TRANSIENT MODEL FOR THERMOELECTRIC GENERATOR SYSTEMS HARVESTING FROM THE NATURAL AMBIENT TEMPERATURE CYCLE

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Abstract: Until now ambient temperature thermoelectric energy harvesting was considered a steady state problem. In this paper we show that this assumption is not justifiable and leads to output power losses of the harvester. We present a new simple model for thermoelectric generator (TEG) systems in transient mode. It suggests designing the thermal contacts (e.g. a heat sink) of the TEG based on the time constant rather than on load matching. Field measurements of four different systems clearly show that the output power of a time constant matched system is several times larger than that of a load matched system and thus confirm the use of a transient model for the layout of the thermal contacts.

Keywords: Thermoelectric generator, ambient temperature gradient, transient model

INTRODUCTION

A TEG usually interfaces a thermal input and an electrical output. Thermal loads connect it to the heat source and sink. An electrical load at the output consumes the converted electrical power. For maximum electrical power output, these loads have to be thermally [1] and electrically [2] adapted to the generator: This so-called load matching refers to steady state conditions. Research on TEGs in the time domain has been done on short-term scale [3] and is lately emerging on larger time scales for automotive [4] and ambient energy harvesting applications. Especially in the case of ambient energy harvesters, the overall temperature difference ΔT is very small (1-10 K). Most of this system ΔT is lost on the thermal contacts rather than on the TEG, which makes thermal load matching essential. Despite this fact, only a few groups dealing with ambient energy harvesters have been considering the thermal contacts at all [5,6]. Their system models are based on stationary heat conduction, since their thermal contacts are modeled solely resistive. As we will show within this work the ambient temperature is a much faster changing source than the fundamental 24 h period suggests. The resulting temperature gradient cannot be treated in stationary mode with constant hot/cold side temperatures at the TEG. Thus, capacitances of the thermal contacts cannot be neglected.

The overall aim of our work is to develop a self-sufficient power supply based on thermoelectric energy conversion for a sensor network monitoring tunnels. A naturally fluctuating temperature gradient exists in tunnels between air and wall. The temperature gradient in a tunnel has the same shape as the one outside between ambient air and soil, but is slightly smaller in amplitude [7]. A simple system harvesting from this gradient consists of an air contact - namely a heat sink - and a TEG.

TRANSIENT MODEL

In general there are two ways to solve heat transfer problems:

- the heat diffusion equation and
- the lumped parameter approximation.

Heat diffusion equation

The heat diffusion equation is the governing equation for transient heat transfer in 1-D given by

$$\frac{\partial T(x,t)}{\partial t} = \frac{\lambda}{\rho c} \cdot \frac{\partial^2 T(x,t)}{\partial x^2}, \qquad (1)$$

with the temperature $T(x,t)$, space and time variables x and t, thermal conductivity λ, density ρ and specific heat capacity c (at constant pressure). It is a partial differential equation (PDE) of second order that describes the exact temperature distribution in a domain over time. Its solution is difficult and strongly depending on geometry and boundary conditions. In our case, we want to know the hot T_h and cold side temperature T_c of the TEG to determine its electrical power output. If T_h and T_c are known, the resulting heat fluxes q_h and q_c can be evaluated by the thermoelectric coupling equations. The cold side temperature is given by the wall temperature. The hot side temperature is the heat sink/TEG interface temperature. In order to obtain this surface temperature of the heat sink we would have to solve the heat diffusion equation on the heat sink geometry. However, the knowledge of the temperature distribution in the heat sink (domain) is not important since we only need the surface temperature (boundary of the domain).

Lumped parameter approximation

The lumped parameter model approximates the spatial temperature distribution of the heat sink by its surface temperature [8]. This simplification is only valid if the dimensionless Biot number is smaller than 1/10. The Biot number is defined as ratio of conduction to convection resistance

$$Bi = \frac{\frac{L_{char}}{\lambda A}}{\frac{1}{hA}} < \frac{1}{10}, \qquad (2)$$

with characteristic length L_{char}, area A and heat transfer coefficient h. Within this work, we use four types of heat sinks. The largest Biot number is obtained for heat sink SK414100SA (Tab. 1). It is $Bi_{SK414100SA} \approx 3.6 \cdot 10^{-5}$ which is by far smaller than 1/10. So the lumped parameter model can be used. The hot side temperature of the TEG T_{HS} (= surface temp. of the heat sink) is then given by Newton's law of cooling

