ON THE CONCEPTS OF ELECTRICAL DAMPING AND STIFFNESS IN THE DESIGN OF A PIEZOELECTRIC BENDING BEAM ENERGY HARVESTER

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Abstract: This paper presents and demonstrates a simple analytical design procedure for a piezoelectric energy harvester that determines the beam geometry and load impedance for a maximum vibration to electricity conversion. The proposed design procedure is established based on the notions of the electrical damping and stiffness which are accurately re-defined herein via the frequency domain analysis of the piezoelectric model. Embedding the electrical damping and stiffness in the mechanical model of the beam decouples the electrical and mechanical dynamics which significantly facilitates the design procedure. Therefore, the procedure includes two independent steps: matching the load with the capacitance of the piezoelectric beam; and matching resonant frequency of the beam with the vibration frequency of the source. Validity of the design procedure is numerically and experimentally verified by using a test setup for piezoelectric bending beam generators.

Keywords: Piezoelectric, Energy Harvesting, Electrical Damping, Bending Beam Generator

INTRODUCTION

A piezoelectric bending beam energy harvester typically consists of two layers of a piezoelectric material which are interleaved between metallic electrodes. To obtain a desired power and voltage from a vibrating beam, the conventional design methods mainly consider the effect of an electrical load as an electrical damping in the design equations [1, 2]. Based on the existing design methods, a mechanical to electrical power conversion will be maximized when: i) the so-called electrical damping ratio equals the mechanical damping ratio [1], or ii) the load resistance is selected as a function of mechanical damping ratio which is obtained from the exact solution of electromechanical model [2]. In this research work, we first clarify that: i) the condition which states electrical and mechanical damping ratio should be equal for maximum power conversion is not a valid statement since for a fixed load the extracted power is increased by reducing the mechanical damping; and ii) matching between the electric load and the piezoelectric impedance is the dominant factor in maximizing the extracted electric power, therefore, finding the exact solution is not necessary. Then, we establish a simple and yet accurate design procedure which formulates the design problem in two steps: tuning the resonance frequency of the beam at the frequency of the vibration source; and matching the load and the piezoelectric beam impedances. The proposed design procedure was validated by investigation of measured data obtained from an experimental test setup compared with the expectations from the design procedure.

MODEL OF A PIEZOELECTRIC BEAM

Figure 1 shows schematic diagram of a vibrating piezoelectric energy harvester (cantilever beam) which is connected to a resistive load. The small-deflection electromechanical model of a vibrating piezoelectric beam is [3]:

$$\begin{aligned} &M\ddot{u} + C_m\dot{u} + K_m u - \theta v = -M\ddot{y}, \\ &Q = \theta u + Cv, \end{aligned} \tag{1}$$

where M, C_m, K_m, θ and C denote the effective mass, mechanical damping, mechanical stiffness, the coupling coefficient and electrical capacitance of the beam, respectively. The mechanical and electrical quantities y, u, Q and v denote the base vibration, tip deflection of the beam, electrode charges, and electrode voltage, respectively. Having the piezoelectric material properties, an estimation of the model coefficients (parallel mode) are given by [1]:

$$C = \frac{4\varepsilon_{33}^T LW}{T}, K_m = \frac{WT^3}{4s_{11}^E L^3}, \theta = \frac{3d_{31}LW}{4\varepsilon_{33}^T T}, \tag{2}$$

where L, W and T are the length, width, and total thickness of the beam.

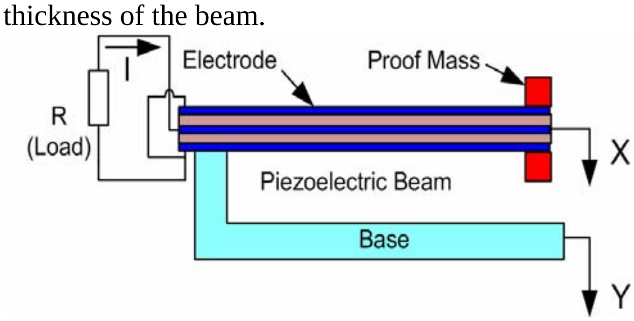

Fig. 1:Piezoelectric energy harvester (Parallel mode).

