Abstract: In this work we investigate the thermodynamic cycle of a resonant, MEMS-based, micro heat engine. The micro heat engine is made of a cavity encapsulated between two membranes. The cavity is filled with saturated liquid-vapor mixture working fluid. Heat is added/rejected from the engine at a frequency equal to its resonant frequency. Both pressure-volume and temperature-entropy diagrams of the resonant engine are used to investigate the thermodynamic cycle of the resonant micro heat engine. The measured second law efficiency of the micro heat engine is 16%.

Keywords: micro heat engine, MEMS engine, resonant heat engine

INTRODUCTION

Development of micro-scale heat engines based on the Rankine cycle [1], Otto cycle [2], Brayton cycle [3], Stirling cycle [4], and Humphrey cycle [5] has been proposed by different research groups. Our group has developed a resonant micro-scale dynamic heat engine that is not based on these standard cycles. The resonant micro heat engine is an external combustion engine with a design based on stacking 2D structures. In previous work we have characterized the performance of the engine off-resonance and demonstrated power production [6]. The resonant frequency of the engine was determined to be on the order of 100 Hz [7].

In this work we look at the thermodynamic cycle of this resonant micro heat engine. In previous work we have shown that resonant operation of the micro heat engine is beneficial [7]. That is, for a resonant operation pressure and volume in the engine cavity are decoupled, i.e. cyclically rise and fall 90 degrees out of phase [7]. To characterize the cycle of the resonant engine both pressure and volume inside the engine cavity are measured. Assuming saturation conditions and using pressure measurements both temperature and entropy inside the engine cavity are estimated. Pressure-volume and temperature-entropy diagrams are then used to characterize the thermodynamic cycle of the resonant engine.

EXPERIMENT

Figure 1 shows a schematic of the resonant micro heat engine. The cavity of the engine is filled with a two-phase mixture of 3M™ PF5060DL fluid. The cavity is encapsulated between two thin membranes. The top membrane acts as an expander/compressor. The bottom membrane acts as an evaporator. A capillary wick fabricated on top of the evaporator membrane controls the thickness of the liquid-phase layer inside the cavity. A small mass is placed on top of the expander membrane. The purpose of the added mass is to reduce the resonant frequency of the engine to the order of 100Hz. Lastly, a thermal switch is used to control the timing and duration of the heat addition and heat rejection processes. Details on fabrication of engine components are provided in [6-8].

Figure 2 illustrates the experimental apparatus used to characterize the thermodynamic cycle of the engine. The engine is mounted on top of a sealed chamber. The chamber is filled with low thermal conductivity gas. The thermal switch consists of a liquid-metal micro-droplet array deposited on a silicon die. The die is bonded to a micro heat exchanger mounted on a piezo-stack actuator. A frequency generator delivers a sine wave voltage to control the stack actuator’s displacement amplitude and frequency of operation. The motion of the piezo-stack actuator is transmitted to the micro heat exchanger through a push-pull device that seals against vacuum.

Heat is transferred to or from the engine when the micro-droplet array contacts the bottom evaporator membrane. In the experiments described in this paper, a thermal switch was used to control the heat rejection from the engine. However, a resistance heater was used for heat addition to the engine to enable precise control of the magnitude and duration of the heat addition.
addition process. The motion of the expander and evaporator membranes is measured with a laser vibrometer. The displacements of both expander and evaporator membranes are sampled with a digital oscilloscope and stored on a personal computer.

To characterize the thermodynamic cycle of the micro heat engine at resonance pressure \( P \), volume \( V \), temperature \( T \), and entropy \( S \) inside engine cavity are determined. It is assumed that working fluid inside engine cavity is saturated and that the pressure and temperature inside the cavity are uniform. The pressure and volume measurements are decoupled during resonance operation. That is they are out of phase by 90 degrees. Because of this the pressure and volume changes are determined independently for a resonance operation. This is done by detecting the pressure and volume changes inside the engine using the deflection of the evaporator and expander membranes respectively. The volume underneath a membrane is determined by performing a double integral on the membrane deflection profile equation given by

\[
\Delta V(\delta) = \frac{\delta}{a^2} \int_{-a}^{a} \int_{-a}^{a} \left[ 1 + \frac{R}{a^2} \left( x^2 + y^2 \right) \right] dx dy \quad (1)
\]

where \( \delta \) is the deflection of the midpoint of the expander membrane in micrometer, \( a \) is \( \frac{1}{2} \) the side length of the membrane, \( x \) and \( y \) are coordinates on the membrane, and \( R = 0.34 \) is a constant determined by matching the deflection profile of the equation to experimental data.

Pressure \( P \) is related to the deflection of the evaporator membrane through a calibrated pressure deflection curve of the form [9]

\[
\Delta P = C_1 \delta + C_2 \delta^3 \quad (2)
\]

where pressure \( P \) is in kPa, \( \delta \) is the evaporator membrane deflection in micrometer. The constants \( C_1 \) and \( C_2 \) are determined by matching the deflection profile of the equation to experimental data. For the 2 \( \mu \)m thick, 64 mm\(^2\) silicon evaporator membrane used in this work the constants \( C_1 \) and \( C_2 \) are 1.4197 \( \times \) 10\(^{-2}\) and 8.4 \( \times \) 10\(^{-6}\), respectively.