$$T_{HS}(t) = T_{air} + (T_{HS}\big|_{t=0} - T_{air}) \cdot e^{-\frac{t}{\tau}}, \qquad (3)$$

with the time constant τ of the heat sink

$$\tau = K_{HS} \cdot C_{HS} = \frac{1}{hA} \cdot c\rho V, \qquad (4)$$

that contains the product of the convective resistance K_{HS} and the heat capacity C_{HS}. Introducing a characteristic length as ratio of volume V to area A

$$L_{char} = \frac{V}{A}, \qquad (5)$$

the time constant of the heat sink reduces to

$$\tau = \frac{\rho c L_{char}}{h}. \qquad (6)$$

The lumped parameter model now allows to reduce the heat sink and the TEG to simple RC elements. Calculating the heat capacities for heat sink SK414100SA (*Fischer*) and TEG241-120-15 (*thermalforce*), we get $C_{HS} \approx 157.5$ J/K and $C_{TEG} \approx 4.3$ J/K. Since C_{TEG} is relatively small (only 2.7% of C_{HS}) and we use one TEG model throughout the measurements, we can neglect C_{TEG} and approximate the time constant of the whole system by that of the heat sink given in eq. (6). The total thermal network thus reduces to an RC element for the heat sink and a resistor for the TEG (Fig. 1).

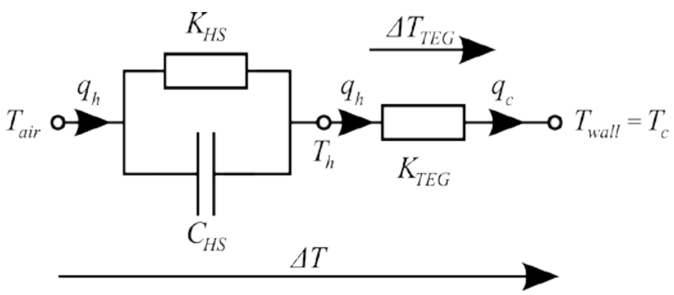

Fig. 1: RC equivalent thermal network for the transient modeling of TEG and heat sink (HS).

This simple thermal network shows that the time constant τ of the heat sink determines how quick the hot side temperature T_h follows changes of the air temperature T_{air} The smaller the time constant, the better the coupling of T_{air} to the TEG. This fact will also be shown in the following laboratory and field measurements.

Thermoelectric coupling equations

The open circuit output voltage of the TEG U_{TEG} is given by [9]

$$U_{TEG} = \alpha \cdot \Delta T_{TEG}, \qquad (7)$$

with the Seebeck coefficient α. Because of contact resistances at the interfaces heat sink/TEG and TEG/wall, a temperature divider between K_{TEG} and $K_{contact}$ must be considered

$$\Delta T_{TEG} = \frac{K_{TEG}}{K_{TEG} + K_{contact}} \cdot (T_{HS} - T_{wall}). \qquad (8)$$

An electric load R_L closes the circuit. The resulting electric current I_L through the circuit then becomes

$$I_L = \frac{U_{TEG}}{R_{TEG} + R_L}, \qquad (9)$$

with an electrical power output P of

$$P = I_L^2 \cdot R_L. \qquad (10)$$

Combining equations (3), (7), (8), (9) and (10), the output power of the TEG can finally be calculated by

$$P = \left(\frac{K_{TEG}}{K_{TEG} + K_{contact}} \cdot \frac{\alpha \cdot (T_{HS} - T_{wall})}{R_{TEG} + R_L} \right)^2 R_L. \qquad (11)$$

Note that the heat sink temperature T_{HS} and the wall temperature T_{wall} are functions of time which is consequently also true for the output power P. The dependence of the space variable x has been cancelled out by the lumped parameter model.

LABORATORY MEASUREMENTS

Four different heat sinks were chosen depending on their thermal resistances K_{HS} and time constants τ. One TEG model (TEG241-120-15, *thermalforce*) was used throughout all measurements for comparability. The thermal resistances of the heat sinks and the time constants of the systems 1-4 were measured on a laboratory setup at steps of 1 K with ΔT of 0 K up to 10 K under natural convection. The temperatures of the ambient air and the heat exchanger (= temperature source) as well as the open circuit voltage of the TEG were logged with a multimeter (Fig. 2).

