 \tag{11}$$

The opposite limit of $\omega_{el}\to\infty$ ($R\to 0$) can be solved in an analogous manner but nearly vanishing phase shift between displacement and charge. The result for G in this limit equals that in the opposing limit (eq. (12)).

$$G=\frac{1}{2}\quad \omega_{el}\to\infty \tag{12}$$

To confirm the validity of the approximations, the approximate figure of merit G is compared to exact numeric results of the equations (2). Figure 2 shows the numeric results and the approximate limiting cases for G in dependence on ω_{el} and for different values of the coupling constant k.

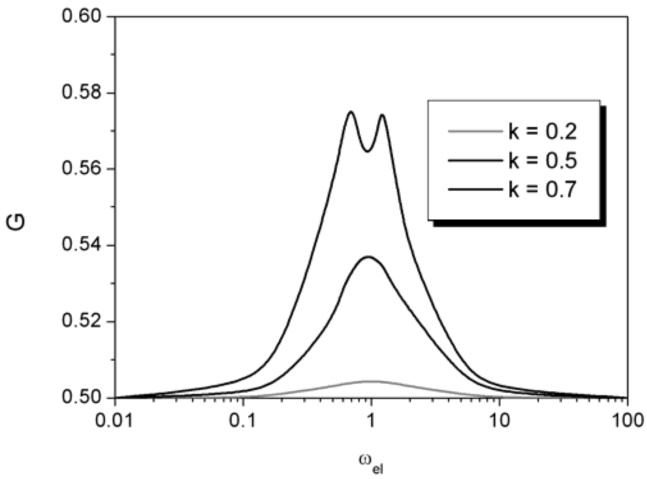

Fig. 2: Figure of merit G for an idealized converter in dependence on the electrical damping frequency ω_{el}.

The graph justifies the approach of this paper. The simple limiting values from equation (11) and (12) match to the exact solution.

G increases slightly with increasing coupling constants k at $\omega_{el}\approx 1$. This effect is quite small an may only be significant when using a bimorph with pure piezoelectric material (e.g. PZT $k_{33}\approx 0.6$), but typically a passive substrate is used for piezoelectric beam harvesters and the effective coupling constants are much smaller. In consequence using $G = 1/2$ as rule of thumb is a good choice for assessing an idealized harvester.

2.3 *G* for real converters

Real harvesters suffer from a finite structural damping or air damping. Assuming a velocity proportional damping, this can be modeled by introducing a damping term into the first equation of (2).

$$\Delta z'+\frac{1}{\omega_d}\cdot\Delta\dot{z}'+\frac{k^2}{\omega_{el}}\cdot\dot{Q}'+\Delta\ddot{z}'=\omega'^2\cdot\sin(\omega'\cdot t) \tag{13}$$

The effect of damping on G is demonstrated by the numeric solution of (2) with (13) for different structural and electrical damping (fig. 3).

As expected the power bandwidth product is decreased with increasing structural damping (decreasing ω_d). Furthermore the range of nearly constant G is drastically reduced, i.e. for $\omega_d < 10^3$ an optimized value of $\omega_{el}\approx 1$ exists while for lower damping (higher ω_d) a broad range of electrical loadings can be chosen.

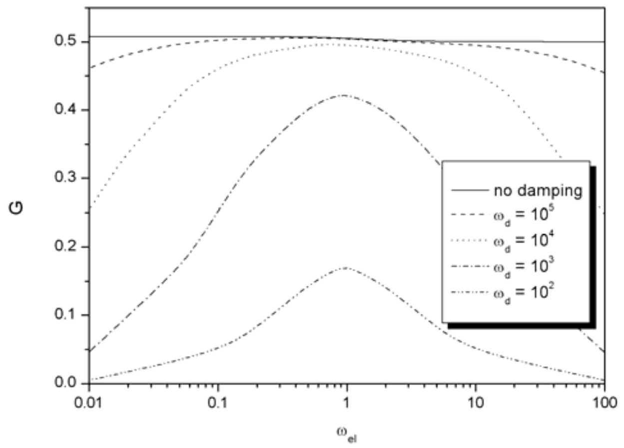

Fig. 3: Figure of merit G in dependence on ω_{el} and ω_{dl} for a structurally damped converter with k = 0.1.

This is due to the fact that for low structural damping two points of maximum power exist while for high structural damping only one optimum at $\omega_{el} \approx 1$ exists as shown in figure 4.

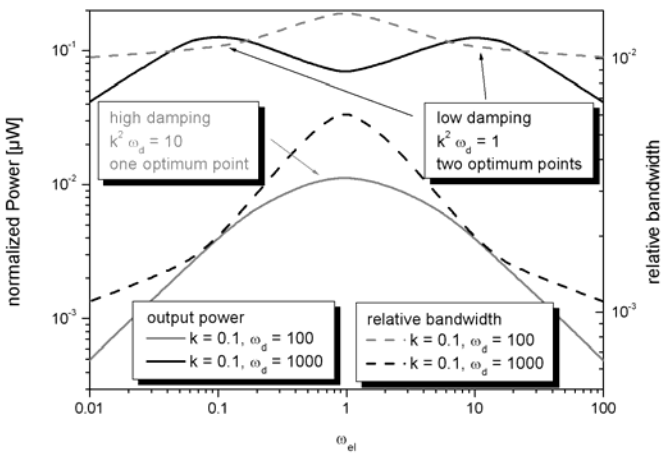

Fig. 4: Power and bandwidth for a real harvester with high and low structural damping.

3. DISCUSSION

It is suggested in this publication that piezoelectric harvesters should be optimized not only on the basis of maximum power but also on the basis of their power bandwidth. The figure of merit *G* is introduced for this purpose. It turns out that the criteria for a good harvester are a high coupling constant *k* and a low structural damping. This finding is nothing new compared to the typical optimization concerning maximum peak power. However, as *G* is finite for an undamped system it is a reasonable number for benchmarking. The designer can easily judge the effect of changes in damping and coupling. In an ideally undamped system changing the load alters the maximum extracted power as well as the power bandwidth, while its product remains virtually unchanged. This observation also holds for slightly damped systems for loads in the range of $\omega_{el} \approx 1$ (see figure 3). Therefore matching the load may not be very critical for real applications, because a mismatch concerning maximum power is compensated by an increased bandwidth. When designing a load adapting circuitry for a piezoelectric harvester, it may approximately be modeled as a generator with constant *G*. This approach drastically reduces the simulation effort compared to more sophisticated models.

4. CONCLUSION

A new benchmarking scheme based on the finite power bandwidth product *G* is proposed. As rule of thumb *G* = 1/2 is the optimum case for low coupling. Further work will concentrate on benchmarking different harvesters from the literature to judge the ultimate limits of piezoelectric power harvesting from vibration.

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