Abstract: Piezoelectric vibration energy harvesters (PVEH) are a relevant alternative to electrochemical batteries for the electrical powering of the nodes in wireless sensor networks. Usually, PVEH based on a resonant principle are designed in such a way that their resonance frequency matches the dominant harmonic of the input vibration. In some conditions it is possible to adopt a different optimization rule making profit of the frequency-dependant electrical impedance of the PVEH and its resonant/anti-resonant behavior. For a similar generated power, this allows tuning the voltage or current delivered by the device, which is of importance for the design of rectifying electronics. The theoretical description of this optimization method and its experimental validation are presented. In the case of microfabricated PVEH based on AlN, it is shown that optimum and almost constant power can be obtained for load impedances ranging from $100 \, \Omega$ to $1 \, M\Omega$.

Keywords: piezoelectric, impedance matching, MEMS

INTRODUCTION

Recently, wireless sensor networks opened the way for a large panel of applications such as industrial monitoring [1] or body area networks [2]. Harvesting energy from environmental mechanical vibrations is a promising alternative to electrochemical batteries for supplying electrical power to the nodes of the networks.

MEMS vibration energy harvesters (PVEH) based on the piezoelectric effect were developed by our team [3]. As illustrated by Fig. 1, the proposed PVEH consist of a silicon cantilever attached to a large proof mass and supporting a piezoelectric capacitor. AlN is used as piezoelectric material. The device is encapsulated in vacuum conditions ($< 1 \, mbar$) by a top and bottom glass layer to limit parasitic damping mechanisms.

![Schematics of the fabricated MEMS harvesters.](image)

Fig.1: Schematics of the fabricated MEMS harvesters.

If the input vibration is simplified to a sinusoidal excitation, two basic conditions are necessary for the optimization of the power transferred by a PVEH to a passive load circuit:

- The impedance of the load circuit should match the one of the piezoelectric vibrator.
- The frequency of the input vibration should match a high response frequency of the harvester.

It is often assumed that the frequency of highest response of a PVEH corresponds to its fundamental resonance frequency $\omega_0^R$, for which the device has minimum electrical impedance. However, if some particular conditions are met, high responses are also observed for frequencies ranging from $\omega_0^R$ to $\omega_0^A$, the latter being the anti resonance frequency of the PVEH, for which the device has maximum electrical impedance. The difference between the minimum and maximum impedance of PVEH can be large. On the other hand, assuming realistic piezoelectric benders, it is shown in this paper that the amount of power delivered to a load circuit with optimum impedance remains in the same range of magnitude for input vibration frequencies ranging from $\omega_0^R$ to $\omega_0^A$.

Therefore, by adjusting the frequency of the input in between resonance and anti resonance of the PVEH, it is possible to tune the voltage or current at which the power is delivered. This is an important consideration for the design of conditioning electronics implying for example rectifier diodes.

In the following, the optimization of PVEH making use of their resonance and anti resonance characteristics is first described from a theoretical point of view. An experimental validation of the method is then performed on the MEMS harvesters described previously.

THEORY

The model of PVEH described in [4] and [5] is taken as a theoretical basis for the derivations proposed in this part. To obtain this model, the linear constitutive equations of piezoelectricity are
transformed into a set of equations relating the macroscopic variables relevant for the analysis of piezoelectric cantilever (proof mass deflection, mechanical force, current and voltage) by use of kinematic, equilibrium and Maxwell’s equations. Small deformations, plane stress and quasi-static (both from an electromagnetic and mechanical point of view) behaviors are assumed.