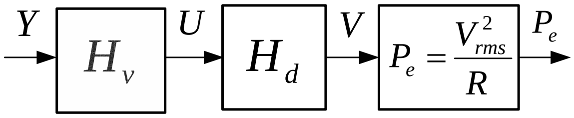

Fig. 2: Block diagram of the energy harvester model.

Equation (1) shows that when the electrodes are electrically shorted (i.e. v=0), the piezoelectric beam behaves as a simple second order mass-spring-damper system. However, when the beam delivers electric power to a load, the non-zero coupling term θv obviously impacts on dynamic behavior of the beam which can be characterized in terms of the electrical damping and stiffness as follows.

Concepts of Electrical Damping and Stiffness

Assume that the electrodes of the beam are connected to a resistive load (*R*). Considering the direction of current on Fig. 1 and from (1), we obtain:

$$v = -Ri = -R\frac{dQ}{dt} = -R\theta\dot{u} - RC\dot{v} \tag{3}$$

Solving (3) for *v* in the Laplace domain leads to the voltage transfer function, H_v, expressed as:

$$H_v = \frac{V(s)}{U(s)} = -\frac{R\theta s}{1+RCs}\ , \tag{4}$$

where *V(s), U(s)* are representation of voltage and deflection in the Laplace domain and *s* is the Laplace operator. A piezoelectric energy harvester is often designed for harvesting energy from a sinusoidal vibration source characterized by an amplitude (Y_0) and frequency (ω) as: $y(t) = Y_0 Sin(\omega t)$. Therefore, (4) can be represented in frequency domain by substituting $s = j\omega$ as:

$$V(j\omega) = -\frac{j\omega R\theta}{1+j\omega RC}U(j\omega), \tag{5}$$

Using (5), the term θv in the frequency domain can be represented as:

$$\theta V = -(K_e + j\omega C_e)U(j\omega), \tag{6}$$

where K_e and C_e can be defined as electrical stiffness and damping, given by:

$$K_e = \frac{(RC\omega)^2\left(\frac{\theta^2}{C}\right)}{1+(RC\omega)^2},\quad C_e = \frac{RC\left(\frac{\theta^2}{C}\right)}{1+(RC\omega)^2}, \tag{7}$$

Using (6) with (1) in the frequency domain, we obtain the deflection transfer function as:

$$H_d = \frac{U}{Y} = \frac{\omega^2 M}{(K_m + K_e - \omega^2 M) + j\omega(C_e + C_m)}. \tag{8}$$

DESIGN OF A PIEZOELECTRIC ENERGY HARVESTER

Using voltage and deflection transfer functions in (4) and (8), the dynamic of a piezoelectric energy harvester can be represented by a block diagram as shown on Fig. 2. The inputs to this diagram are the amplitude and frequency of a vibration source, and the design problem is to determine the geometry (K_m and C) and the amount of resistive load (R) such that maximum electric power will be delivered to the load. Using the deflection and voltage transfer functions, one can look for an optimal solution that maximizes the power for a given vibration amplitude and frequency. Such a solution must be obtained numerically since the combined transfer function of the system is a third-order dynamic system and closed form solution for such a system is too complicated to be useful for an analytical design. Herein, we present an analytical design method in two steps: maximizing the extracted electric power for a fixed deflection by matching the resistive load, and then maximizing the deflection transfer function for the matched resistor. These steps facilitate the design problem and it will be shown in the next section that the method provides a solution adequately close to the exact optimum.

Step 1: Maximizing the Extracted Electrical Power

The average electrical power that is delivered by a vibrating beam is:

$$P_e = \frac{|V(j\omega)|^2}{2R} = \frac{R\omega^2\theta^2U_0^2}{2(1+(RC\omega)^2)}\ , \tag{9}$$

The electric power P_e is maximum with respect to R when $R_{opt} = 1/(C\omega)$ and:

$$P_{e,\max} = \frac{\omega\theta^2U_0^2}{4C} \quad at \quad R_{opt} = \frac{1}{C\omega}, \tag{10}$$

Equation (10) presents a sub-optimal condition for a matched resistive load, since U_0 is also a function of R based on the deflection transfer function (8) but we assumed that it is not a function of *R* for now.