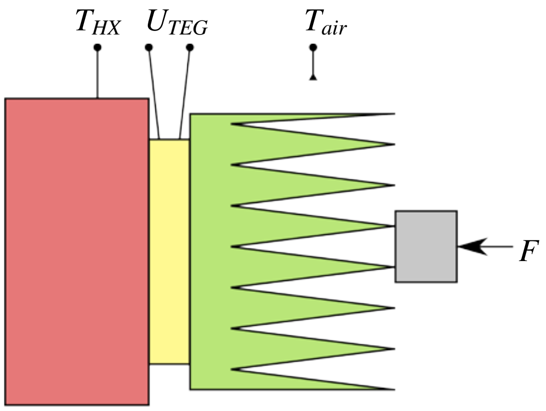

Fig. 2: Laboratory setup for measuring the thermal resistance K_{HS} and time constant τ. From left to right: Heat exchanger, TEG, heat sink, compressive load.

The sampling rate was 5 s and the compressive force *F* 330 N. The measurement results are shown in Tab. 1. A step response of system 1 would need approx. 4 min to reach 63,2% (= $1\text{-}e^{-1}$) of its steady state temperature.

Tab. 1: Measured thermal time constants τ for the systems 1-4. The thermal resistance K_{HS} only refers to the heat sink and was measured under natural convection conditions.

System No.	**Heat sink *Fischer***	**τ / s**	**K_{HS} / K/W @ ΔT = 1K-10K**
1	ICKPEN45W	239	9.3-7.1
2	SK47550SA	374	10.3-7.6
3	ICKPEN3XE1	402	3.4-2.5
4	SK414100SA	416	5.6-4.3

For the given TEG (Tab. 2) the literature suggests a thermally matched load of approx. 2.4 K/W. The heat sink which matches this thermal resistance of the TEG best is that of system 3.

Tab. 2: TEG data sheet properties (TEG241-120-15, thermalforce).

K_{TEG} / K/W	**α / V/K**	**R_{TEG} / Ω**
2.44	0.095	9.7

FIELD MEASUREMENTS

Field tests of the systems 1-4 were performed in a 1.1 km long road tunnel (Hugenwaldtunnel near Freiburg, Germany) at a tunnel depth of 303 m for a period of two weeks. A mounted system consists of a clamping bar, stamp, heat sink, TEG and heat conducting foil as thermal interface material to the wall (Fig. 3). Mechanical fixation of the clamping bar was done with screws. The air and wall temperatures and the TEG open circuit voltages were recorded with a data logger at 5 s sampling rate (Fig. 5).

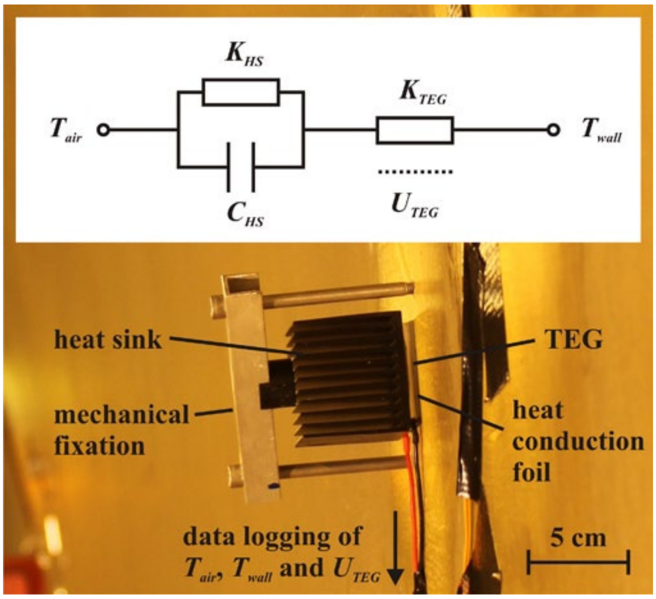

Fig. 3: System 4 during the field measurement in the Hugenwald road tunnel at 303 m tunnel depth.

If we go back to the assumption of Newton's law of cooling, we are now able to calculate the heat sink temperature from the measured air temperature (Fig. 4). Equation (3) must be converted to an iterative form:

$$T_{HS,calc}(t+1) = T_{air}(t+1) + (T_{HS}(t) - T_{air}(t+1))e^{-\frac{\Delta t}{\tau}}, \tag{12}$$

with the time step Δt representing the sampling rate. This conversion is necessary since the measured T_{air} is no explicit function of time, but a vector with discrete temperature values, that varies significantly with time. The solution of $T_{HS,calc}$ plotted in Figure 4 can therefore only be calculated numerically, but not analytically.