According to this model, the power delivered by a PVEH to a load resistor \( R \) can be written as Eq.1 in which \( A_0 \) and \( \omega \) are respectively the amplitude and frequency of the input acceleration, \( m \) is the effective mass of the bender, \( C_p \) the clamped capacitance, \( Q_m \) the mechanical quality factor, and \( K \) the generalized (also called effective) electromechanical coupling factor. \( K \) is a macroscopic equivalent for bending structures of the piezoelectric material constant \( k_{31} \). \( \Omega \) corresponds to the frequency of the input normalized to \( \omega_0^2 \) and \( \Psi \) is the normalized load parameter. The optimum load parameter \( \Psi_{opt} \), equal to the absolute value of the PVEH impedance normalized to \((\omega_0^2 C_p)^2\), can be written as (5).

\[
P = \frac{m A_0^2}{2 \omega_0^2} \frac{\Omega^2 K^2 \Psi}{\left(\Omega^2 - 1\right)^2 + \Omega^2 \Psi^2 \left(\Omega^2 - K^2\right)^2} + D
\]

\[
D = \frac{1}{Q_m} \left( \frac{1}{Q_m} + 2 \Psi \left( K^2 + \frac{\Omega \Psi}{2 Q_m} \right) \right)
\]

\[
\Omega = \frac{\omega}{\omega_0}
\]

\[
\psi = RC_p \omega_0^R
\]

\[
\Psi_{opt} = \frac{1}{\Omega} \sqrt{\frac{1 + \left(\Omega^2 - 1\right)^2 Q_m^2}{1 + \left(\Omega^2 - K^2\right)^2 Q_m^2}}
\]

Analysis of (1) reveals that a single maxima of the power in terms of \( \Psi \) and \( \Omega \) is observed when the figure of merit \( M = K^2 Q_m \) [6] is below low a certain value, determined numerically to be approximately equal to 2.5. In this case, the maximum power is obtained slightly above the resonance of the bender for \( \Omega = 1 = \Omega^A \) and \( \Psi_{opt} = 1 \), i.e. the optimum load resistor is equal to \((\omega_0^R C_p)^{-1}\).

However, if \( M \) is above 2.5, two maximum are found [5][7]. This situation is illustrated by Fig. 2 in which (1) normalized to \( m A_0^2 / 2 \omega_0^R \) is plotted for \( K^2 = 0.0027 \) and \( Q_m = 1200 \), which are values representative of the experiments presented later. The optimum load parameter for each frequency \( \Psi_{opt} \) and the optimum frequency for each load parameter \( \Omega_{opt} \) are also plotted by respectively the thick grey and black lines. The first maximum of power is found around \( \Omega^R \) while the second is obtained slightly below \( \Omega = (1 + K^2)^{1/2} = \Omega^A \), which corresponds to the anti-resonance frequency of the PVEH. It can be seen from Fig. 2 that the optimum load parameter for the resonance (anti-resonance) peak is close from the minimum (maximum) of \( \Psi_{opt} \). The optimum values of the power and of the load at \( \Omega^R \) and \( \Omega^A \) can be approximated by (6) till (9).

\[
\Psi_{opt}^R = \frac{1}{\sqrt{1 + K^4 Q_m^2}} \quad \Psi_{opt}^A = \frac{1}{\Psi_{opt}^R \sqrt{1 + K^2}}
\]

\[
p_{opt}^R = \frac{m A_0^2}{4 \omega_0^R} \frac{Q_m}{1 + \sqrt{1 + \frac{1}{K^4 Q_m^2}}}
\]

\[
P_{opt}^A = (1 + K^2)^{5/2} p_{opt}^R
\]

For \( K^2 = 0.0027 \) and \( Q_m = 1200 \), (8) and (9) are almost equal. Following one of the optimum paths (thick grey and black lines of Fig. 2) between resonance and anti-resonance, the power remains very close from its maxima (deviations from this observation may occur if \( K^2 \) becomes large). On the opposite, the corresponding impedance of the load circuit varies greatly. Therefore, it is possible to tune the voltage and current at which power is delivered by the PVEH by adjusting the frequency of the input vibration between \( \Omega^R \) and \( \Omega^A \) and by implementing an appropriate value of the load impedance.