Step 2: Maximizing the Deflection Transfer Function

To improve the harvested electric power, it is important to design the beam such that maximum deflection at the vibration frequency occurs, which means maximizing the $H_d = U/Y$ ratio. Using the matched resistance $R = 1/(C\omega)$ in (7), the deflection transfer function (8) is simplified as:

$$H_d(j\omega) = \frac{\omega^2 M}{(K_m + \frac{\theta^2}{2C} - \omega^2 M) + j\omega(C_m + \frac{\theta^2}{2C\omega})}, \quad (11)$$

The maximum of H_d approximately occurs at a frequency close to its natural frequency: the frequency at which the real part of the denominator is zero. Thus, it is logical to design the beam geometry (L, W, T) such that the frequency of the vibration source matches with the natural frequency of the beam. At this frequency, the magnitude of deflection transfer function becomes:

$$H_d = \frac{\omega_n M}{C_m + \frac{\theta^2}{2C\omega_n}}, \quad \omega_n = \sqrt{\frac{K_m + \theta^2/(2C)}{M}}, \quad (12)$$

Substituting for K_m, θ and C from (2) in (8), we obtain:

$$H_d = \frac{1}{2\varsigma_m\sqrt{1-\varsigma_e} + \varsigma_e}, \quad \varsigma_m = C_m \big/ 2\sqrt{K_m M}, \quad (13)$$

where ς_e is defined as the electrical damping ratio as:

$$\varsigma_e = (\frac{\theta^2}{C\omega_n})/(2M\omega_n). \quad (14)$$

Equation (13) simply shows that when the resistor load is matched with the piezoelectric capacitance, the deflection transfer function is only a function of the mechanical and electrical damping ratios (ς_m and ς_e). Typically the mechanical damping ratio is about 0.01 for a millimeter scale rectangular beam and for a matched resistive load, H_d ranges from 15 to 25.

Summary of the Design Methods

The formulations of this section and the precedent section suggest the following two design methods:

Optimal Design: This method numerically solves the model equations (4)-(9) to determine the best geometry (K_m, C) and resistive load such that the harvested electrical power is maximized for a given vibration amplitude and frequency.

Sub-optimal Design: This method is an analytical method based on a simplifying assumption which has been elaborated in two steps in this section. The method splits the design problem into uncoupled electrical and mechanical problems to facilitate the design procedure. First, a matched resistive load is selected for a fixed tip deflection, and then by using this resistance value, the geometry of the beam is designed such that the deflection transfer function will be maximized.

The rest of this paper verifies the validity of the second method and it will be shown that the proposed design method has an excellent match with both experimentally measured data and the exact numerical solution for the model.

DESIGN METHOD VERIFICATION

To verify the design procedure, a test setup was arranged that includes two piezoelectric bending beams (*Q220-A4-303YB* of *Piezo System Inc.)* which one of them was used as a shaker and the other beam was used as the bending beam generator with the dimensions: *L=31.8*, *W=12.7*, and *T=0.51 mm*. The piezoelectric bending generator was installed at the tip of the shaker. Two linear accelerometer sensors (*LIS244AL* of *ST Microelectronics*) were installed at the tip and at the base of the beam. The frequency and amplitude of the shaker were controlled via a function generator connected to a power amplifier. Vibration at the based (*y* on Fig. 1) was measured via one of the accelerometers and the other accelerometer measures vibrations at the tip (*x* on Fig. 1). Tip deflection was calculated as *u=x-y*. The load was a variable resistor which its voltage and current were measured using a digital oscilloscope (33220 of *Agilent Technologies*) and a digital multi-meter (570 *EXTECH Instruments*). A proof mass was also attached at the end of the beam, providing a total effective mass of *M=0.96 gr*.

The first test experimentally determines the deflection transfer function by exciting the shaker with a sinusoidal waveform at various frequencies. Then, the accelerations at the tip and base of the beam (a_x, a_y) were measured. Since the vibrations were sinusoidal, $|a_x| = \omega^2 X_0$ and $|a_y| = \omega^2 Y_0$. Therefore, the deflection transfer function at ω is calculated as:

$$H_d(j\omega) = \frac{U}{Y} = \frac{X-Y}{Y} = \frac{|a_x|}{|a_y|}\left(\angle a_x - \angle a_y\right)\text{-}1$$

where magnitudes and phases of a_x and a_y were determined from the measured outputs of the accelerometers.