Based on pressure \( P \) and volume \( V \) measurements, the boundary work of the engine is defined as

\[
W = \int P dV \quad (3)
\]

The mechanical efficiency is then defined based on the boundary work \( W \) in (3) and the input energy delivered to the engine per cycle \( E \), given by

\[
\eta_{\text{mech}} = \frac{\int P dV}{E} \quad (4)
\]

The Carnot efficiency is defined based on the maximum and minimum temperatures of the working fluid and given by

\[
\eta_{\text{Carnot}} = 1 - \frac{T_{Li}}{T_{Hi}} \quad (5)
\]

where \( T_{Li} \) and \( T_{Hi} \) are the minimum and maximum engine cavity temperatures, respectively. Using pressure measurements \( P \), temperature inside the cavity \( T \) is interpolated from thermodynamic tables of 3M™ PF5060DL working fluid.

To specify the entropy \( S \) inside the cavity, two independent intensive properties are needed. Entropy \( S \) is obtained using quality and pressure information.

The quality of the mixture \( \dot{Q} \) along with pressure \( P \) serve as the two intensive properties needed. The quality of the mixture \( \dot{Q} \) is calculated based on volume \( V \) and density \( \rho \):
\[ Q = \frac{V \ast \rho}{m_{total}} \]  

The product \((V \ast \rho)\) represents the mass of vapor inside the cavity. The total mass of liquid-vapor mixture is given by \(m_{total}\). The vapor density \(\rho\) is interpolated using thermodynamic tables of 3M™ PF5060DL working fluid. Entropy \(S\) is then given by

\[ S = S_f + Q \ast S_{fg} \]  

\[ S_{fg} = S_g - S_f \]  

where \(S_g\) and \(S_f\) are entropies of saturated vapor and saturated liquid, respectively.

RESULTS AND DISCUSSION

Throughout this paper, the expander and evaporator are 300 nm thick silicon nitride and 2 µm thick silicon membranes, respectively. The side length of the expander membrane is 10 mm. The size of the saturated-vapor bubble is 9 mm. The thickness of the cavity is 150 µm. The mass added to the engine is, \(m_{added} = 3.9\) g. The wick structures are 5 µm thick, 10 µm high, and spaced 90 µm apart. The resonant frequency of this particular engine is determined to be \(f_n = 115\) Hz. Details on measurement of resonant frequency are in [7]. In the experiment, the micro heat engine is operated at its resonant frequency, \(f_n = 115\) Hz. Input energy per cycle, delivered to the evaporator, is \(E = 1.0\) mJ and it is delivered with a 1% duty cycle. The thermal switch, controlling heat rejection, is maintained at 4°C during the operation of the engine. The experiment is carried out at room temperature, \(T_0 = 294\) K. The saturation pressure of the two-phase mixture inside the engine cavity is \(P_{sat} = 25351\) Pa gage at room temperature \(T_0 = 294\) K.

Figure 3 shows the time history of pressure and volume inside the cavity when the engine is operated at resonance, \(f_n = 115\) Hz. Volume and pressure inside engine cavity are obtained from (1) and (2), respectively. The figure shows that pressure and volume in the engine cavity are decoupled and cycle work is produced.

The corresponding pressure-volume and temperature-entropy diagrams are shown in Figure 4 and 5, respectively. The cycle work is represented by the area enclosed by the pressure-volume and temperature-entropy diagrams. The maximum pressure rise and volume change inside the cavity are 45 Pa and 0.55 mm³, respectively. The engine is operating over a very small temperature difference, on the order of 1 K. At this condition the mechanical work produced by the micro engine in one cycle is 0.018 µJ. The small operating temperature difference is due partly to the small amount of heat added and to the large thermal storage of the engine structure, the membranes and the wicks. That is, very little of the heat that is added to the engine goes to vaporizing working fluid, less than 10%.

To calculate the Carnot efficiency, the engine may be viewed as operating between the two temperature reservoirs at \(T_H\) and \(T_L\), respectively. The second law efficiency, the ratio of the thermal efficiency to the Carnot efficiency is then found to be 16%. The thermal efficiency is evaluated from the ratio of the work produced by the working fluid to the energy added to the membrane.

From the pressure-volume and temperature-entropy diagrams it can be seen that the cycle is roughly rectangular in shape and consists of two constant temperature processes and two constant volume processes. Heat is transferred from the engine structure to vaporize the working fluid in a constant temperature process. During this process the membrane is moving outward and the vapor volume increases. After vaporization is completed, due to inertial effects, the membrane continues to move outwards for a short time and the temperature and pressure drop slightly. Once the expander membrane reaches its maximum deflection point, the thermal switch closes the gap with the evaporator membrane and heat is conducted away from the engine structure. Engine pressure and temperature decrease over a very small volume change. During resonant operation of
The cycle of the resonant MEMS-based micro heat engine has been acquired experimentally and has been characterized using both pressure-volume and temperature-entropy diagrams. The working cycle is nearly rectangular in shape and consists of two constant temperature processes and two constant volume processes. The measured second law efficiency of the micro heat engine is 16%.

**REFERENCES**