If one is not concerned by the current or voltage at which the generated power is supplied, it is possible to use the aforementioned effect for broadening the frequency response of the PVEH. Practically, this may be realized with an active tracking system varying the impedance of the load when a variation of the dominant frequency of an input vibration is detected.
The analysis proposed in this section is focused on PVEH with a low effective electromechanical coupling and a large mechanical quality factor. These characteristics are typical of the MEMS harvesters that we developed. Because of the small value of $K$, the range of frequencies for which voltage/current tuning is possible remains limited. In the case of commercially available benders based on sintered piezoelectric ceramics, $K^2$ can reach a value of 0.1. The frequency shift between resonance and anti-resonance is then more pronounced. However, parasitic dissipations are also larger [4]. This situation is currently under investigation.

The theoretical predictions that have been described above are confirmed by experimental measurements in the next section.

**EXPERIMENTS**

The presented experimental results correspond to a device with a total length (beam and proof mass) of 4.4 mm and a width of 3 mm. The thickness of the silicon beam and piezoelectric capacitor are respectively 50 μm and 1.2 μm. The effective mass $m$ of the sample is equal to 14 mg and its resonance frequency to 1082.4 Hz. Its clamped capacitance is 360 pF. A value of 1200 is measured for $Q_m$ by using the half bandwidth method on the square of the short circuit current. $K^2=0.0025$ is found from the difference between $\omega_0A$ and $\omega_0R$.

For sake of clarity, the experimental results are not presented in terms of the normalized input frequency $\Omega$ and of the load parameter $\Psi$ used in the theoretical section, but rather in terms of the frequency $f$ and load resistor $R$.

In a first series of experiments, the value of $R$ is fixed and the frequency of the sinusoidal vibration is varied. The amplitude of the input vibration is set to 0.32 g. For each value of $R$, the frequency of maximum power is reported in Fig. 3. It corresponds to the black thick line of Fig. 2. As explained before, the minimum and maximum of the optimum frequency are respectively equal to the resonance and anti resonance frequencies of the PVEH. It can be seen that the fit between the model and the theory is good, even if a small but almost constant shift is observed when $R$ is above 100 kΩ.

The power corresponding to the optimum frequency for each load is given in Fig. 4. Again, a good fit is observed between theory and measurements. By adjusting the frequency of the input vibration, a power of 3 μW can be generated for load impedances ranging from 100 kΩ to 1 MΩ. In the former case, the current and voltage are respectively 6 μA and 0.5 V. In the latter, they are 1.5 μA and 2 V. Also, approximately 90% of the optimum power can be generated from 50 kΩ to 2 MΩ, so that it is possible to tune further the generated voltage and current.
deformations (up to about 2.5 g) where the power output will be significantly higher, even exceeding 100 µW.

![Graph showing output power at optimum load vs. frequency]

**Fig.6:** Output power at optimum load vs. the frequency. Solid line: theory, markers: experiments. The inset is a magnification of the section where maximum power is obtained.

In a second series of experiments, the sample is excited by a sinusoidal vibration with constant frequency and amplitude (0.32 g) while the load resistor $R$ is varied. The experiments are repeated for several frequencies. These measurements correspond to the grey thick line of Fig. 2. The optimum value of the load resistor and the corresponding power versus the frequency are given in respectively Figure 5 and 6. The model and the experiments are again in good agreement.

**CONCLUSION**

It is shown in this article that it is possible in some conditions to make use of the resonance and anti-resonance characteristics of piezoelectric energy harvesters for tuning the voltage or current at which power is generated, which is of importance for the design of rectifying electronics. The theoretical aspects of the proposed optimization method and its experimental validation are presented. In the case of a MEMS harvester, it is shown that a constant amount of power can be delivered over load impedances ranging from 100 kΩ to 1 MΩ. The generated voltage is multiplied by a factor 4 between the two extremes. The tested devices have a low effective electromechanical coupling factor. Therefore, the range of frequencies for which voltage/current tuning is possible remains limited. The presence of this effect in high coupling devices is being investigated.

**REFERENCES**