Figure 3 compares the analytically calculated H_d with the experimentally measured one at different frequencies which shows an excellent match between the analytical model for H_d and measured data.

The second test verifies the electrical model of the beam by means of measuring the total extracted power (P_e on Fig. 2) versus R. The power was also

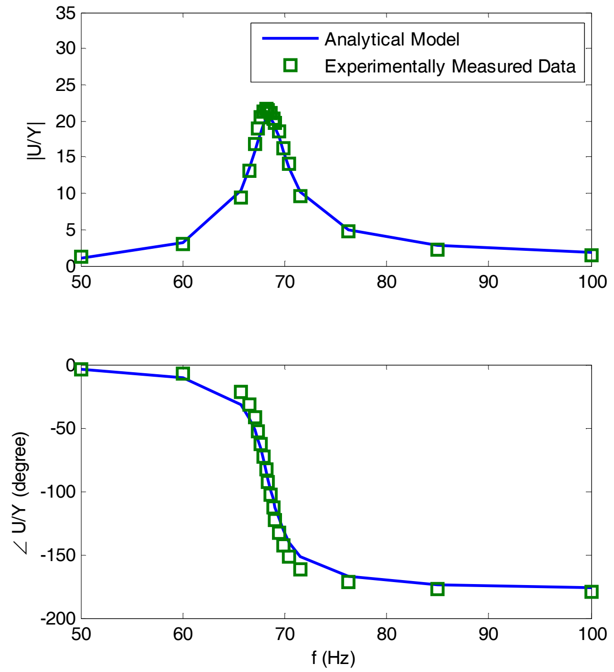

Fig. 3: Verification of the model for H_d.

calculated using the analytical model. The good match between measured and calculated data on Fig. 4 verifies the accuracy of the electrical model. Finally, to verify the validity of the proposed design procedure the optimum load resistance based on exact numerical solution of the analytical model (Eqs. (4)-(9)) was calculated and compared with $R = 1/(C\omega)$ at different frequencies as shown on Fig. 5. The close match of graphs on Fig. 5 shows that optimizing the electric power assuming a fixed deflection at the resonance frequency of the system does not severely impact the accuracy of the calculation when it is compared with the exact numerical solution of the model.

CONCLUSION

A simple design procedure for piezoelectric bending beam energy harvesters has been proposed that includes two steps: matching the load with the capacitance of the beam; and matching the resonant frequency of the beam with the frequency of the vibration source. Also, the concepts of electrical damping and stiffness have been accurately re-defined and used within the design procedure in matching the resonant frequency of the beam with the vibration frequency.

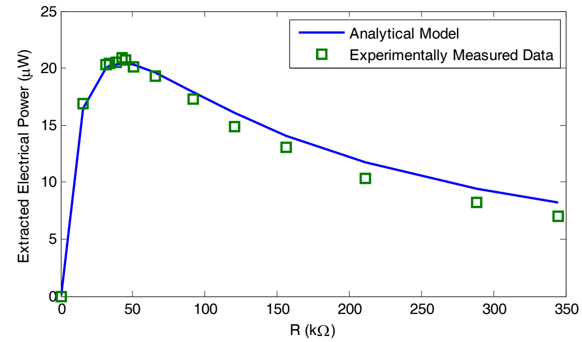

Fig. 4: Verification of the model for H_v.

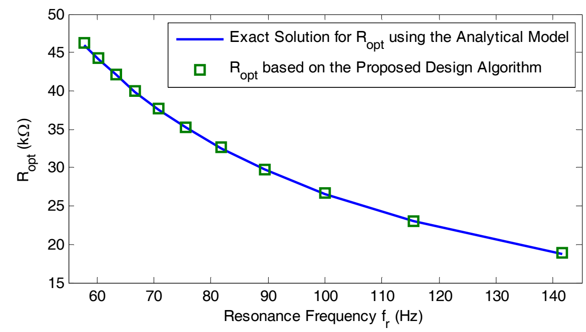

Fig. 5: Verifying the design procedure's assumption.

Validity of the proposed design procedure has been verified based on experimental test results and numerical solution of the piezoelectric bending beam model. The proposed algorithm facilitates the design of piezoelectric bending beam energy harvesters based on an accurate analytical model.

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