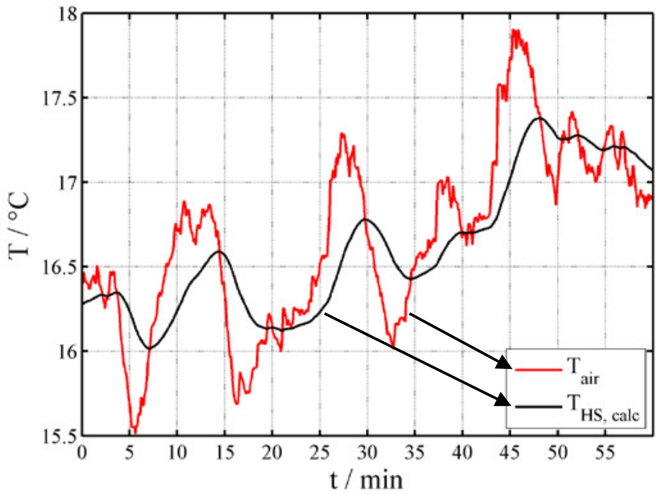

Fig. 4: Measured air temperature in the tunnel field measurement and calculated heat sink temperature from the model for system 1. Even the heat sink with the smallest τ is still thermally inert against quick air temperature changes.

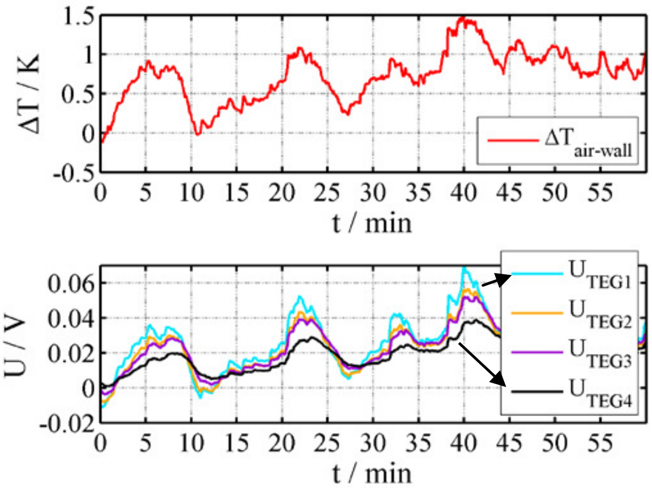

Fig. 5: Measured temperature difference between the air and wall temperatures and TEG open circuit voltages of the systems 1-4 in the tunnel measurement.

Figure 5 shows that the ambient temperature gradient ΔT is a highly transient phenomenon. Within the first ten minutes ΔT goes from 0 K up to 1 K and back again. System 1 generates the largest voltage from this transient ΔT. Since it offers the smallest time constant, it is able to quickly follow source variations - time constant of source and generator match well. Therefore system 1 produces the largest output voltage (Fig. 5) and electrical power (Tab. 3). By contrast, the thermal load matched system 3 produces only half of this power. Its large time constant dampens the ΔT fluctuations instead of harvesting from them.

Tab. 3: Comparison of thermal system properties and resulting output power P_{mean} and energy E_{day}. *calculated from the measured voltages U_1-U_4.*

System No.	τ / s	$K_{HS\text{-}mean}$ / K/W	P_{mean}* / µW $R_{TEG} = R_L$	E_{day} / J
1	239	8.3	20.1	1.74
2	374	8.5	15.3	1.32
3	402	2.8	10.1	0.87
4	416	4.9	7.9	0.68

SUMMARY

A transient model for thermoelectric generator systems, harvesting from the natural ambient temperature gradient, has been introduced. It is based on the lumped parameter approximation and enables the calculation of the TEG hot and cold side temperatures from a transient system ΔT. The energy harvester is modeled by an RC element for the heat sink and a resistor for the TEG. The model states that the TEG hot side temperature can be evaluated from the heat sink temperature using Newton's law of cooling. The time constant of the heat sink was then calculated and measured. The model as well as the measurements show, that a thermal load matched TEG/heat sink combination produces much less power (factor 2) than a thermal time constant matched one. A sufficiently small time constant enables the air contact to quickly follow source variations. Thus, the model implies that thermal load matching is not an option for the layout of a thermoelectric energy harvester in transient mode. The energy harvester must be designed with respect to thermal resistance and heat capacity of its components, summed up in the time constant. The thermal time constant of the harvester must be as small as possible compared to the 24 h period of the source.

OUTLOOK

Improvements in the model can be achieved by setting up the heat fluxes including the Peltier effect for closed loop configurations. The PDE and the heat capacity of the TEG could be used to obtain more exact solutions and to confirm the lumped system approximation.

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